Unique equilibria in a Diamond-Dybvig Model* †

Marcelo Catena
BCRA
mcatena@bcra.gov.ar

George McCandless
BCRA
gmccandless@bcra.gov.ar

May 31, 2000

*Both authors are members of the Research Deparment of the Central Bank Argentina. The views expressed in this paper are entirely personal and do not necessarily reflect the views of the Central Bank of Argentina.

†The authors wish to thank the seminar participants at Di Tella University, CEMA University and the BCRA for useful comments and suggestions. All remaining errors are ours.

Abstract

Standard Models of Bank runs are notorious not only for featuring Nash Equilibria (i.e. for given set of fundamentals, a bank can close or stay open depending upon strategic interactions among depositors) but also for lacking a satisfactory way of choosing among equilibrium outcomes or assigning a probability distribution to these outcomes. As a consequence it is not possible to study the relationship between fundamentals and bank failure nor the effect prudential policies have on the latter.

We develop a Diamond-Dybvig style model with stochastic bank fundamentals (bank asset returns) which depositors observe imperfectly. Using iterated dominance arguments a lá Carlsson and van Damme (1993), we find a unique equilibrium over the space of fundamentals. In particular we are able to partition the set of fundamentals into two connected subsets: for fundamentals in the first set the bank always closes and for fundamentals in the second set it always stays open. Hence, for any given stochastic distribution of bank fundamentals we can calculate a probability distribution over the different equilibrium outcomes.

This model unifies two strands of the bank run literature. Here both liquidity demand and asset returns pay explicit roles in determining whether a bank is run or not. Thus, we can show, as historical experience has suggested, that banks are more likely to close in periods of higher perfectly anticipated demands for liquidity. We also find that reserve requirements have ambiguous results that depend on the underlying probability distribution of portfolio returns. A version of capital requirements can reduce the probability of bank closings even though, in this model, banks fail primarily for liquidity reasons.

1. Introduction

Since its publication, Diamond and Dybvig (1983) and the host of variants that it generated have served as the principal models of bank runs. These models have had the virtue of being simple: three periods, a one period storage technology, and a two period investment technology that forces one to take a loss if liquidated after only one period. Individuals are of two types (but they don't know which type they are in period zero) and one type will need to withdraw their funds from the bank in period one while the rest can wait until period two (but they do not need to). A mutual bank can offer returns that improve individual welfare by aggregating the storage problem for those who will need to withdraw in period one and remove the necessity to liquidate ongoing investment projects. However, the model has two Nash equilibria. In one, long term projects go to term and the promised payouts are made. In the second equilibria, individuals believe that others, who do not need to withdraw their funds from the bank, will do so and sufficiently reduce the payouts in period two so that they will also withdraw their funds from the bank. The second Nash equilibria is the *bank run* equilibrium and a variety of policies have been proposed to prevent it.

The main difficulty with the Diamond-Dybvig model for a policy maker is that it offers no hints as to when or how the bank run equilibrium will occur. Any bank can be run or not, simply if individuals decide that others will run the bank. This situation is sometimes referred to as a sunspot equilibria although exactly what is the sunspot that individuals are using is left unknown. Chari and Jagannathan (1988) have developed a model in which the 'sunspot' is the withdrawals by type one individuals who type twos cannot identify as type ones and interpret as running the bank.

Substantial empirical evidence suggests that bank runs are not sunspot events or at least that the choice of which banks are run is not a sunspot event. Calomiris and Gorton (1991). Calomiris and Mason (1997) review the Chicago Banking Panic of 1932 and argue that only the weakest (i.e., insolvent) banks were run. A Central Bank study (D'Amato, Grubisic and Powell (1997)) of the 1995 Tequila crisis suggests that it was the weakest banks that were run and closed (although there is some evidence to suggest that bank size was also important, independent of the condition of the bank and that small banks were run disproportionately).

Here we develop a model in which banks are run or not because of the realized and expected returns that they are receiving on their portfolio. Banks with high realized and expected returns are never run while those with low realized and expected returns are. We try to keep our model as close as possible to the classic model of Diamond-Dybvig. The two important changes are in the addition of random portfolio returns and in the noisy signal that depositors receive about the returns that their bank is getting. The noisy signal that the depositors receive is crucial to the results, without it we cannot get a single line dividing the returns space into a region where banks are run and one where they are not. However, this noise does not need to be large and such an assumption seems to us to be very realistic.

The solution strategy is that of a game with iterated dominance. A basic reference for models with iterated dominance is Carlsson and Damme (1993) and an example of how this kind of model has been used with respect to runs on a currency is in Morris and Shin (1998). We begin with two regions of returns: in one the optimal strategy of a depositor will be to always run, independent of what the other depositors and in the other her optimal strategy will be to never run, independent of what the others are doing. Given that individuals who receive signals inside these regions have determined strategies, we can determine the optimal strategy for individuals who receive signals just outside these regions. It turns out that they have dominant optimal strategies. The always run and never run regions expand until they meet at a single boundary. We calculate that boundary for risk neutral depositors.

Section 2 describes the model for the simple case where there are no Diamond-Dybvig type ones. Section 3 gives a heuristic explanation of the iterative dominance result and shows the calculated boundary. Section 4 relates the strategy boundary to the regions of returns under which banks close or do not. Section 5 introduces Diamond-Dybvig type ones and considers reserve requirements. Section 6 considers a type of capital requirement. We end with some comments on work yet to be done.

2. Basic Model

2.1. Setup

There are three periods in this economy. In period 0, each of a continuous mass of measure 1 of individuals deposit 1 unit of goods in a bank. The bank is mutual and offers to pay r_d in period 1, where $r_d > 1$, or equal shares of the returns on the investment in period 2. The first period interest rate offered by the bank is determined by a market and is outside the control of the bank. Individuals are

