

The M1 Vector-Error-Correction Model: Some Extensions and Applications

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Abstract

In this paper, we present a vector-error correction forecasting model for inflation based upon disequilibrium in the money market. Excess supplies of money, measured as the deviation of actual money from an estimated long-run cointegrating vector, can be quite persistent and create strong inflationary pressures. We use the model to identify monetary policy shocks and perform a number of conditional forecasting exercises. We also use a bootstrapping simulation method to derive the distribution of inflation forecasts based upon parameter and future shock uncertainty.

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Conducting monetary policy . . . is akin to driving without full vision—perhaps like driving in a rainstorm with defective windshield wipers. It can be done, but only very carefully. (John Crow, Eric J. Hanson Memorial Lecture, 1988)

The Fed's dilemma is like that of a tugboat captain pushing a long string of barges in a dense fog; the awkward load is difficult to pilot. He needs to start his turn half a mile before the bend. But he can't see the bend until it's too late. (Allan Murray, *The Wall Street Journal*, 1989)¹

Introduction

Monetary policy makers face a difficult task when evaluating the current state of the economy and deciding what actions are needed to achieve their objectives, such as keeping inflation within a given range. Because long and variable lags exist between a monetary policy action and its effects on economic variables, policy-makers need a way to assess whether their actions are having, or indeed will have, the desired effect.

Economists at the Bank of Canada use inflation forecasts, in addition to other variables considered to have leading-indicator properties, to inform policy-makers' views on the current and future state of the economy. Some of these variables are new orders and shipments, housing market activity, inflation, and various money and credit aggregates. Since no one single indicator is superior to all others, a good strategy is to monitor many variables to try to ensure that the best signals about the economy are being considered. This strategy also extends to models. Given that a model is simply a collection of assumptions or behavioural rules about the way an economy works, economists maintain several models for forecasting or conditional projections.²

Economists at the Bank have pursued modelling strategies along a continuum anchored at one end by purely theoretical approaches and at the

1. Many thanks to Pierre Duguay for assistance in finding the John Crow quotation. The Murray reference is from Dorfman (1999).

2. Engert and Selody (1998) and Berk (1997) made excellent arguments for the use of multiple models in formulating monetary policy. No single model can capture all aspects of the economy, so it is useful for policy-makers to have several different models summarizing different views or aspects of the economy.

other end by purely empirical approaches.³ The model we use in this paper is an M1 vector-error-correction model (VECM), which could be described as lying somewhere between the middle of the continuum and the purely empirical end. At the heart of this model is a long-run money-demand function. Several extensions have been made to the basic model Hendry (1995) presented to add more theory (or structure) to it in order to make it more useful for conducting counter-factual analysis: for asking “what if” questions.

Sections 1 and 2 provide some background and context for the M1 VECM and discuss the changes made to Hendry’s original model. Section 3 details why gross M1 is no longer the preferred measure of narrow money for the VECM and outlines how adjusted M1, the preferred measure, is constructed. Section 4 discusses the identification of policy shocks in the VECM, and section 5 lays out the framework for using information from models to inform policy. Section 6 provides some direction for future work.

1 Background

Inflation is essentially a monetary phenomenon. In the long run an excess creation of money is bound to lead to inflation. In the short run the links may not be as tight. After an unsuccessful attempt at using money-growth targets to reduce inflation in the 1970s both here in Canada and in the United States, many models now used to guide policy advice assume that money plays only a passive role and may be ignored for all practical purposes because the central bank and commercial banks are assumed to simply supply money passively in accordance with agents’ demand. Consequently no causal role is given to money in these models of inflation. This paper uses an active-money paradigm in which money causes inflation. However, in this model, money’s causal effect on inflation does not depend on using money as the instrument of monetary policy, as is the case in many theoretical models. A very short-term interest rate such as the overnight rate can be considered to be the instrument of monetary policy, consistent with the Bank of Canada’s operating procedure. In responding to changes in the overnight rate, financial intermediaries make loans to agents in the economy and hence create deposits. These agents then transact with other agents using the newly created balances, leading to changes in the level of aggregate activity and prices. The underlying premise is that agents have a long-run demand for money, and the amount of money an agent actually holds fluctuates around these desired money holdings. For the purposes of the discussion below, the difference between actual money supply and estimated long-run money

3. Thanks are due to Kevin Moran and Jack Selody for suggesting this analogy.

demand (using the long-run parameters but evaluated at the current values of the variables in the long-run demand function) is called the money gap.⁴

Laidler (1999) discussed the passive- and active-money views, drawing on the buffer-stock theory to explain why people may temporarily be off their long-run money-demand function.⁵ For example, suppose an agent receives an unanticipated lump sum of money. Information and transaction costs are involved in deciding what to do with the money and then doing it. Hence it is optimal to take some time to arrive at a decision, and consequently at any time an agent's actual holdings of transactions money might differ from his or her desired long-run holdings of money. That is, actual money holdings fluctuate around the desired level (i.e., long-run demand for money), much as a firm's inventories fluctuate around some level of desired inventory holdings. A firm or individual with money holdings exceeding the desired level of money balances will act to get rid of these excess balances by transacting with other agents in the economy. An excess aggregate supply of money can translate into inflationary pressure in much the same way that an excess demand for goods does; too much money chases too few goods. Hence a positive money gap, where the stock of money exceeds the aggregate long-run demand for money, is associated with periods of rising inflationary pressure, and a negative gap, where the stock of money is less than long-run demand for money, is associated with disinflationary pressures.

The adjusted-M1 VECM presented in this paper is an extension of work done by Hendry (1995), who estimated a unique long-run cointegrating vector between M1, output, prices, and a short-term interest rate. The vector can be thought of as a long-run money-demand function. Since money demanded does not have to be equal to money supplied at each point in time (though they must be equal in the steady state), the error-correction term of the VECM can be thought of as a money gap, which has been shown to have predictive power for inflation.⁶

4. In this sense, what is being evaluated is the difference between actual money and today's value of the long-run demand for money. The long-run money-demand function could also be evaluated at the long-run values of the variables in the long-run demand function, but doing so causes the money gap to lose most of its predictive power. This is likely because rather than evaluating money supply at its actual value, the model should include some notion of long-run supply. Identifying the money-supply process is an area for future work.

5. Laidler's (1999) discussion focuses on narrow, or transactions, money.

6. Armour et al. (1996) and Engert and Hendry (1998) found the VECM to be a good inflation-forecasting model at horizons of one to two years. As well, Fung and Kasumovich (1998) found that following an expansionary monetary policy shock, a positive money gap opens up, followed by an increase in prices.

2 Details of the VECM

The model used in this paper is similar to Hendry's original model in that it estimates a unique and stable long-run cointegrating vector between quarterly data for nominal M1, real output, the consumer price index, and a short-term interest rate. This vector can be also considered to be a long-run money-demand function. (See the appendix for more details on the model.)

The Johansen–Juselius (1990) methodology was used to estimate the long-run cointegrating vector from a VECM of the form

$$\Delta X_t = \Gamma(L)\Delta X_t + DZ_t + \alpha\beta'[X_{t-1}], \quad (1)$$

where X_t is a vector of endogenous variables (i.e., money, output, prices, and interest rates), $\Gamma(L)$ is a matrix of parameters for a fourth-order lag process, Z_t is a vector of stationary exogenous variables including seasonal dummies, and D is the matrix of parameters associated with the exogenous variables. The α parameters measure the speed at which the variables in the system adjust to restore a long-run equilibrium, and the β vectors are estimates of the long-run cointegrating relationships between the variables in the model.

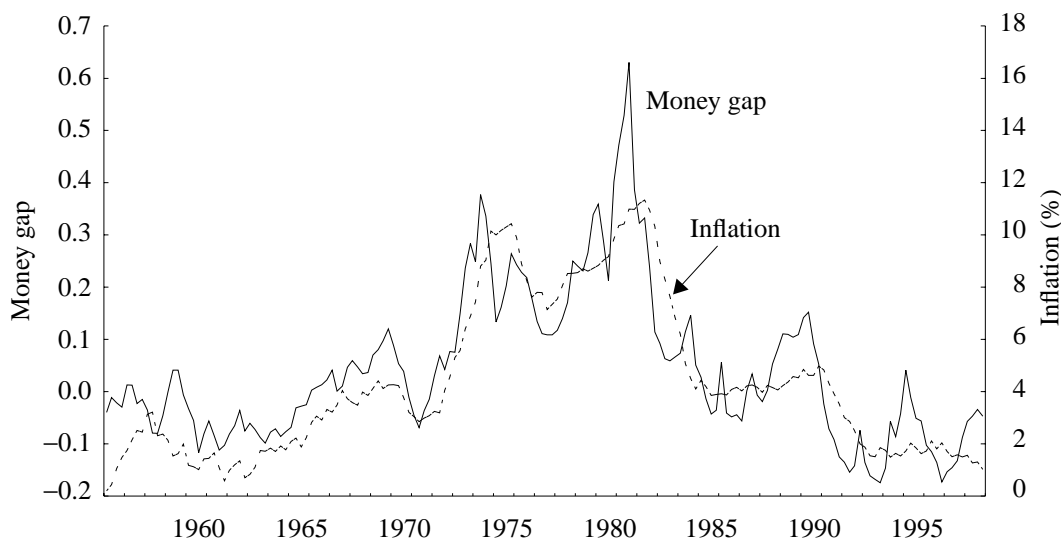
This system was found to have a unique stable long-run cointegrating relationship between money, inflation, output, and interest rates. Unitary price elasticity is imposed (i.e., prices move one for one with changes in money), and the long-run coefficient on output is around 0.5 while that on the interest rate is about -0.04 , both of which are consistent with previous work.⁷