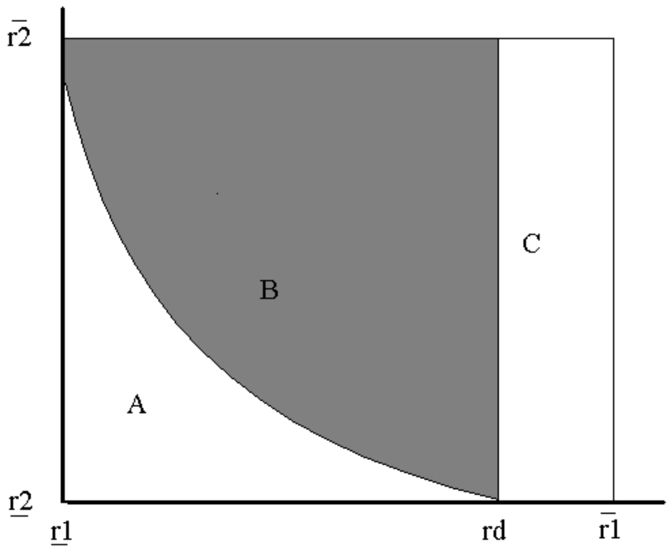

Figure 2.1: Space of possible returns

risk neutral and receive a utility of $c_1 + c_2$ where c_i is their consumption (equal to withdrawals from the bank) in period i. Individuals are free to withdraw their deposits in either period 1 or 2. The bank invests the deposits in a portfolio which, when viewed from period 0, generates an expected return less than r_d in period 1 and more than ${r_d}^2$ in period 2. The full set of possible net returns $\Omega = \Omega_1 \times \Omega_2 = \{r_1, r_2\}$ is an closed rectangular subset of $\mathbf{R}^2$ and is bounded by $\{\underline{r_1}, 1\}$ below and by $\{\overline{r_1}, \overline{r_2}\}$ above. It is assumed that $0 < \underline{r_1} < 1$, and that $\overline{r_1} > r_d$, $2\underline{r}_1 > r_d$ and $\overline{r_2} > r_d$. The density function of these returns is continuous and nowhere zero on Ω.

In period 1, both r_1 and r_2 are realized: call this $\widehat{r} = \{\widehat{r}_1, \widehat{r}_2\}$. However, while $\widehat{r}_2$ is observed perfectly, $\widehat{r}_1$ is observed with noise. Specifically, depositor i does not observe $\widehat{r}_1$ directly but observes an independent signal s_1^iwhich is distributed uniformly around a 2ε interval centered on $\widehat{r}_1$ (i.e. $s_1^i \in [\widehat{r}_1 - \varepsilon, \widehat{r}_1 + \varepsilon]$ and $cov\left(s_1^i, s_1^j\right) = 0$ for $i \neq j$). Based on the observed value of period 2 return and the signal of period 1 return, each depositor decides in period 1 either to attempt to withdraw r_d or to wait until period 2 and receive their share of what remains of the bank.

The box in Figure 2.1 shows the space Ω of possible returns. The shaded area, B, we call the Diamond-Dybvig area, since it contains the returns that correspond to those in their classic paper. The curved line is the set of $\{r_1, r_2\}$ where $(r_1)(r_2) = r_d$. For any signal, s_1^i, that is inside area A , the expected returns in period 2 will be less than r_d and individual i will want to withdraw her deposit even if she believes that no one else will withdraw theirs. Therefore, for any signal pair $\widehat{r} = (r_1)$ more than ε inside this area, everyone will always attempt to withdraw their deposits in period 1.[1] Let $g(r)$ represent the fraction of individuals who will withdraw deposits at realization r, then for realizations more than ε inside area A, $g(r) = 1$. If the signal s^i is inside area C, then the expected return in period 2 will always be more than r_d and individual i will leave her deposits in the bank even if she believes that everyone else will withdraw theirs.[2] For any realization of $\widehat{r}$ more than ε inside area C, no one will withdraw their deposits in period 1 and for realizations in that region, $g(r) = 0$. For any other signals, including those in the Diamond-Dybvig area, a depositor's optimal strategy will depend on her expectation of the strategies of all the other depositors.

Let $\varphi(s_1, r_2)$ be the proportion of the depositors who observe the pair (s_1, r_2) which run the bank in period 1. Here, $\varphi(s_1, r_2)$ is best considered the strategy of an individual who observes (s_1, r_2) and has a value of 1 when the individual runs the bank and 0 when she does not and a fractional value if the individual chooses a mixed strategy. Let $g(r, \varphi)$ be the portion of depositors who end up withdrawing their funds in period 1 when the realized return is r. Since the signal s_1 is uniformly distributed inside a 2ε interval centered around r_1, which we will denote r_1^ε,

$$g(r, \varphi) = \frac{1}{2\varepsilon} \int_{r_1^\varepsilon} \varphi(s_1, r_2) ds_1.$$

where $r \equiv (r_1, r_2)$. A bank closes when it cannot meet the demands of the depositors in period one. This occurs when the fraction of depositors who withdraw is bigger than $\alpha(r)$, where

$$\alpha(r) = \frac{r_1}{r_d}.$$

Given the strategies $\varphi(s)$, the bank closes for realizations in the set $A(\varphi) \subset \Omega$,

[1] With a realization within ε of the boundry of A, some individuals will get signals outside of A and these might choose not to run the bank.

[2] For signals inside C, individuals will never run since, with $r_2 > 1$, expected returns from not running so are always better than for running. When $r_1 > r_d$, the more that run, the more there is for those who don't run.

where

$$A(\varphi) = \{r\,|g(r,\varphi) \geq \alpha(r)\}\,.$$

The payoff of the depositors depend on whether they run the bank or not and whether the bank closes or not. The expected payoff from running the bank when the realization is r and when fraction of other depositors running is $\varphi(s)$ is equal to

$$h_r(r,\varphi) = \begin{cases} r_d \text{ when } r \notin A(\varphi) \\ \frac{r_1}{\varphi(r)} \text{ when } r \in A(\varphi) \end{cases}.$$

Note that r_1/φ is the expected return from attempting to withdraw your deposits. Each individual who withdraws gets r_d, but it may be the case that not everyone attempting to withdraw gets something. This is a result of the assumption of sequential servicing of depositors. Those late in the line might get nothing.

With the same realization and strategy set, the payoff for keeping ones deposits in the bank (not running the bank) is equal to

$$h_{nr}(r,\varphi) = \begin{cases} \frac{r_1 - r_d\cdot\varphi(r)}{1-\varphi(r)}(r_2) \text{ when } r \notin A(\varphi) \\ 0 \text{ when } r \in A(\varphi) \end{cases}$$

Those who did not attempt to withdraw when the bank closed get nothing.

For an individual who gets a signal s, the expected net payoff for running the bank instead of staying is

$$\begin{aligned} u(s,\varphi) &= \frac{1}{2\varepsilon}\int_{s_1^\varepsilon} \left[h_r(r,\varphi) - h_{nr}(r,\varphi)\right] dr_1 \\ &= \frac{1}{2\varepsilon}\int_{s_1^\varepsilon \cap A(\varphi)} \frac{r_1}{\varphi(r)} dr_1 \\ &\quad + \frac{1}{2\varepsilon}\int_{s_1^\varepsilon \cap \overline{A(\varphi)}} \left(r_d - \frac{r_1 - r_d\cdot\varphi(r)}{1-\varphi(r)}\cdot r_2\right) dr_1 \end{aligned}$$

where s_1^ε is an interval of length 2ε centered on s_1 and $\overline{A(\varphi)}$ is the compliment of $A(\varphi)$ on Ω.

2.2. Equilibrium in Threshold Strategies

Denote by $\varphi^*(s_1, r_2, \varphi)$ the optimal strategy for individual who receives a signal s_1 when the realization of period 2 return is r_2 and the aggregate strategy is given by φ. Then,

$$\varphi^*(s_1,r_2,\varphi) = \begin{cases} 1 \text{ when } u(s_1,r_2,\varphi) > 0 \\ 0 \text{ when } u(s_1,r_2,\varphi) \leq 0 \end{cases}.$$

We will restrict our search of Nash equilibria (NE) to the set of symmetric strategies. Consequently, a symmetric NE must satisfy:

$$\varphi(s_1, r_2) = \varphi^*(s_1, r_2, \varphi)$$

We will further restrict ourselves to "threshold" strategies. By a threshold strategy we mean that a depositor will run the bank if and only if the signal s_1 he observes is below some (unique) threshold which is a function of r_2.

Definition 2.1 (Threshold Strategy). *Given some function $f : \Omega_1 \to \Omega_2$ a (symmetric) threshold strategy has the form:*

$$\varphi_f(\widetilde{s}_1, r_2) = \begin{cases} 0 \text{ when } \widetilde{s}_1 \geq f(r_2) \\ 1 \text{ when } \widetilde{s}_1 < f(r_2) \end{cases}.$$

We will denote the function f in the definition of the threshold strategy as the "threshold" function.

An important feature of threshold strategies is that:

Proposition 2.2. *When $\varepsilon \to 0$, there exists a unique equilibrium among the class of (symmetric) threshold strategies. Moreover, the implied threshold function is continuously differentiable and decreasing.*

Proof. First, notice that a necessary condition for an equilibrium in threshold strategies to exists is that when the realization of period 2 return is given by r_2 a depositor who observes a signal $\widetilde{s}_1 = f(r_2)$ must be indifferent between running or not. With some abuse of notation, we can write this as $u\left(f(r_2), r_2, \varphi_f\right) = 0$. Consequently, the equilibrium will be unique if there is a unique function f which satisfies the former condition.

Given any threshold strategy, the fraction the fraction of depositors who end up withdrawing their funds in period 1 when the realized return is r is given by:

$$g(r, \varphi_f) = \begin{cases} 1 & \text{if } r_1 \leq f(r_2) - \varepsilon \\ \frac{f(r_2)+\varepsilon-r_1}{2\varepsilon} & \text{if } f(r_2) - \varepsilon < r_1 < f(r_2) + \varepsilon \\ 0 & \text{if } r_1 \geq f(r_2) + \varepsilon \end{cases}$$

Given φ_f the bank will close for realizations of $r = (r_1, r_2)$ which satisfy $g(r, \varphi_f) = \frac{f(r_2)+\varepsilon-r_1}{2\varepsilon} \geq \frac{r_1}{r_d}$ which implies that:

$$A(\varphi_f) = \left\{ r \in \Omega \,\middle|\, r_1 \leq r_d \cdot \frac{f(r_2) + \varepsilon}{r_d + 2\varepsilon} \right\}$$

Consequently,

$$\begin{aligned}
u\left(f\left(r_2\right), r_2, \varphi_f\right) &= \int_{f(r_2)-\varepsilon}^{r_d \cdot \frac{f(r_2)+\varepsilon}{r_d+2\varepsilon}} \frac{r_1}{f\left(r_2\right) - r_1 + \varepsilon} dr_1 \\
&+ \frac{1}{2\varepsilon} \int_{r_d \cdot \frac{f(r_2)+\varepsilon}{r_d+2\varepsilon}}^{f(r_2)+\varepsilon} \left(r_d - \frac{r_1 - r_d \cdot \frac{f(r_2)+\varepsilon-r_1}{2\varepsilon}}{1 - \frac{f(r_2)+\varepsilon-r_1}{2\varepsilon}} \cdot r_2 \right) dr_1 \\
&= \left[f\left(r_2\right) - \varepsilon\right] - r_2\left[f\left(r_2\right) + \varepsilon\right] \\
&+ \left(f\left(r_2\right) + \varepsilon\right) \ln \left[\frac{r_d + 2\varepsilon}{f\left(r_2\right) + \varepsilon} \right] + r_2\left[r_d + \varepsilon - f\left(r_2\right)\right] \ln \left[\frac{r_d + 2\varepsilon}{r_d + \varepsilon - f\left(r_2\right)} \right]
\end{aligned} \tag{2.1}$$

It can be shown that, for a given r_2, $u\left(f\left(r_2\right), r_2, \varphi_f\right)$is strictly decreasing in $f\left(r_2\right)$ so that for any r_2 there is a **unique** number $f\left(r_2\right)$ which satisfies $u\left(f\left(r_2\right), r_2, \varphi_f\right) = 0$. Moreover, setting the right hand side of (2.1) equal to 0 we can solve explicitly for the inverse of $f\left(r_2\right)$:

$$r_2 = m\left(s_1\right) \equiv f^{-1}\left(s_1\right) = \frac{\left[s_1 - \varepsilon\right] + \left(s_1 + \varepsilon\right) \ln \left[\frac{r_d+2\varepsilon}{\widetilde{s_1+\varepsilon}}\right]}{\left[\widetilde{s}_1 + \varepsilon\right] - \left[r_d + \varepsilon - \widetilde{s}_1\right] \ln \left[\frac{r_d+2\varepsilon}{r_d+\varepsilon-\widetilde{s_1}}\right]}$$

Moreover, letting ε be small, we get that

$$m(s_1) = \frac{s_1 \ln \left[1 + \ln \left(\frac{r_d}{\widetilde{s_1}}\right)\right]}{s_1 - \left[r_d - s_1\right] \ln \left[\frac{r_d}{r_d - \widetilde{s_1}}\right]}$$

and

$$\frac{df\left(r_2\right)}{dr_2} = \left[\frac{dm\left(s_1\right)}{ds_1}\right]^{-1} = \frac{\frac{s_1}{r_2}\left(1 - \ln\left[\frac{s_1}{r_d}\right]\right)}{-1 + r_2\left[1 + \frac{r_d}{s_1} \ln\left[1 - \frac{s_1}{r_d}\right]\right]} < 0$$

which implies that the f is continuously differentiable and decreasing.