However, the estimated short-run parameters of Hendry's original model (i.e., the α parameters as well as the coefficients on the lagged endogenous variables) were unstable in that they varied greatly over the sample period. Consequently a number of exogenous variables were added to improve the estimates. One of the exogenous variables included is a measure of the output gap calculated using potential output from the Bank's Quarterly Projection Model (QPM), which measures potential using an extended multivariate filter (see Butler 1996 for details on this approach). Other exogenous variables added are the Can\$/US\$ spot exchange rate, the U.S. 90-day commercial paper rate, the U.S. inflation rate, the change in non-personal notice deposits post-1980, and a permanent shift dummy for the early 1980s. The 1980s shift variable is interpreted as a proxy for the financial innovations that occurred at chartered banks at that time.⁸ Many other variables, such as daily-interest account rates and dollar values, the

7. The restriction of unitary price elasticity in an unrestricted regression was not rejected.

8. See Freedman (1983), Gomme (1998), and Aubry and Nott (2000) for discussions of some of these innovations.

Figure 1
Money gap vs. 8-quarter inflation rate



yield curve, and the volatility of long-term rates were also tried as proxies for these innovations; however, none of them successfully eliminated the need for the 1980s dummy variable.

Including non-personal notice deposits in the model as an exogenous variable was an attempt to internalize the shift between demand and notice deposits that has been occurring over the 1990s. Reserve requirements on demand and notice deposits were eliminated in the early 1990s, leading to some redistribution of funds on the part of agents (mainly business customers) out of notice deposits and into demand deposits. The redistribution has increased the growth rate of M1 in recent years; however, while the increase in M1 growth has been associated with some acceleration of real GDP growth, it has not spilled over into increased inflation to date. The shift between notice and demand deposits is not the only innovation to affect the M1 aggregate,⁹ and this is why adjusted M1, rather than gross M1, is used in the model. The construction of adjusted M1 is discussed in section 3.

Some equilibrium conditions have been imposed on the model to force it to a particular steady state. In the steady state, potential output growth is assumed to be 2.3 per cent, inflation is 2 per cent, and money growth is 3.2 per cent, as implied by the long-run money-demand parameters and the assumptions on output and price growth. The steady-

9. See Bank of Canada (1998), Atta-Mensah and Nott (1999), and Aubry and Nott (2000) for discussions of the innovations affecting M1 in the 1990s.

state overnight rate is about 4.9 per cent and is based on the historical relationship with U.S. interest rates.

As discussed in section 1, the difference between actual money supply and estimated long-run money demand is called the money gap. The money gap has moved very closely with actual inflation over the last 40 years (see Figure 1) and helps the model to predict inflation.

The main differences between the M1 VECM presented in this paper and Hendry's original are that the current model uses adjusted M1 (discussed in section 3) rather than gross M1 and the overnight interest rate rather than the 90-day commercial paper rate. The overnight rate has been the policy instrument in Canada since 1994, and there is evidence that the overnight rate provides a good way to measure monetary policy in Canada over a much longer period (see Armour, Engert, and Fung 1996). Moreover, because the overnight rate is highly correlated with the 90-day commercial paper rate (the correlation since 1956 is 0.98), changing the short-term interest rate used in the model had no significant effects on the estimated money-demand function or the model's forecasting performance.

3 Dealing with Financial Innovations in Narrow Money

In Canada, changes in real M1 growth are correlated with changes in real GDP growth about two quarters in the future.¹⁰ M1 growth is also correlated with changes in prices about eight quarters in the future.¹¹ However, in the 1990s the relationship between money and other economic variables appears to have shifted, possibly in relation to the "restructuring" of the Canadian economy after the 1990–91 recession. This shift could also be related to the change in monetary policy to a regime of explicit inflation targeting. A last explanation could be that the many financial innovations in the 1990s appear to have changed the nature of deposit accounts.

Given this last possibility, the definition of narrow money for use in the VECM was re-examined.¹² Narrow money is generally considered to be money used in transactions for goods and services. In Canada, narrow money is currently defined as M1, which comprises currency, personal chequing accounts, and current accounts. Over the period 1992 to 1994, reserve requirements on accounts were phased out, reducing the distinction between notice and demand accounts.¹³ Also, the improvement in electronic financial services in recent years and the increased popularity of debit cards,

10. See, for instance, Bank of Canada (1999, 29).

11. As discussed in Hendry (1995) and Armour et al. (1996).

12. Aubry and Nott (2000) examined the conceptual issues of what should be included in a measure of narrow money.

ATMs, and telephone/PC banking have led agents to economize on their cash balances and enabled them to more easily access non-M1 accounts for transactions purposes. These technological improvements seem to have increased the degree of substitutability between cash and demand or notice deposit accounts, and consequently a broader definition of transactions money might be more appropriate in an electronic world. As well, most of the products financial institutions currently offer have joint transactions and savings characteristics. Thus some proportion of these balances does not really belong in a transactions money measure and should be excluded. The problem is to come up with a reasonable way of approximating this proportion.

In Canada over the period 1980 to 1982, another series of financial innovations introduced instability into the parameters of the model's cointegrating vector. However, the 1980s innovations tended to simply move money from (M1) demand deposits to (M2) notice deposits. In order to deal with an environment of high interest rates and changing reserve requirements, banks offered customers incentives to move their accounts from ones that were costly for the banks to maintain (demand deposits) to ones that were more cost-effective for the banks (notice deposits). Banks introduced innovations such as daily-interest savings and daily-interest chequing accounts to motivate consumers to switch from non-interest-bearing demand accounts to interest-bearing notice accounts. The shift in the 1990s has not only been related to a switch back from notice to demand accounts owing to the reduction and eventual removal of reserve requirements for demand accounts, but also to the advent of technological changes, as more types of accounts now have the characteristics of transactions money.

Adjusted M1 is a model-based measure of money that was constructed for this paper specifically to correct the VECM instability and estimate the size of the distortion in M1. This was done in two steps.

First, the money-forecasting equation from a gross-M1 VECM (estimated from 1956 to 1993) was used to forecast M1 growth from 1992Q1 to 1999Q1 using actual values for all other variables in the model. It yielded a time series we called "distortion-free" money. This series is an

13. Reserve requirements were higher on demand deposits included in M1 than on notice deposits excluded from M1. As a consequence, beyond the essentially irrelevant withdrawal-notice requirement, the distinction between demand and notice accounts has become meaningless. So far, this innovation seems to have affected mainly business accounts. Banks have begun to pay more attractive rates of interest on current accounts, and businesses have shifted some of their funds into those accounts.

Figure 2
Income elasticity from rolling regression

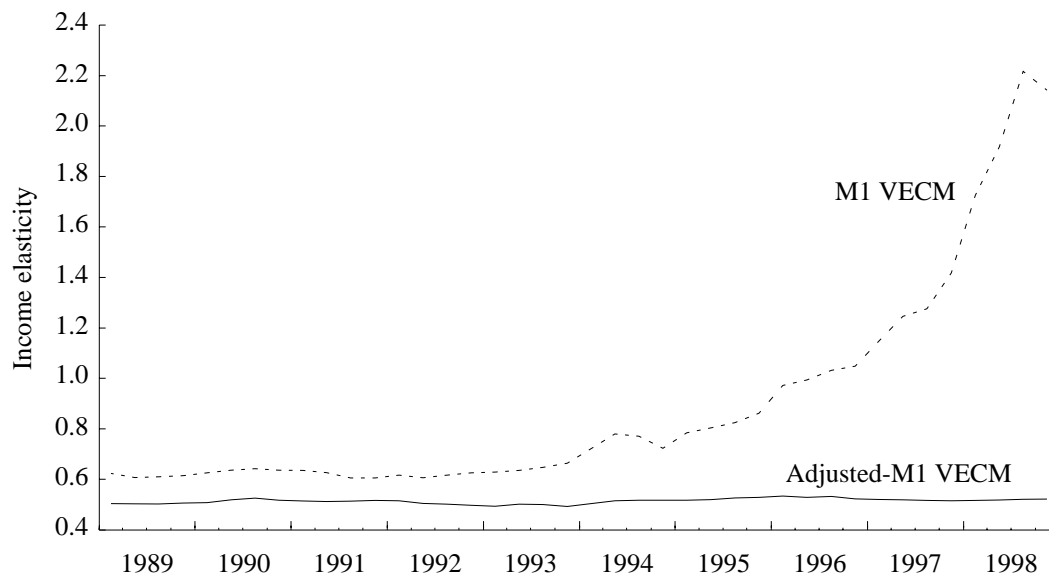


Figure 3
Interest rate semi-elasticity from rolling regression

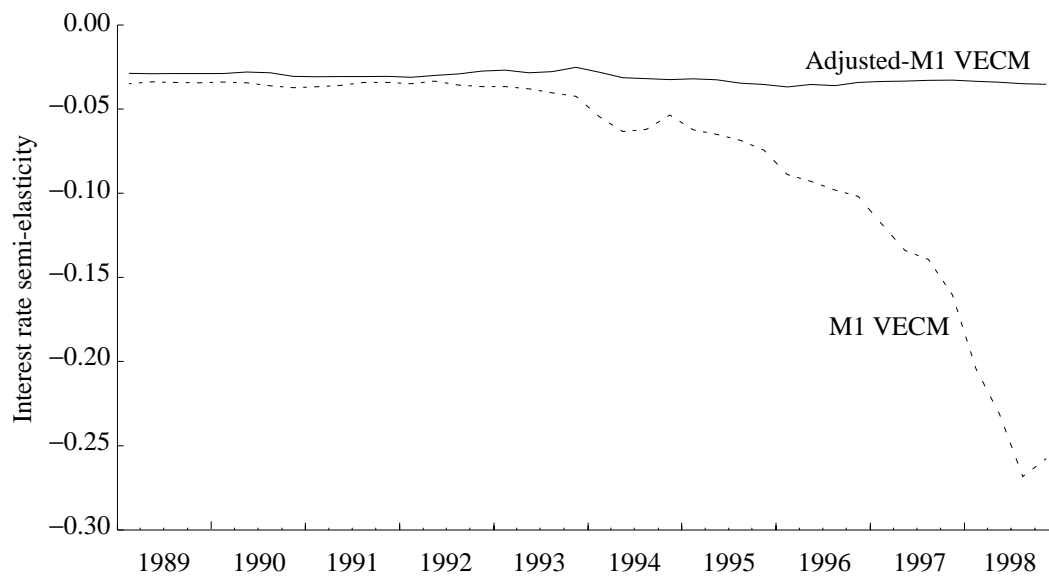


Figure 4
Year-over-year growth rate of gross M1 vs. adjusted M1

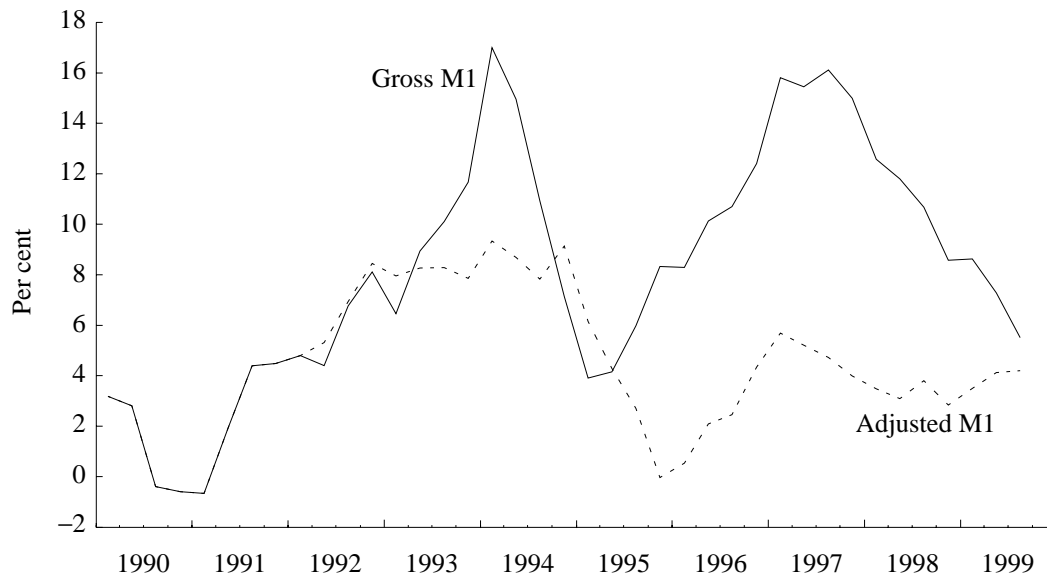
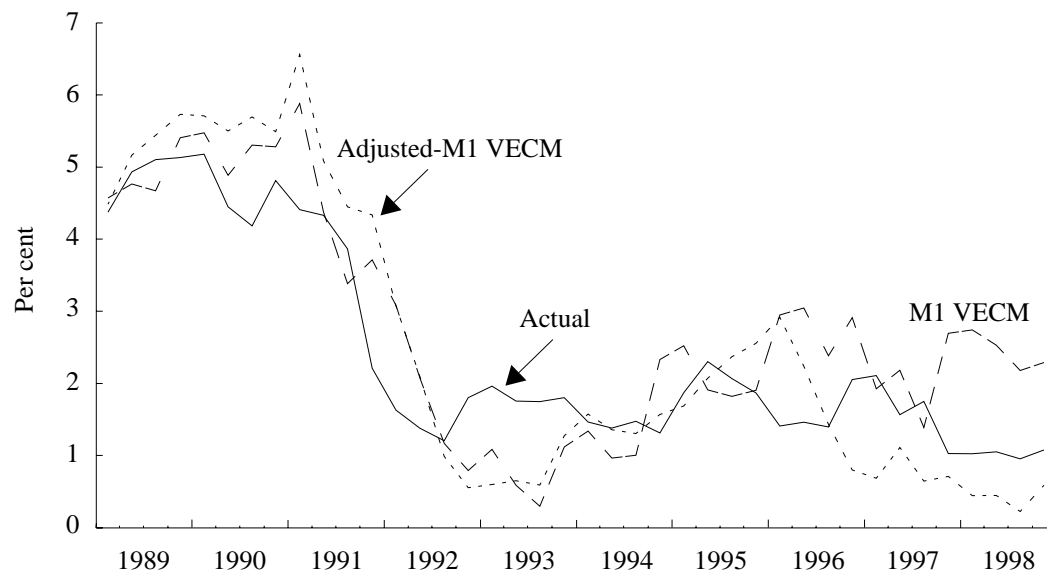


Figure 5
Forecast and actual 4-quarters-ahead 4-quarter inflation rate



estimate of what M1 would have been had the data-generating process not changed in the 1990s.

Second, in order to relate the distortion-free money series to the observable money data, it was regressed on all the components of M1++ (gross M1 plus all notice deposits). Because the coefficients were similar on some components that could reasonably be thought of as having the same sort of characteristics or users, these components were grouped together to reduce the number of parameters to estimate in order to improve efficiency, given the small sample size.

3.1 Calculating adjusted M1

Adjusted M1 is calculated as follows:

$$\text{adjusted M1} = 1.58 (\text{currency}) + 0.28 (\text{non-personal}) \\ \text{for 92Q1 to 94Q3}$$

$$\text{adjusted M1} = 1.19 (\text{currency}) + 0.22 (\text{non-personal}) \\ + 0.15 (\text{personal}) \text{ for 94Q4 to 99Q1,}$$

where *non-personal* is the sum of current accounts and non-personal notice deposits, and *personal* is all personal notice deposits.¹⁴

Adjusted M1 differs from M1 in two respects:

1. Choice of components: Adjusted M1 includes notice accounts but not personal chequing accounts (PCAs) because the latter include investment dealer accounts (which today represent more than half of PCAs). The investment dealer accounts appear to be held predominantly to purchase financial assets such as mutual funds, stocks, and bonds, rather than to buy goods and services, and therefore should probably be classified within some broader aggregate that is defined as store-of-wealth money rather than in our measure of transactions money.
2. Choice of weights: M1 uses fixed weights of 1 on each of its components, whereas the weights of adjusted M1 differ from 1 based on the estimation results. Adjusted M1 also permits the weights to change at discrete points in the sample.

Given the small size of the sample, as well as the extent to which the parameters have shifted over time, the weights reported here should be treated with caution. This issue will be discussed in more depth later.

14. The sample was divided into two subperiods to reflect the fact that the parameter estimates after 1994Q3 are substantially different from those prior to it.

The primary purpose of this exercise was to correct the VECM instability, and Figures 2 and 3 clearly show that the adjusted-M1 VECM has more stable parameters, by design, than the original M1 VECM.

Figure 4 shows that the growth rate of adjusted M1 has been much weaker than that of gross M1 over much of the 1990s. However, by design, adjusted M1 is more consistent with the actual movements of prices, output, and interest rates.

In spite of this lower growth rate the inflation forecasts of the adjusted-M1 VECM are similar to those of Hendry's original M1 VECM (see Figure 5). This is likely because the M1 VECM version had shifted parameters to offset the high M1 growth and still obtained moderate inflation forecasts. The adjusted-M1 VECM uses lower money growth but more-stable parameters to obtain a reasonably similar forecast through most of the sample, with a root-mean-squared error (RMSE) of 0.91 compared to an RMSE of 0.94 for the original M1 VECM.

3.2 Why choose this approach?

The instability in the long-run parameters in Hendry's model could have been corrected with dummy variables, but this would not have provided any information about the sources of the instability.¹⁵ The approach taken in our paper is also more flexible than the dummy-variable approach in that it is not necessary to impose a priori when the distortion should end. As Figure 4 shows, the distortion to M1 (the difference between the growth rate of gross M1 and adjusted M1) has been about 6 to 7 per cent per year over the last three years. In previous work done at the Bank by Aubry and Nott (2000), distortions related to specific institutional events were examined (reclassification of current accounts at some banks from notice to demand, the effects of dealer accounts in PCAs, and the introduction of the \$2 coin), and the estimate of the size of the distortion was found to be only about 2 per cent per year.¹⁶

It can probably be assumed that these numbers provide upper- and lower-bound estimates of the size of the distortion. Since Aubry and Nott's work examined only three institutional events, it seems reasonable to conclude that their estimation of the size of distortion would be considered as a lower bound. Adjusted M1 can be considered as an upper bound because all possible sources of instability are attributed to distortions in M1.