Finally, to show uniqueness, notice that applying Leibnitz' rule,

$$\frac{du\left(s_1, r_2, \varphi_f\right)}{ds_1} = -\left(s_1 - \varepsilon\right) + r_d - \left(s_1 + \varepsilon\right) \cdot r_2$$

so that:

$$\lim_{\varepsilon \to 0} \frac{du\left(s_1, r_2, \varphi_f\right)}{ds_1} = r_d - r_1\left(1 + r_2\right) < r_d - 2r_1 < 0$$

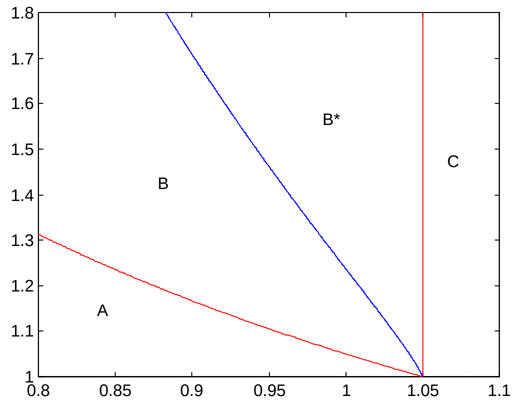

Figure 2.2: Calculated fixed points for rd=1.05

since $2\underline{r}_1 > r_d$. This means that the proposed threshold strategy is indeed an equilibrium because it is optimal to run iff $s_1 \leq f(r_2)$. ■

Figure 2.2 shows the calculated results for an economy with $r_d = 1.05$. Regions B and B^* are the Diamond-Dybvig regions. If the realized returns are in region B, the bank will be run. If the realized returns are in region B^*, the bank will not be run.

2.3. Unique Equilibrium: Iterative Dominance

We argue that the function f is the unique NE of the game even if do not restrict ourselves to threshold strategies. The fact that individuals receive a noisy signal about the realized returns is important in generating this result. We know that when they get signals in region A, everyone's best strategy is to run the bank. For signals in region C, it is everyone's best strategy not to run the bank. Beginning with this knowledge, we can find out what individuals who receive a signal inside region B will do. We begin with an individual who receives a signal in region B but close to the frontier with region C. Figure 2.3 shows one such signal s_1.

An individual receiving this signal believes that realization is uniformly distributed an interval of 2ε centered around point s_1 and that the rest of the population are receiving signals which are distributed symmetrically over an 4ε interval centered around that point. As shown in the figure, this interval includes points inside region C (shaded grey). Individuals who receive those signals will never

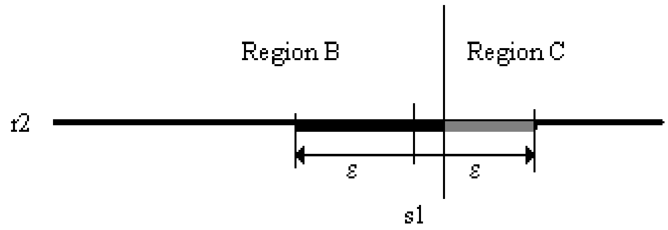

Figure 2.3: Interval of radius epsilon near C

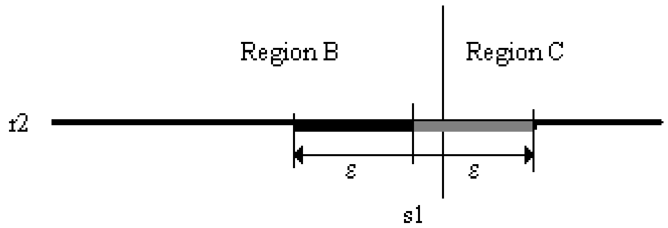

Figure 2.4: The 'no run' boundary has advanced

run the bank, so completely independently of whatever strategy those in the white portion of the circle choose to do, those in the grey part will not run.

Suppose that the individual at s believes that all the other individuals in the white part of the circle will choose to run the bank[3] If s is close to the vertical boundary between regions B and C, slightly more than $1/2$ of the population will run the bank. Since the observed r_1 is close to r_d, the bank will have enough resources so that it will be able to remain open and pay the individual at r a return in period 2 greater than r_d. Since r_d is the most one could get by running the bank in period one, the individual at s_1 will not run the bank but will choose to wait until period two and get the higher return

[3]In one sense, this is the worst case. If everyone in the white section chose not to run the bank, then if the expected returns in period 2 were greater than r_d, neither would the individual at r.

One can apply the same reasoning to individuals who gets a signal between s_1 and the boundary of C. None of them will choose to run the bank.. Assuming that the same thing happens at other points near C, the boundary indicating which individuals will not run the bank moves out as the shaded line in Figure 2.4. The boundary moves out to the set of r's for which $u(r, \varphi_v) = 0$, where φ_v is defined by the vertical line separating regions B and C. The region of signals where individuals will not run the bank has advanced into the B portion of the returns space. Iterating on this process will move the boundary until $u(r, \varphi) = 0$ for a point r on the boundary when that same boundary is used to determine φ.

In a similar fashion, one can advance from the boundary between regions A and B, knowing that all individuals who receive signals inside A will run the bank and assuming that those outside do not. This will move the run boundary further into the B region. Iterating on this procedure eventually gives a boundary where $u(r, \varphi) = 0$ for a point r on the boundary when that same boundary is used to determine φ. There the boundary ceases to advance.

According to our argument, the two boundaries are in fact the same. The iterative process moving up and to the right from the A region and that moving down and to the left from the C region terminate in the same function. Given the strategy set determined by this common function, the points where $u(r, \varphi) = 0$ all fall on the same function.

2.4. Bank Closure given the Equilibrium Strategy

Given the function shown in figure 2.2 that determines the individual strategies, we need to determine under which realizations a bank will close in period one because of a run and under which it will not. Recall that individuals are receiving a signal about the realization of period 1 return that is an interval of length 2ε centered around that realization. For realizations more than ε to the left of the line in figure 2.2, everyone will receive signals to the left of the line and all will follow the same strategy: they will all run the bank. For realizations in this region, the bank will always close. For realizations more than ε to the right of the line in figure 2.2, everyone will receive signals to the right of the line and all will follow the same strategy: they will not run the bank. In the region $\pm\varepsilon$ of the line the fraction who will run is determined by the fraction of the ball of radius ε that is to the left of the line. The bank will close when this fraction exceeds $\alpha(r) = r_1/r_d$ which is the maximum amount of withdrawals the bank can attend.