15. One problem with the dummy-variable approach is that it would treat the distortion in all components as equal. Our approach allows for the possibility that the amount of distortion in the components is different.

16. See Bank of Canada (1998), Atta-Mensah and Nott (1999), and Aubry and Nott (2000).

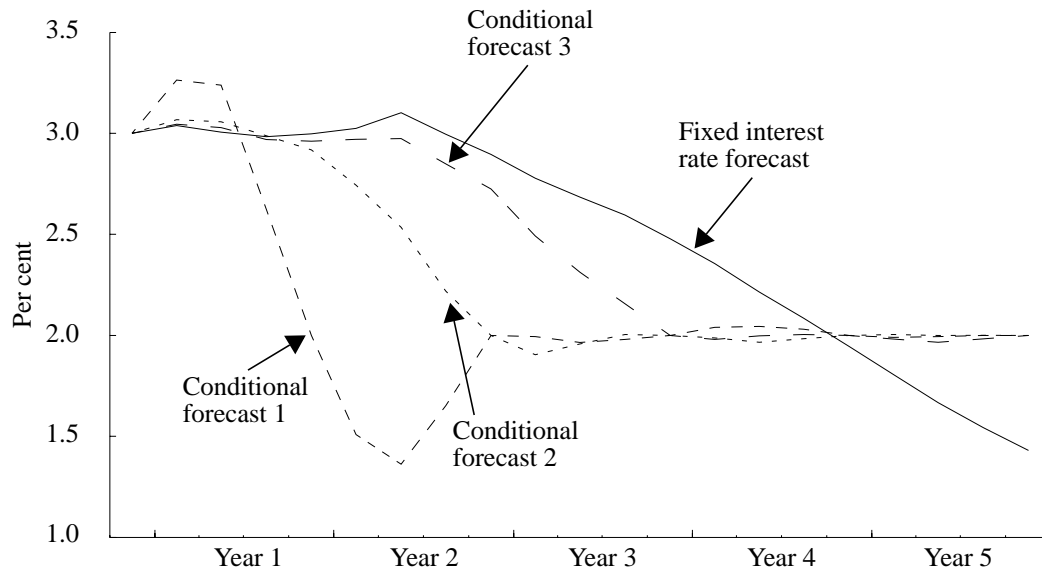
Hence it can be argued that our approach might provide too much of a correction, as the methodology may also be attributing structural changes in the economic relationship between M1 and output and inflation to distortions in M1 related to financial innovations.

The alternative measures of money M1+ (M1+ is M1 plus chequable notice deposits) and M1++ were also tried as the money variable in VECMs, but a stable money-demand function could not be estimated using either definition. This result probably reflects the fact that neither M1+ nor M1++ adequately measures transactions money over history. To address this, “extended M1+” and “extended M1++” series were constructed using the level of M1 up to 1990 and then using the growth rate of M1+ (or M1++) to calculate the level of “extended M1+” (or “extended M1++”) post-1990. However, even the extended definitions did not lead to a stable money-demand function, probably because the added components are not completely transactions-oriented, but also include some money held as a liquid store of value.

Since the weights on the components of adjusted M1 have changed because of financial innovations that occurred over time and not within a single quarter, assuming fixed weights on the components could be problematic. A time-varying parameter model with Kalman filtering may seem appropriate, but estimating such a model has proven difficult given the small data sample available. The results are quite sensitive to the initial assumptions, and we do not have a ready technique to restrict the weights to be positive in this environment. As a compromise, the single break in the weights is allowed.

To sum up, adjusted M1 can be thought of as the money growth that should have been observed over the last few years if the relationship between money, output, interest rates, and prices had remained unchanged from the past. Of the three possible reasons for the observed instability in money’s relationship with other economic variables in the 1990s, the first reason has to do with institutional changes and difficulties with our current data-reporting system that imply we may no longer be measuring the appropriate data. The second reason has to do with the changing nature of money demand in an electronic world. Finally, the instability may reflect the economy’s structural changes that are not specifically related to financial innovations. The M1 distortion estimated using the VECM incorporates all of these elements, so even though adjusted M1 can be related to a measure of transactions money, one should be careful about making inferences based on this aggregate. In fact, we consider adjusted M1 to be an interim step on the path to finding a new narrow aggregate. However, given that some economic interpretation can be put on the components of adjusted M1 and

Figure 6
Inflation forecasts



its relation to distortion-free M1, adjusted M1 is, by design, the best aggregate now available for use in the VECM.

4 Identifying Policy Shocks

Another change from the original VECM Hendry described is that policy shocks have now been identified in our model as the structural shocks to the interest rate equation as derived from a Choleski decomposition.¹⁷ That is, policy shocks are identified as unanticipated innovations to the overnight interest rate.

Previously the models were generating a “price puzzle,” in that a policy-induced increase in the nominal interest rate was accompanied by a rise in inflation. The puzzle arose because an increase in the interest rate caused a decline in the estimated long-run demand for money, and the decreased demand in turn created a positive money gap that led to a persistent inflation bubble. One possible explanation for this outcome is that changes in the interest rate have been more closely correlated over history with changes in expected inflation rather than with monetary policy shocks, and the model has not yet properly identified all of the movements in expected inflation. However, it seems reasonable that long-run money

17. The ordering of the variables in the decomposition is: U.S. interest rate, U.S. prices, overnight rate, adjusted M1, non-personal chequable accounts, output, prices, and the exchange rate.

Figure 7
Money-growth forecasts

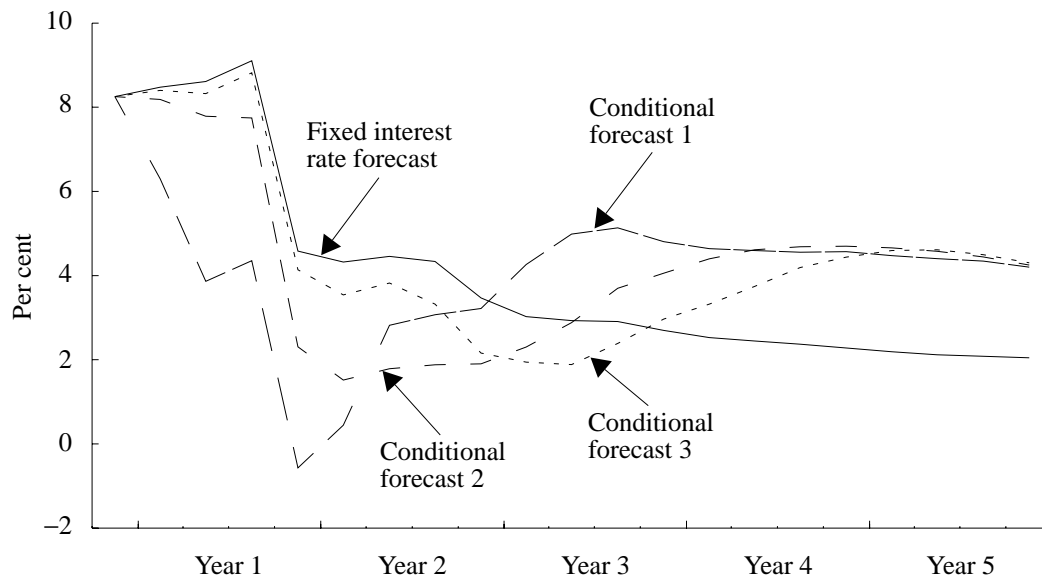
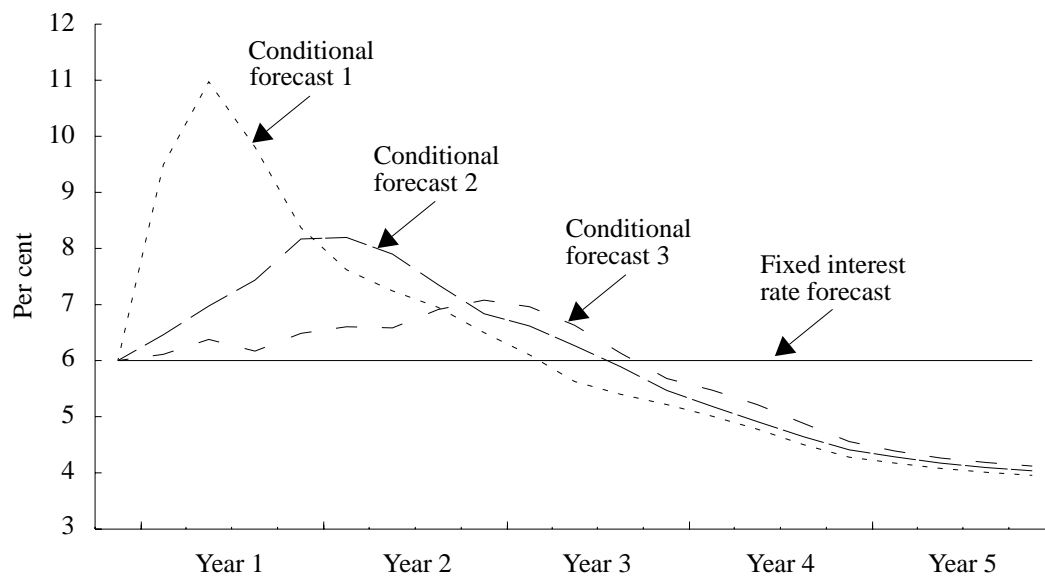


Figure 8
Overnight interest rate forecasts



demand should be based on a smoother measure of the opportunity cost of money. It is unlikely that the long-run demand for money will move substantially with every transitory change in the interest rate. Consequently an “unanticipated policy-free” interest rate series was computed by removing the model’s estimated structural policy shocks from the overnight rate, basing its removal on the argument that agents would not immediately adjust their long-run money demand to the latest interest rate policy shock. In a world with limited information regarding policy shocks, agents would respond slowly to policy innovations as they learned about the nature of the latest change in interest rates. It is this unanticipated policy-free rate that enters into the calculation of the long-run money-demand parameters and the money gap.

Using our unanticipated policy-free interest rate implies that a policy tightening will leave money demand unchanged in the quarter of the shock, thereby removing the price puzzle from the model. As well, in the first few quarters following the monetary policy tightening, the interest rate increase slows money growth by more than money demand, causing an excess demand for money, and that in turn causes inflation to fall.

The Bank is also continuing to investigate how best to measure the output and interest rate variables used in calculating money demand. Empirically the money gap that is calculated from current values of output and interest rates is the best predictor of inflation. Theoretically, however, long-run money demand calculated from long-run measures of output and interest rates—for instance, potential output and equilibrium interest rates—makes more sense (see Gerlach and Svensson 1999). We hope to examine these issues in future work.

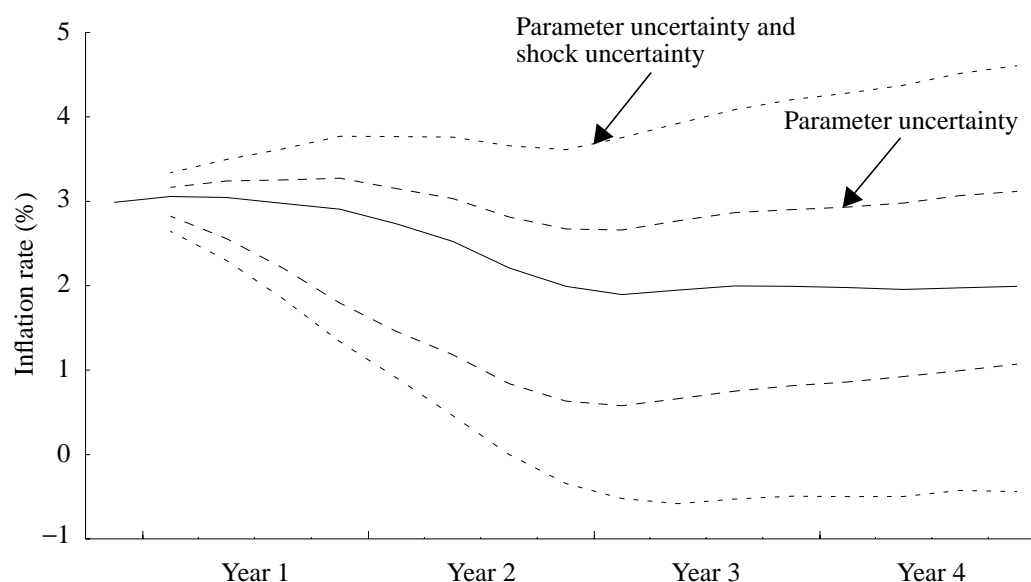
With policy shocks as they are now identified in our model, it is possible to back out the shocks to interest rates that will move inflation to the midpoint of the Bank’s inflation-control target range over a given horizon.

5 Using the Adjusted-M1 VECM for Forecasting

As an example of how the adjusted-M1 VECM could be a useful model for policy-makers, assume a set of initial conditions with 3 per cent inflation, 8 per cent money growth, 3.5 per cent output growth, and 6 per cent interest rates. These conditions were also chosen so that when the interest rate was held fixed at 6 per cent, the inflation rate would be stable around 3 per cent for the first two years out of sample.

Four separate forecasts could be provided with this model given these initial conditions. The first is a fixed interest rate forecast.¹⁸ The other three

Figure 9
Conditional inflation forecast 2 with error bands



are conditional forecasts in which we estimated the series of interest rate shocks necessary to move the 4-quarter inflation rate to the midpoint of the inflation-control target range over 4, 8, or 12 quarters, and maintain inflation at 2 per cent in the fourth quarter of each year thereafter.

Figure 6 illustrates these forecasts. In the base case, in which the overnight rate is held fixed at 6 per cent (which is above the model's steady-state value for the overnight rate), inflation will eventually decline to a point below 2 per cent in year 4. Because the overnight rate is fixed above its steady-state value, the inflation rate will converge to some new lower steady state. The line representing conditional forecast 1 shows the forecast in which the overnight rate is increased to move inflation back to 2 per cent in only 4 quarters, and the line representing conditional forecast 2 shows the forecast in which the overnight rate is increased to return inflation to 2 per

18. We did not construct an explicit “inflation-forecast indicator” as Svensson (1999) did by using the deviation of the conditional unchanged-interest-rate inflation forecast from the inflation target, though we have done so implicitly.

Figure 10
Distribution of the 4-quarter conditional inflation rate forecasts
based on parameter uncertainty for conditional forecast 2

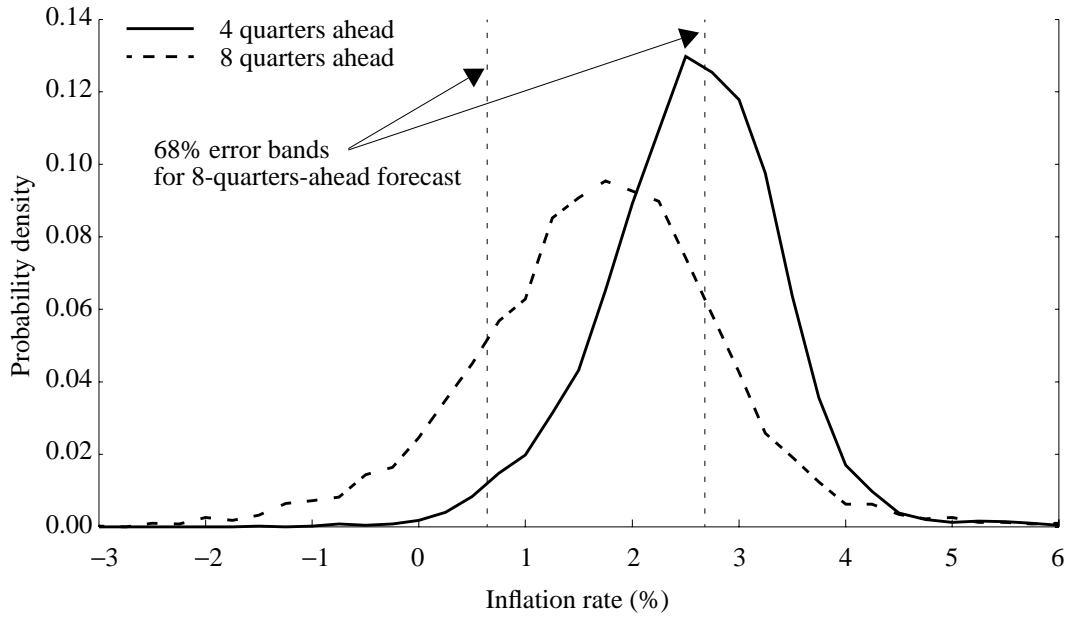


Figure 11
Distribution of the 4-quarter conditional inflation
rate forecasts based on parameter and future shock
uncertainty for conditional forecast 2

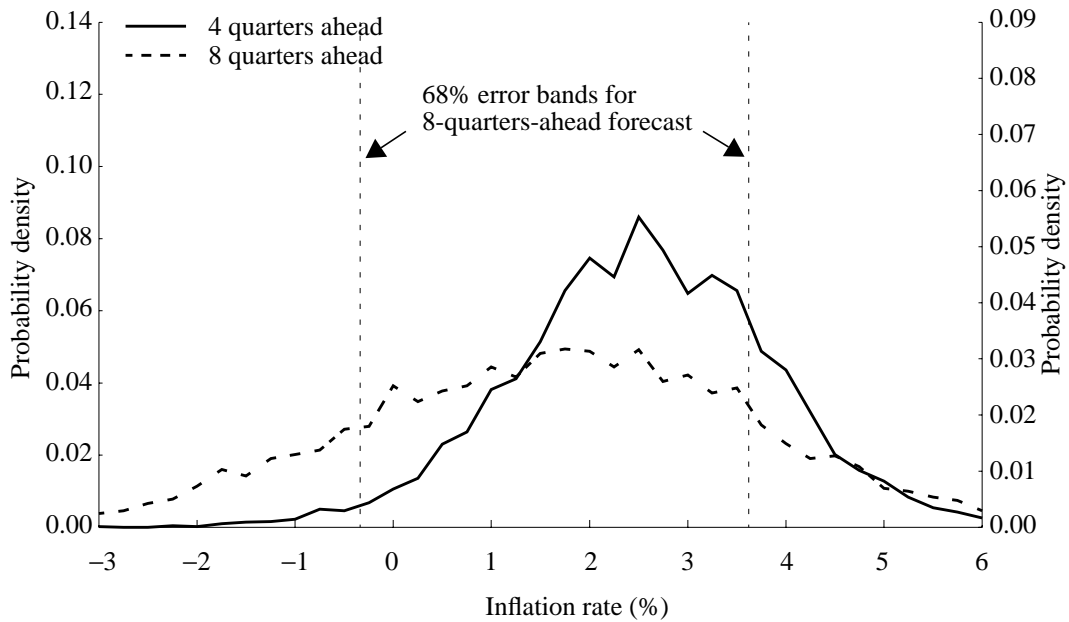


Table 1
Inflation probabilities from conditional forecast 2
with parameter and future shock uncertainty

Date	Probability of inflation between 1 and 3 per cent	Probability of inflation between 1.5 and 2.5 per cent
Year 1Q1	50%	8%
Year 1Q2	55%	24%
Year 1Q3	58%	30%
Year 1Q4	52%	27%
Year 2Q4	37%	19%
Year 3Q4	32%	17%

cent in 8 quarters. Conditional forecast 3 moves the overnight rate to bring inflation back to 2 per cent in 12 quarters.