Specifically, recall that given the equilibrium threshold strategy, the fraction of people who run when the realization is r is given by:

$$g(r,\varphi_f)=\begin{cases}1 & \text{if } \ r_1\leq f(r_2)-\varepsilon\\ \frac{f(r_2)+\varepsilon-r_1}{2\varepsilon} & \text{if } \ f(r_2)-\varepsilon<r_1<f(r_2)+\varepsilon\\ 0 & \text{if } \ r_1\geq f(r_2)+\varepsilon\end{cases}$$

Since $r_1 \leq r_d$,[4] the bank will close when:

$$g(r,\varphi_f)=\frac{f(r_2)+\varepsilon-r_1}{2\varepsilon}\geq\frac{r_1}{r_d}$$

which occurs when:

$$r_1\leq\frac{r_d\left(f(r_2)+\varepsilon\right)}{r_d+2\varepsilon}$$

3. With deterministic fraction of Type Ones

3.1. Setup

The standard Diamond-Dybvig model requires a fraction of the population to encounter a surprise need for consumption in period one and for that reason need to withdraw deposits. In fact, this need is the reason that banks exist. For simplicity, we have not included type ones in the development of the current version of the model. However, this does not pose any particular problem. If the expected returns for the first period are less than one, a risk neutral bank will choose to hold in storage (reserves) the amount of deposits required to meet the (perfectly) expected payments to type ones. The model is written so as to permit a reserve requirement in excess of the withdrawals of the type ones.

Suppose that there are two production technologies: one storage and the other a long term project. Storage returns one unit of the good for every unit stored, between periods 0 and 1 and between 1 and 2. The long term technology offers a return of r_1 in period 1 and a return of $r_1 r_2$ in period 2. Goods that were in the storage technology between periods 0 and 1 cannot be put into the long term technology from period 1 to 2. As before, the returns on the long term technology are random and $r=\{r_1,r_2\}$ is realized in period 1.

A certain fraction of the depositors, θ, are type ones and only get utility from consuming in period one. Each type one will withdraw r_d from the banks so that their net withdrawals will be $\theta\cdot r_d$. The other $1-\theta$ depositors see r_2 perfectly and

[4] When $r_1 > r_d$, the bank never closes in the first period.

get a noisy signal for r_1. The signals are uniformly and independently distributed on an interval of length 2ε centered on the realized r_1. As before a unit mass of depositors deposit one unit of good in the bank. The bank promises a return of $r_d > 1$ between each period.

If a bank puts δ of the deposits into storage, then it closes in period 1 if

$$\delta + (1-\delta)r_1 \leq (1-\theta)g(r,\varphi)r_d + \theta \cdot r_d$$

where $g(r,\varphi)$ is the fraction of the type two population who run the bank if the realization is r and their strategies are φ. Define the set of realizations over which the bank closes as:

$$A(\varphi) = \left\{ r \left| \frac{\delta + (1-\delta)r_1}{r_d} \leq (1-\theta)\varphi(r) + \theta \right. \right\}.$$

If we restrict our search for threshold strategies,

$$g(r,\varphi_f) = \frac{f(r_2) + \varepsilon - r_1}{2\varepsilon},$$

and the boundary for the A region is equal to

$$r_1 = \frac{r_d(1-\theta)(f(r_2)+\varepsilon) + 2\varepsilon(r_d\theta - \delta)}{r_d(1-\theta) + 2\varepsilon(1-\delta)}.$$

If the bank doesn't close in period 1, it will have resources equal to

$$\beta(r) = \min \left[\begin{array}{c} \delta - ((1-\theta)g(r,\varphi)+\theta)\cdot r_d + (1-\delta)r_1 r_2, \\ [\delta + (1-\delta)r_1 - ((1-\theta)g(r,\varphi)+\theta)\cdot r_d]r_2 \end{array} \right]$$

in period 2. The first term is the amount of resources the bank will have in period 2 when $((1-\theta)g(r,\varphi)+\theta)\cdot r_d \leq \delta$, and the second term is when $((1-\theta)g(r,\varphi)+\theta)\cdot r_d > \delta$. The division between these two sets occurs when

$$r_1 = f(r_2) + \varepsilon - \frac{2\varepsilon(\delta - \theta r_d)}{r_d(1-\theta)}.$$

Notice that if $\delta < \theta r_d$, that implies that the storage is insufficient to cover the number of type ones who will run for certain and that the first item in the $\beta(r)$ equation will never occur. With $\delta > \theta r_d$, the r1 given above is where the first item in the $\beta(r)$ equation is relevant. In what follows, we will assume that the bank has sufficient reserves so that $\delta > \theta r_d$.

As before, the payoffs to a depositor depend on whether they run a bank or not. The payoff for running a bank with realization r and fraction of other depositors running $(1-\theta)\varphi(r)+\theta$ is

$$h_r(r,\varphi)=\begin{cases} r_d \text{ when } r\notin A(\varphi) \\ \frac{\delta+(1-\delta)r_1}{(1-\theta)g(r,\varphi)+\theta} \text{ when } r\in A(\varphi)\end{cases}.$$

The payoff from keeping ones deposits in the bank is

$$h_{nr}(r,\varphi)=\begin{cases} \min\left[\frac{\delta-((1-\theta)g(r,\varphi)+\theta)r_d+(1-\delta)r_1r_2}{1-((1-\theta)g(r,\varphi)+\theta)},\frac{[\delta+(1-\delta)r_1-((1-\theta)g(r,\varphi)+\theta)r_d]r_2}{1-((1-\theta)g(r,\varphi)+\theta)}\right] \text{ when } r\notin A(\varphi) \\ 0 \text{ when } r\in A(\varphi)\end{cases}.$$

For a signal s, the expected net payoff for running the bank as opposed to staying in is

$$\begin{aligned} u(s,\varphi) &= \frac{1}{2\varepsilon}\int_{s_1^\varepsilon}\left[h_r(r,\varphi)-h_{nr}(r,\varphi)\right]dr && (3.1)\\ &= \frac{1}{2\varepsilon}\int_{s_1^\varepsilon\cap A(\varphi)}\frac{\delta+(1-\delta)r_1}{(1-\theta)g(r,\varphi)+\theta}dr \\ &\quad +\frac{1}{2\varepsilon}\int_{s_1^\varepsilon\cap\overline{A(\varphi)}} r_d-\min\left[\begin{array}{c}\frac{\delta-((1-\theta)g(r,\varphi)+\theta)r_d+(1-\delta)r_1r_2}{1-((1-\theta)g(r,\varphi)+\theta)}, \\ \frac{[\delta+(1-\delta)r_1-((1-\theta)g(r,\varphi)+\theta)r_d]r_2}{1-((1-\theta)g(r,\varphi)+\theta)}\end{array}\right]dr. && (3.2)\end{aligned}$$

As in the previous model, one can find an equilibrium in threshold strategies. In this case we have:

$$\begin{aligned} g(r,\varphi_f) &= \begin{cases} 1 & \text{if } r_1\le f(r_2)-\varepsilon \\ \frac{f(r_2)+\varepsilon-r_1}{2\varepsilon} & \text{if } f(r_2)-\varepsilon<r_1<f(r_2)+\varepsilon \\ 0 & \text{if } r_1\ge f(r_2)+\varepsilon\end{cases}\\ s_1^\varepsilon\cap A(\varphi_f) &= \left[f(r_2)-\varepsilon,\frac{r_d(1-\theta)(f(r_2)+\varepsilon)+2\varepsilon(\theta r_d-\delta)}{r_d(1-\theta)+2\varepsilon(1-\delta)}\right]\\ s_1^\varepsilon\cap\bar{A}(\varphi_f) &= \left[\frac{r_d(1-\theta)(f(r_2)+\varepsilon)+2\varepsilon(\theta r_d-\delta)}{r_d(1-\theta)+2\varepsilon(1-\delta)},f(r_2)+\varepsilon\right]\end{aligned}$$

Plugging into 3.2, solving the integrals explicitly for $s_1=f(r_2)$ and setting it equal to zero we can find the inverse of $f(r_2)$:

$$r_2=f^{-1}(s_1).$$

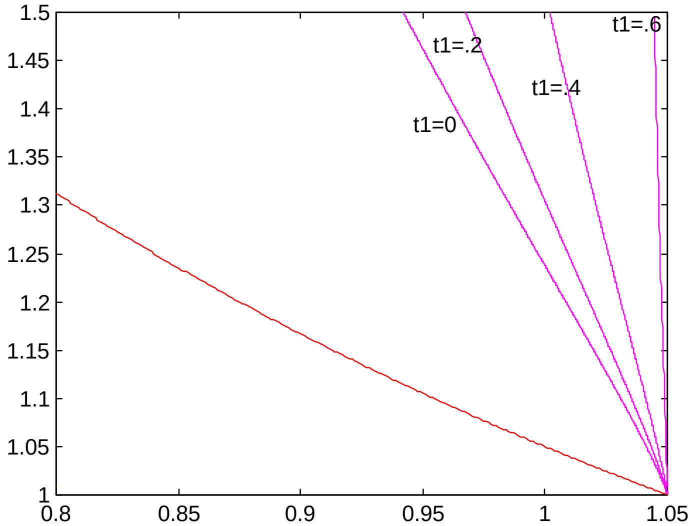

Figure 3.1:

This function can be written out as

$$r_2 = \frac{(1-\delta)(s_1-\varepsilon) + \frac{D}{(1-\theta)}\ln\left(\frac{F}{D}\right) - (r_d-\delta)\ln\left(\frac{r_d-\delta}{r_d(1-\theta)}\right)}{(1-\delta)(s_1+\varepsilon) - G\ln\left(\frac{(r_d-\delta)}{r_d(1-\theta)}\frac{F}{G}\right) - ((s_1-\varepsilon)(1-\delta))\ln\left(\frac{r_d-\delta}{r_d(1-\theta)}\right)}$$

where

$$\begin{aligned} D &= (1-\delta)(s_1(1-\theta)+\varepsilon(1+\theta))+\delta(1-\theta), \\ F &= r_d(1-\theta)+2\varepsilon(1-\delta), \text{ and} \\ G &= (r_d-\delta-(s_1-\varepsilon)(1-\delta)). \end{aligned}$$

Notice that r_2 is defined only over the set of $(s_1, r_d, \theta, \delta)$ where the denominator of the f^{-1} function is positive. This occurs when θ is not too big relative to δ and when s_1 is not too small relative to r_d.

3.2. Some properties of this model

Figure 3.1 shows the strategy lines for four values of θ when $\delta = 0$. The higher is the quantity of depositors who will be type ones, the smaller the set of returns

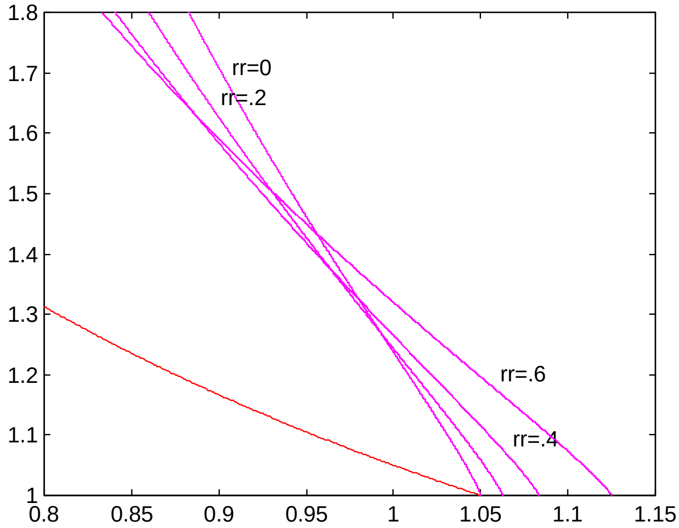

Figure 3.2: Strategy space for different reserves

over which the bank will not close. One can prove that as $s_1 \to r_d$ and $\varepsilon \to 0$, $r_2 \to 1$ for all values of $\theta \in [0, 1)$.

Figure 3.2 shows the strategy lines for four values of δ (reserves) when $\theta = 0$ (when there are no type ones). Increasing δ has two effects on the strategy line. First, The value of the line at $r_2 = 1$ increases as d increases and is equal to $(r_d - \delta)/(1 - \delta)$. Second, the slope of the line declines. The tradeoff between these two effects determines if increasing d results in increasing the set of returns over which the bank is not run. This set increases at higher r_2, so the larger is $\overline{r_2}$, the more likely is it that increased reserves will result in higher probability of the bank staying open.

3.3. Results when reserves just cover type ones

Figure 3.3 shows the strategy line for three situations. The base line (0 t1, 0 rr) has no type ones and is without a reserve requirement. The line parallel (.2 t1, .21 rr) has 20% type ones and reserves of 21% (to cover the promised $r_d = 1.05$ for the 20%). The third line (.4 t1, .42 rr) has 40% type ones and a reserve requirement of 42% which is just sufficient to cover the promised $r_d = 1.05$ for these type ones.