Figure 7 shows the money-growth forecasts associated with the inflation forecasts shown in Figure 6. In the fixed interest rate forecast, money growth is initially higher than in the forecasts in which inflation is lowered to 2 per cent more quickly. Money growth subsequently falls in the base case because the overnight interest rate is held fixed at a contractionary level above steady state.

Figure 8 plots the path of the overnight rate for the same set of forecasts. Moving inflation to 2 per cent in this example requires tightening policy through an increase in the overnight rate. However, by year 3 the policy tightening is completely reversed to keep inflation from falling below 2 per cent. The closer the target horizon (4 quarters in conditional forecast 1 but 12 quarters in conditional forecast 3), the more the interest rate must be increased to achieve the target. Similarly, money growth is more volatile for closer target horizons. Choosing the appropriate policy requires considering both the target horizon and the required interest rate or money-growth movements.

Another way of conveying information about the state of the world and possible future outcomes is to provide confidence intervals or probabilities of these outcomes. For instance, a “reference range” or “monitoring range” can be constructed for money growth that is consistent with achieving 2 per cent inflation over a given horizon. This reference range would have some associated probability of inflation remaining within the target range or, alternatively, some tighter bands. As actual money growth becomes known, deviations of growth from the range should then give early warning of any impending deviations of inflation from the target range. One advantage of using a money-growth reference range as an information variable, in addition to the inflation forecasts themselves, is that it helps to gauge the change in inflationary pressures in the months since the

Figure 12
Inflation forecasts under alternative decompositions

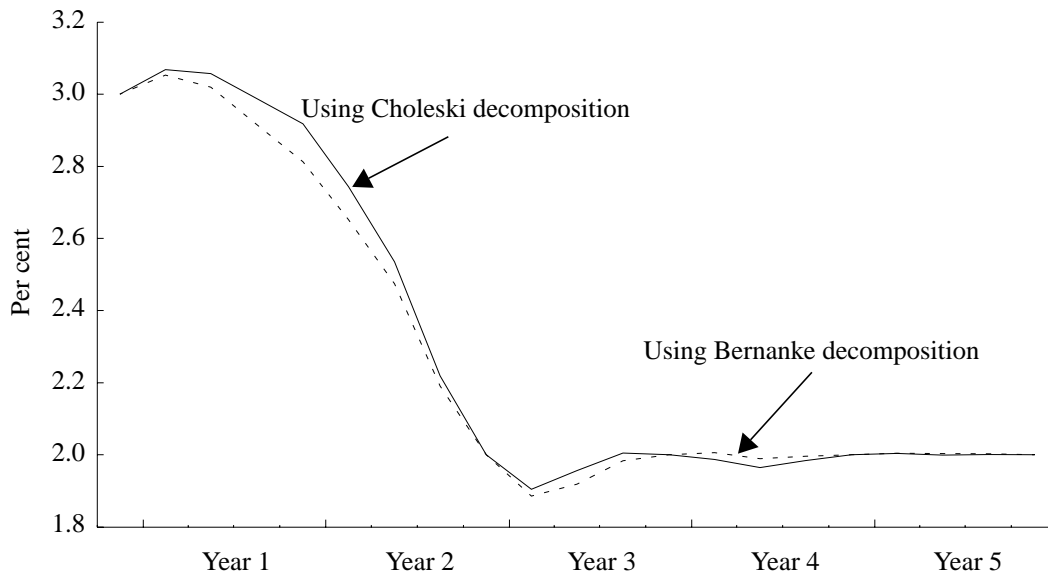


Figure 13
Money-growth forecasts under alternative decompositions

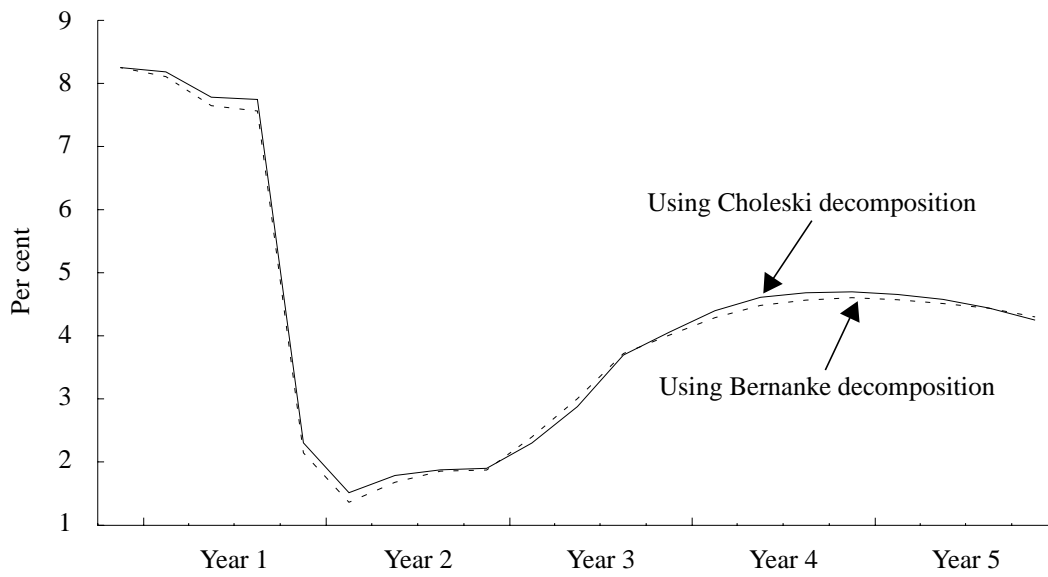
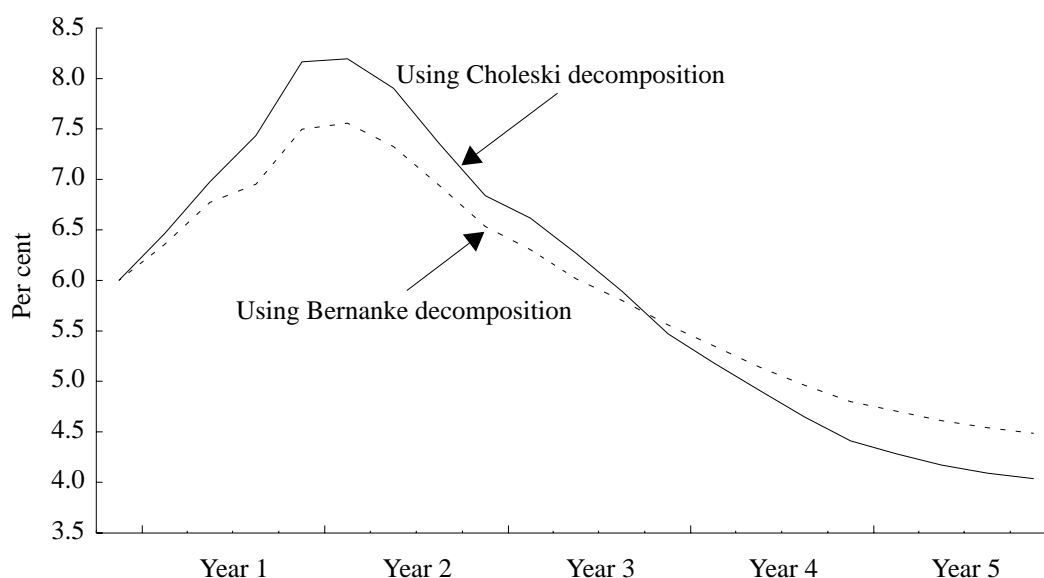


Figure 14
Overnight rate forecasts under alternative decompositions



reference range was derived.¹⁹ Gerlach and Svensson (1999) found that the information from a money-growth indicator is subsumed by a money gap. Our findings generally support this claim. However, an attraction of money growth rate indicators and reference ranges is that they are perhaps easier to explain than a money gap and thus may help a central bank explain its reasons for a policy action.

The European Central Bank (ECB) uses the 3-month growth rate of the 12-month moving average of M3 growth as one of the pillars of its two-pillar strategy of achieving and maintaining price stability.²⁰ Although the ECB uses a broad aggregate to allow for the possibility of shifts such those due to financial innovations, we have explicitly accounted for the shifts that have occurred in Canada in our narrow aggregate.

19. The main distinction between an intermediate target and an information variable lies in the degree of correlation between the intermediate target/information variable and the goal variable (GV). An intermediate target (IT) is assumed to have a tight correlation with the GV, whereas the correlation of an information variable with the GV is much looser. A related concern is the degree of controllability between the IT and the GV. If policy-makers are unable to control the IT, it seems unlikely that they would be able to control the GV.

20. The ECB's stability-oriented monetary policy is based on: (i) a prominent role for money and (ii) a broadly based assessment, using financial and other indicators, of both the outlook for price developments and the risks to price stability.

Alternatively, the probabilities of inflation remaining within the target range can be calculated without using a reference range for money growth. Figure 9 shows the conditional inflation forecast and two possible 68 per cent confidence intervals (about 1 standard deviation) when the interest rate is set as in conditional forecast 2 to achieve 2 per cent inflation in eight quarters. The confidence intervals were calculated from a bias-corrected bootstrap technique proposed by Kilian (1998) and discussed in Sims and Zha (1995).²¹ The distribution of inflation forecasts shown by the inner bands is based on uncertainty about the model's parameters. The distribution shown by the outer bands is based on uncertainty about both the model's parameters and possible future exogenous shocks.

The complete distributions for the 4-quarter inflation forecast four and eight quarters ahead are plotted in Figures 10 and 11. The vertical lines represent the 68 per cent error bands for the 8-quarters-ahead forecast as shown at the end of year 2 in Figure 9. From distributions such as these we can calculate various probabilities that may interest policy-makers. The probabilities that inflation will be within the 1 to 3 per cent official inflation-control target range or within a tighter range of 1.5 per cent to 2.5 per cent are given in Table 1. Similar distribution functions can be computed for each of the forecast variables. For instance, monitoring ranges for the money-growth rate or the interest rate can be computed that are consistent with achieving the target inflation rate.

In summary, our model can generate the point forecasts and their associated probabilities for any number of starting-point assumptions and/or policy scenarios, providing significant information about possible outcomes and desirable policy scenarios.

6 Future Work

Decompositions other than the Choleski could have been used to identify monetary policy shocks. The estimate of the response of money growth to an interest rate shock has varied over time, and thus it may be appropriate to include an overidentifying restriction in the decomposition of shocks (cf. Bernanke 1986 or Sims 1986). Such a restriction would change the contemporaneous response of money growth to a variation in interest rates from the average response estimated over history with the Choleski decomposition. When one examines recent data, money's response to an

21. This technique requires one bootstrap to first obtain an estimate of the bias in the model's coefficients. Bias-corrected coefficients are calculated and used for a second bootstrap simulation to generate the error bands. We computed 5,000 bootstrap samples at each stage.

interest rate shock appears to have increased. (The greater contemporaneous response of money growth to movements in interest rates might be explained by a change in how rapidly banks respond to policy-induced shocks to interest rates to alter the amount of liquidity they provide to agents in the economy.) The average historical change in money growth for a 100-basis-point change in the overnight rate has been about 0.6 per cent. Using a Bernanke decomposition in another version of the model used below, this response has been increased to 1 per cent.²²

For the conditional forecast in which policy responds to push inflation to 2 per cent in eight quarters (conditional forecast 2), the inflation and money-growth rates are essentially the same using either the Bernanke or Choleski decomposition. However, the size of the interest rate decrease needed is smaller in the Bernanke case than in the Choleski case. Money growth does relatively more to move inflation towards its target, leaving less work to be done via the interest rate channel.

Our future research will also examine more-sophisticated alternative monetary policy identification techniques, such as those employed for Canada by Fung and Kasumovich (1998) and Fung and Yuan (2000). Varying this aspect of the model will perform a robustness check of its predictions. The Bank is also investigating definitions of transactions money as well as the identification of desired money supply for financial institutions.

Conclusion

The M1 VECM has predicted inflation reasonably well over history and still appears to be a good forecasting model, especially in light of modifications like using adjusted M1, identifying policy shocks, and deriving probabilities for inflation outcomes.

Forecasts from the VECM can augment the information coming from other models used at the Bank. They can provide alternative views of what could happen in the economy and give some information about the “balance of risks.” Multiple models could be especially helpful to policy-makers during times of extreme uncertainty and/or structural shifts, but even in relatively stable times, advice from different models helps to balance risks about the outlook for the future.²³

Different models that yield similar predictions would tend to lessen policy-makers’ uncertainty regarding possible outcomes, *ceteris paribus*,

22. The 1 per cent estimate of the contemporaneous response of money to a 100-basis-point change in the overnight rate is within 2 standard deviations of the estimate of the money-response parameter.

making policy judgments somewhat easier. However, relying on multiple models has the greatest value when a model relying on one set of variables and assumptions forecasts one outcome and another model with a different set of variables and assumptions forecasts another—perhaps quite different—outcome. In any event, advice based on multiple models should give policy-makers more information and hence allow them to achieve their desired goals. As Alan Blinder (1998, 12) advises, “Use a wide variety of models and don’t ever trust any one of them too much.”

23. See, for example, Engert and Selody (1998) and Berk (1997). Different models are used to entertain a possible shock, such as a change in policy or a real-side shock, and evaluate its impact on the forecasts of relevant variables. Such an approach may also help in assessing the uncertainty associated with particular shocks.

References

- Armour, J., J. Atta-Mensah, W. Engert, and S. Hendry. 1996. "A Distant Early Warning Model of Inflation Based on M1 Disequilibria." Bank of Canada Working Paper No. 96-05.
- Armour, J., W. Engert, and B.S.C. Fung. 1996. "Overnight Rate Innovations as a Measure of Monetary Policy Shocks in Vector Autoregressions." Bank of Canada Working Paper No. 96-04.
- Atta-Mensah, J. and L. Nott. 1999. "Recent Developments in the Monetary Aggregates and their Implications." *Bank of Canada Review* (Spring): 5-19.
- Aubry, J.-P. and L. Nott. 2000. "Measuring Transactions Money in a World of Financial Innovation." In *Money, Monetary Policy, and Transmission Mechanisms, 00-00*. Proceedings of a conference held by the Bank of Canada, November 1999. Ottawa: Bank of Canada.
- Bank of Canada. 1998. "Recent Developments in the Monetary Aggregates and their Implications." *Bank of Canada Review* (Spring): 3-15.
- . 1999. *Monetary Policy Report* (May).
- Berk, J.M. 1997. *Monetary Transmission: What Do We Know and How Can We Use It?* Rev. ed. Amsterdam: De Nederlandsche Bank. DNB Staff Reports No. 15.
- Bernanke, B. 1986. "Alternative Explanations of the Money-Income Correlation." *Carnegie-Rochester Conference Series on Public Policy* 25: 49-99.
- Blinder, A.S. 1998. *Central Banking in Theory and Practice*. Cambridge, Mass.: MIT Press. The Lionel Robbins Lectures.
- Butler, L. 1996. *The Bank of Canada's New Quarterly Projection Model, Part 4. A Semi-Structural Method to Estimate Potential Output: Combining Economic Theory with a Time-Series Filter*. Technical Report No. 77. Ottawa: Bank of Canada.
- Crow, J. 1988. "The Work of Canadian Monetary Policy." *Bank of Canada Review* (February): 3-17.
- Dorfman, J. 1999. "Punch Bowls, Barges, and Other Bits of Fed Lore." *Bloomberg News*. (20 May 1999)
<URL:<http://www.bloomberg.com/feature.html>> (20 May 1999).
- Engert, W. and S. Hendry. 1998. "Forecasting Inflation with the M1-VECM: Part Two." Bank of Canada Working Paper No. 98-6.
- Engert, W. and J. Selody. 1998. "Uncertainty and Multiple Paradigms of the Transmission Mechanism." Bank of Canada Working Paper No. 98-7.
- Freedman, C. 1983. "Financial Innovation in Canada: Causes and Consequences." *American Economic Review* 73 (2): 101-6.
- Fung, B.S.C. and M. Kasumovich. 1998. "Monetary Shocks in the G-6 Countries: Is There a Puzzle?" *Journal of Monetary Economics* 42 (3): 575-92.
- Fung, B.S.C. and M. Yuan. 2000. "Measuring the Stance of Monetary Policy." In *Money, Monetary Policy, and Transmission Mechanisms, 00-00*. Proceedings of a conference held by the Bank of Canada, November 1999. Ottawa: Bank of Canada.
- Gerlach, S. and L.E.O. Svensson. 1999. "Money and Inflation in the Euro Area: A Case for Monetary Indicators?" BIS and Institute for International Economic Studies, Stockholm University. Draft.
- Gomme, P. 1998. "Canada's Money Targeting Experiment." *Federal Reserve Bank of Cleveland Economic Commentary* (1 February): 1-4.
- Hendry, S. 1995. "Long-Run Demand for M1." Bank of Canada Working Paper No. 95-11.
- Johansen, S. and K. Juselius. 1990. "Maximum Likelihood Estimation and Inference on Cointegration—With Applications to the Demand for Money." *Oxford Bulletin of Economics and Statistics* 52 (2): 169-210.
- Kilian, L. 1998. "Small-Sample Confidence Intervals for Impulse Response Functions." *Review of Economics and Statistics* 80 (2): 218-30.