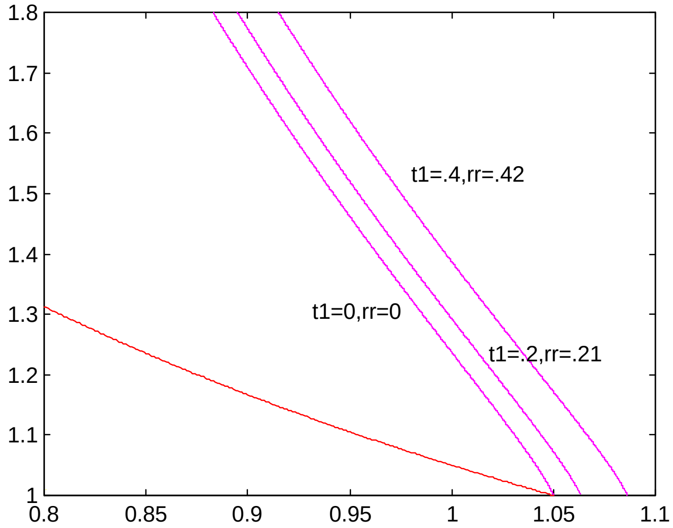

Figure 3.3: Reserves just cover type ones

The value of the line at $r_2 = 1$ increases as it did with the graph with reserves but no type ones but the slope of the line stays approximately constant. The shift out is equal to $(r_d - \delta)/(1 - \delta)$.

It is clear from the graph that when a perfectly anticipated fixed fraction will need to withdraw in the first period, this reduces the set of returns over which the remaining fraction of the population will not run the bank. This is because the type ones increase the set of returns over which a bank will be unable to meet any given amount of withdrawals of the type twos. A fraction θ of type ones require $\theta \cdot r_d$ of storage and since $r_d > 1$, this reduces the amount that remains in period zero that is invested for the type twos. Since this reduction is constant and independent of the returns that result in period one, the higher returns are required to keep banks from being run generate a parallel shift in the strategy function.

3.4. Reserves in excess of those required to cover the type ones

Increasing the reserves beyond the $k \cdot r_d$ required to cover the type ones is allocating time one resources to cover the possible withdrawals by type twos. The strategy

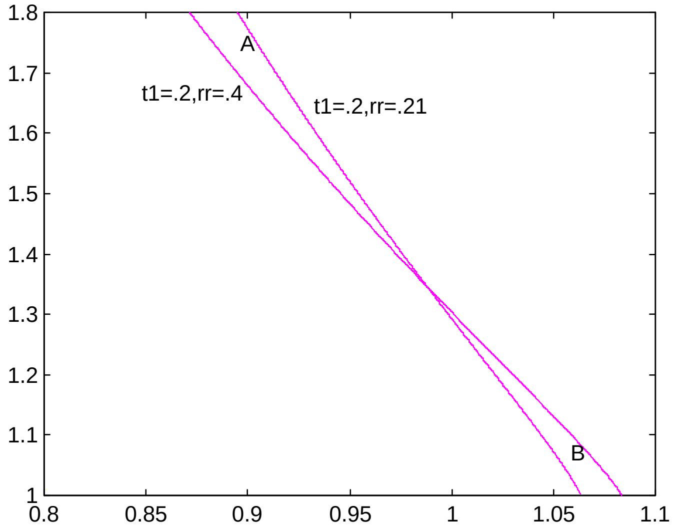

Figure 3.4: Effect of increased reserves

function is shown in Figure 3.4 by the line marked '.2 t1, .4 rr', that is 20% of the population is type ones and .4 units of the time zero good are allocated to storage (reserves). These extra reserves have the effect of reducing the total returns to the type twos (in period two) and increasing the set of returns over which the bank is run in the regions where $r_1 > 1$ and r_2 is low (the region marked B), but it reduces the set of returns over which the bank is run for regions where $r_1 < 1$ and r_2 is relatively high (the region marked A). This is because the reserves increase the period one resources of the bank for realizations of $r_1 < 1$ and decreases them otherwise. The increase in resources changes the probability that the bank will close for any fixed amount of type twos running the bank. The increased chance of surviving the run in period one coupled with high expected returns in period two moves the strategy function.

Whether a reserve requirement is a net gain for the economy or not (from at time zero ex-ante point of view) depends on the underlying probability distribution of the r's (the $\{r_1, r_2\}$ pairs). If the probability of realization occurring in the region marked 'A' in Figure 3.4 is greater than that in the region marked 'B', then it is likely that imposing reserve requirements will increase expected welfare as measured from period zero. Region 'A' is the area added to the no run region by the reserve requirement and 'B' is the area lost.

4. Capital and capital requirements

It is not entirely clear how one should introduce a capital requirement into a Diamond-Dybvig mutual bank. In the standard setup, the mutual owners of the bank who have not taken out their deposits in period one divide evenly among themselves the period two gross returns. In the standard model, depositors are also the owners and bare risk as to the second period returns.

We choose a way of introducing capital that recognizes this ownership role of the depositors but preserves their right to run the bank in period one. The bank can 'borrow' capital in period zero by offering a return in period two so that the expected return is equal to the market rate of interest.[5] If a bank closes in period one because it is run, the bank pays nothing to the investors. If it is not run, the investors are paid in period two before the depositors. We understand that this last assumption is not consistent with normal bank failure rules where the depositors are paid before the owners. However, here there is no distinction between owners and depositors.[6]

Here we consider a version of the models without type ones. Suppose that a bank holds a quantity of capital equal to a fraction k of its deposits and puts all deposits and capital into the investment project (so it is holding capital equal to $k/(1+k)$ of its loans). It will close in the first period if

$$(1+k)r_1 \leq g(r,\varphi)r_d$$

where $g(r,\varphi)$ is the fraction of the population who run the bank when the realization is r and the strategies are φ. Define the set where the bank closes as the set of realizations

$$A(\varphi) = \left[r \left| \frac{(1+k)r_1}{r_d} \leq g(r,\varphi) \right. \right].$$

If the bank does not close in period one, it must first pay the investors $k \cdot r_k$ where r_k is the interest rate that gives an expected value equal to the market rate for

[5] We are assuming a large,external market for capital with risk neutral investors.

[6] A more attractive way of modelling this might be to make some minimum payout to the depositors before the capital is paid off. Then whatever is left after capital is paid is a second payment to the depositors/owners. For example, in period two, depositors might first get r_d^2 before capital gets anything.

risk free investments. If the bank does not close, the depositors receive

$$\min\left(0, [(1+k)r_1 - g(r,\varphi)r_d]r_2 - k \cdot r_k\right).$$

If the bank does not have enough in the second period to pay off the investors, the depositors receive nothing.