- Laidler, D. 1999. "Passive Money, Active Money, and Monetary Policy." *Bank of Canada Review* (Summer): 15–25.
- Sims, C.A. 1986. "Are Forecasting Models Usable for Policy Analysis?" *Federal Reserve Bank of Minneapolis Quarterly Review* 10 (1): 2–16.
- Sims, C.A. and T. Zha. 1995. "Error Bands for Impulse Responses." Federal Reserve Bank of Atlanta Working Paper No. 95–6.
- Svensson, L.E.O. 1999. *Monetary Policy Issues for the Eurosystem*. London: Centre for Economic Policy Research. Discussion Paper No. 2197.

Appendix

Details of the Adjusted-M1 VECM

Step 1: Estimating long-run money demand and the money gap

The Johansen-Juselius methodology is used to test for the existence of a unique long-run cointegrating relationship between money, inflation, output, and interest rates (non-seasonally adjusted data). The model is an error-correction model because deviations of money demanded from money supplied (the money gap) are assumed to be corrected in the long run. The model has the form

$$\Delta X_t = \Gamma(L)\Delta X_t + DZ_t + \alpha\beta'[X_{t-1}, D80a_{t-1}], \quad (A1)$$

where

$$X_t = [MI_t, CPI_t, Y_t, RONf_t]$$

$$RONf_t = \text{level of "policy-free" overnight interest rate} \\ = RON_t - \varepsilon_t$$

ε_t is the residual from the interest rate equation (A8)

$$MI_t = \text{log level of adjusted M1}$$

$$Y_t = \text{log level of real output}$$

$$CPI_t = \text{log level of the consumer price index}$$

$$Z_t = [\text{constant, 3 seasonal dummies, output gap}_{t-1}, \Delta \log(\text{exchange rate}) \text{ from } t \text{ to } t-3, \Delta USCP90_t \text{ rate, } D80b * \Delta NPN_t, D80a_t]$$

$$\text{output gap}_{t-1} = Y_t - \text{Bank of Canada's estimate of potential output from QPM}$$

$$USCP90 = \text{U.S. 90-day commercial paper rate}$$

$$D80b = 0 \text{ for } 1979\text{Q4} \text{ and before, and } 1 \text{ thereafter}$$

$$NPN_t = \text{non-personal notice deposits}$$

$$D80a_t = 0 \text{ for } 1979\text{Q4} \text{ and before, and } 1 \text{ for } 1983\text{Q1} \text{ and after.} \\ \text{Increases linearly from } 0 \text{ to } 1 \text{ from } 1980\text{Q1} \text{ to } 1982\text{Q4.}$$

$$\Gamma(L) = \text{matrix of parameters for a fourth-order lag process}$$

Equation (A1) is estimated from 1956Q1 to 1998Q4.

The money gap is calculated as

$$mgap_t = c + MI_t - CPI_t - \hat{\beta}_{yt}Y_t + \hat{\beta}_{rt}RONf_t + \hat{\beta}_{d81t}D80a_t, \quad (A2)$$

where

c = long-run constant to ensure the gap converges to 0 in steady state

$\hat{\beta}_{yt}, \hat{\beta}_{rt}, \hat{\beta}_{d81t}$ = Johansen estimates of the long-run parameters

Some additional variables need to be calculated before step 3, the forecasting step.

Step 2: The interest rate gap

The interest rate gap, $RGAP_t$, is estimated from the auxiliary equation

$$R_t = k + aUSR_t, \quad (A3)$$

where

$R_t = RON_t - \text{expected inflation}_t$

$USR_t = USCP90_t - \text{expected U.S. inflation}_t$

Expected, $E[inflation_t]$ = actual inflation from $t-1$ to t

Expected U.S. inflation $_t$ = actual U.S. inflation from $t-1$ to t

Therefore the real interest rate gap is $R_t - (k + aUSR_t)$. To obtain a nominal interest rate gap, an expected-inflation gap is added. The resulting nominal interest rate gap is

$$RGAP_t = R_t - (k + aUSR_t) + E[inflation_t] - inflation_{ss}, \quad (A4)$$

where expected inflation is defined as above, and steady-state inflation is assumed to be the average inflation rate for the previous 10 years. In 1993Q1 the steady-state inflation rate shifts to 2 per cent and stays at that level.

Step 3: The forecasting model

Equation 1: *MI*

$$\Delta MI_t = \Gamma_1(L) \begin{bmatrix} \Delta MI_t \\ \Delta CPI_t \\ \Delta Y_t \\ \Delta RON_t \end{bmatrix} + D_1 Z_t + \alpha_1 MGAP_{t-1}, \quad (A5)$$

where

$$Z_t = [\text{constant, output gap}_{t-1}, \Delta \log(\text{exchange rate}) \text{ from } t \text{ to } t-3, \\ \Delta USCP90_t \text{ rate, } D80b * \Delta NPN_t, MONPOL_{t-1}]$$

$MONPOL_{t-1} = 0$ for 1987Q4 and before, the 4-quarter inflation rate less target inflation thereafter. Target inflation is 3 per cent from 1988Q1 to 1992Q4, then declines to 2 per cent in 1995Q4 and stays at that level.

$MGAP_{t-1}$ = the money gap derived above in step 1

Equation 2: *Price*

$$\Delta CPI_t = \Gamma_2(L) \begin{bmatrix} \Delta MI_t \\ \Delta CPI_t \\ \Delta Y_t \\ \Delta RON_t \end{bmatrix} + D_1 Z_t + \alpha_1 MGAP_{t-1}, \quad (A6)$$

where

$$Z_t = [\text{constant, output gap}_{t-1}, \Delta \log(\text{exchange rate}) \text{ from } t \text{ to } t-3, \\ \Delta USCP90_t \text{ rate, } D80b * \Delta NPN_t, D80a, DPOLICY]$$

$DPOLICY = 0$ for 1992Q4 and before, increases to 1 in 1999Q4 and after

$MGAP_{t-1}$ = the money gap derived in step 1

The $DPOLICY$ shift dummy is introduced as a permanent shift dummy in this price equation to represent a shift to a new lower steady-state inflation rate. The equation is restricted in such a manner that it yields a steady-state inflation rate of 2 per cent.

Equation 3: Output

$$\Delta Y_t = \Gamma_3(L) \begin{bmatrix} \Delta MI_t - \Delta CPI_t \\ \Delta Y_t \\ spread_t \end{bmatrix} + D_1 Z_t + \alpha_1 MGAP_{t-1}, \quad (A7)$$

where

$spread_t$ = overnight rate – 10-year-and-over bond rate from QPM

Z_t = [constant, output gap $_{t-1}$, $\Delta USCP90_t$ rate, $D80b * \Delta NPN_t$, $D91$, $D89$]

$D91$ = 0 for 1990Q4 and before and 1 thereafter

$D89$ = 0 prior to 1989Q1, 1 between 1989Q1 and 1996Q2, and 0 thereafter

The equation was restricted so that the steady-state output growth rate is 2.3 per cent and so that the coefficients on prices have the opposite sign but same magnitude as those on money (real money growth, rather than nominal money growth is used in the equation).

Equation 4: The overnight rate

$$\Delta RON_t = \Gamma_4(L) \begin{bmatrix} \Delta MI_t \\ \Delta CPI_t \\ \Delta Y_t \\ \Delta RON_t \end{bmatrix} + D_2 Z_t + \alpha_2 MGAP_{t-1} + \gamma_1 RGAP_{t-1} + \gamma_2 UIP_{t-1}, \quad (A8)$$

where

Z_t = [constant, output gap $_{t-1}$, $\Delta \log(\text{exchange rate})$ at t , $\Delta USCP90_t$ rate from t to $t - 3$, $D80b * \Delta NPN_t$, $MONPOL_{t-1}$]

$RGAP_{t-1}$ = the interest rate gap derived above in step 2

UIP_{t-1} = deviation from uncovered interest rate parity when UIP is defined as

$$UIP_t = RON_t - (usc90_t + 400(lforex_{t+1} - lforex_t) + k + (a - 1)USR_t), \quad (A9)$$

where a and k are from equation (A4) above and

$lforex$ = log level of Can\$/US\$ exchange rate.

Equation 5: Relative purchasing power parity

$$RPPP_t = \Gamma_3(L) \left[\Delta M1_t \Delta Y_t \Delta RON_t RPPP_t \right] + D_2 Z_t + \gamma_2 RGAP_{t-1}, \quad (A10)$$

where

$$RPPP_t = \Delta SPOT - \Delta CPI + \Delta USCPI$$

$$Z_t = [\text{constant, output gap}_{t-1}, \Delta USCP90_t \text{ rate from } t \text{ to } t-2, D60Q1, D73 * RPPP_{t-1}, \Delta \text{lpcom from } t \text{ to } t-2]$$

$D60(Q1)$ = one-period dummy with a value of 1 in 1960Q1

$D73$ = a permanent shift dummy with a value of 1 from 1973Q1 and 0 before

lpcom = log level of commodity prices

$SPOT$ = log level of spot Can\$/US\$ exchange rate

Equation 6: The change in non-personal notice deposits

An AR(4) with constant shift dummies $D80a$, $D87Q3$, where

$D87Q3$ = 1 from 1987Q3 and 0 before.

Equation 7: The U.S. inflation rate

An unrestricted AR(4) on the quarter-over-quarter U.S. inflation rate.

Equation 8: The U.S. 90-day real rate

An AR(2) on the U.S. 90-day real rate with constant shift dummies for 1973Q1 to 1979Q4 and 1981Q1 to 1986Q1.