As before, the payoffs to a depositor depend on whether she runs the bank or not and whether the bank closes or not. The payoff for running a bank with a realization r and fraction of other depositors running equal to $g(r,\varphi)$ is

$$h_r(r,\varphi) = \begin{cases} r_d \text{ when } r \notin A(\varphi) \\ \frac{(1+k)r_1}{g(r,\varphi)} \text{ when } r \in A(\varphi) \end{cases}.$$

The payoffs from keeping ones deposits in the bank are

$$h_{nr}(r,\varphi) = \begin{cases} \min\left[0, \frac{[(1+k)r_1 - g(r,\varphi)\cdot r_d]r_2 - k\cdot r_k}{1-g(r,\varphi)}\right] \text{ when } r \notin A(\varphi) \\ 0 \text{ when } r \in A(\varphi) \end{cases}.$$

For a signal s_1, the expected net payoff from running a bank as opposed to staying in are

$$\begin{aligned} u(s,\varphi) &= \frac{1}{2\varepsilon}\int_{s_1^\varepsilon} [h_r(r,\varphi) - h_{nr}(r,\varphi)]\, dr_1 \\ &= \frac{1}{2\varepsilon}\int_{s_1^\varepsilon \cap A(\varphi)} \frac{(1+k)r_1}{g(r,\varphi)} dr_1 \\ &\quad + \frac{1}{2\varepsilon}\int_{s_1^\varepsilon \cap \overline{A(\varphi)}} r_d - \min\left[0, \frac{[(1+k)r_1 - g(r,\varphi)\cdot r_d]r_2 - k\cdot r_k}{1-g(r,\varphi)}\right] dr_1. \end{aligned}$$

Proceeding as in the previous sections we find the inverse of the unique threshold function:

$$r_2 = f^{-1} = \frac{(f-\varepsilon)(1+k) - (f+\varepsilon)(1+k)\ln\left(\frac{(1+k)(f+\varepsilon)}{2\varepsilon(1+k)+r_d}\right) + kr_k}{(1+k)(f+\varepsilon) + (r_2(\varepsilon - f)(1+k) + r_d r_2 + kr_k)\ln\left(\frac{r_d + k\frac{r_k}{r_2} - (f-\varepsilon)(1+k)}{2\varepsilon(1+k)+r_d}\right)}$$

We calculate the fixed point strategy function for this economy when $k = .2$, when $r_d = 1.05$, and when $r_k = 1.10$. This and the strategy function for the same economy without capital is shown in Figure 4.1. Region A is the increased no run area and region B is the increased run area. Capital has a large effect in reducing the run region and results in banks staying open under many realizations where the bank would have closed without the capital. Whether capital increases the expected returns for the depositors needs yet to be analyzed.

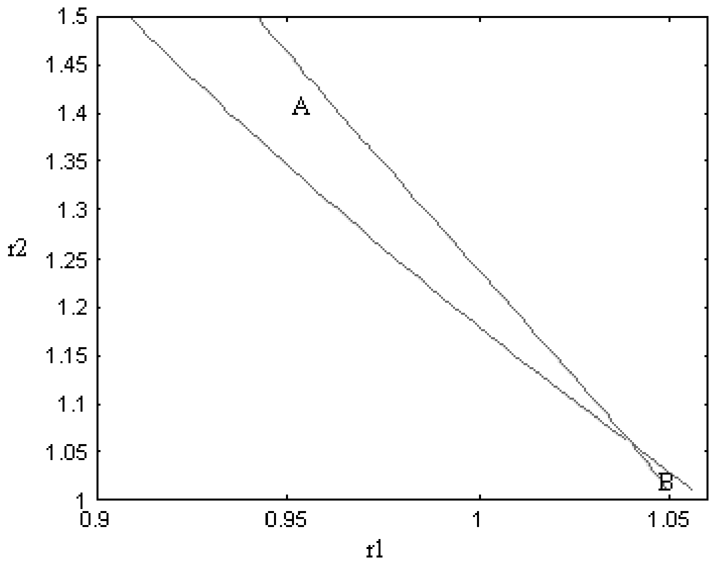

Figure 4.1: Effect of capital requirements

5. Conclusions

We have developed a model of banks very similar to that of Diamond and Dybvig in which noisy information about the condition of the bank results in equilibria in which a bank run is determined by the returns on the bank's portfolio. We find that both reserve requirements and capital change (and can reduce) the set of realizations over which banks can be run.

This model as it stands takes a number of variables as exogenous which might be made endogenous. The interest rate offered to depositors who withdraw in the first period is given outside the bank. Given a distribution over the set Ω of returns, we should be able to calculate the deposit interest rate that maximizes depositor period zero expected utility. Given the distribution of returns, there might be an optimal amount of capital for the bank to take on or an optimal amount of reserves to maximize depositor utility. Study of these problems is projected.

We have not yet shown uniqueness of the fixed point strategy function although for risk neutral depositors our calculations generate one function, independent of our initial conditions. We are trying to determine the characteristics of the game that give this uniqueness.

References

Calomiris, Charles W. and Gorton, Gary (1991). The Origin of Banking Panics, *in* G. Hubbard (ed.), *Financial Markets and Financial Crises*, University of Chicago Press, Chicago.

Calomiris, Charles W. and Mason, Joseph R. (1997). "Contagion and Bank Failures During the Great Depression: The June 1932 Chicago Banking Panic", *American Economic Review*, **87**(5): 863–883.

Carlsson, Hans and Damme, Eric Van (1993). "Global Games and Equilibrium Selection", *Econometrica*, **61**(5): 989–1018.

Chari, V.V. and Jagannathan, Ravi (1988). "Banking Panics, information and rational expectations equilibrium", *Journal of Finance*, **43**: 749–761.

D'Amato, Laura, Grubisic, Elena and Powell, Andrew (1997). *Contagion, Banks Fundamentals or Macroeconomic Shock? An Empirical Analysis of the Argentine 1995 Banking Problems*, Area de Economía Y Finanzas, Documento de Trabajo Número, Banco Central de la República Argentina.

Diamond, Douglas W. and Dybvig, Philip H. (1983). "Bank Runs, Deposit Insurance and liquidity", *Journal of Political Economy*, **91**: 401–419.

Morris, Stephen and Shin, Hyun Song (1998). "Unique Equilibrium in a Model of Self–Fulfilling Currency Attacs", *American Economic Review*, **88**(3): 587–597.