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# A model of working capital with idiosyncratic production risk and firm failure*

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**Abstract**

This paper is a contribution to the literature on possible pro-cyclical effects of capital rules under Basil 2 capital regulations. The addition of both idiosyncratic uncertainty and risk averse managers to a cash-in-advance model with financial intermediaries that finance working capital changes the way that the interest rate paid by borrowers responds to technology shocks and the correlation of these interest rates with output. Without idiosyncratic uncertainty, the interest rate for working capital borrowed by the firms is positively, and highly, correlated with output. Once idiosyncratic technology shocks are added, the correlation between the interest rate for working capital borrowed by the firms and output becomes highly negatively correlated. In stationary states, increases in the idiosyncratic shocks cause the risk averse firm managers to produce at levels where average total costs are well below average output, providing them with a partial cushion against a very low idiosyncratic shock. When absolute risk aversion is high, the effect is to dampen the effects of monetary shocks in the economy with idiosyncratic risk, but when absolute risk aversion is low, the effects of monetary shocks on real variables is higher, the larger the idiosyncratic risk.

## 1 Introduction

There has been a substantial recent literature on the relationship between business cycles, interest rates, and bank capital. Part of the interest in this theme has been generated by the new Basel II rules for bank capital and the fear that these rules will have the effect of increasing the output cycle. To be able to deal with these questions, it is necessary to have a model that can generate the

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counter-cyclical relationship between interest rates and output, and between output and capital (or bank loan loss provisions).

This paper presents a simple model in which a financial intermediary exists to provide working capital for firms. The working capital borrowed at the beginning of a period pays for labor input in that period and is paid back at the end of the period. However, firms are subject to idiosyncratic technology shocks that is know only after the production process has occurred and that will cause some of the firms to fail. The idiosyncratic technology shocks that we add are uniformly distributed is a symmetric band around one.

In order to have a relatively small number of firms fail, we need the managers of the firms to be risk averse. If the managers were risk neutral, half of the firms would fail each period. This seems excessive. The problem is how to make managers risk averse without adding too much heterogeneity to the model. This we do by making managers like everyone else except that they get non-pecuniary rewards (or punishments) for the profits (or losses) that their firm makes. While non-pecuniary rewards are not all that common in the economic literature, they have been used in models like Diamond [1] for the managers of banks.

One question that the model faces is how much risk aversion should we give the managers. If they are too risk averse, the economy pays very high costs in terms of average output and output variance can decline as a function of the idiosyncratic risk. With less risk aversion (values for absolute risk aversion less than one), we find example economies where idosyncratic risk increases the variance in output and where interest rates and loan loss provisions move opposite output in response to technology shocks.

Since the risk averse firm managers are the most complicated element of the model, we deal with them first, followed by the financial intermediaries and then the rather standard household sector. We find stationary states, log-linearize the model and study the standard error and correlation characteristics of simulations.

## 2 Firms

One wants firm managers to be risk adverse. However, the nonlinearity that come from the agent's optimization problem makes aggregating very difficult if individuals have different incomes. One way to handle this is to allow money or goods incomes to be the same and to put the concavity that we need for risk aversion as a non-pecuniary result in the utility function. Firm managers get wages as the rest of the workers, earn returns on their capital holdings and bank deposits just as everyone else, but they get an additional welfare gain or loss as a function of the profits of the firm that they manage. Devices of this form have been used before, for example, in Diamond [1], for the contracts between lenders and firm managers and between depositors and bank managers.

A small fraction of the population are managers of firms. They supply labor to the market for wages and rent capital just as everyone else in the economy does but they also receive (additively) utility from how they manage their firm. Their utility function is of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln c_t^i + B h_t^i + G(\pi_t^k) \right],$$

where

$$\pi_t^k = \lambda_t \varphi_t^k \left(k_t^k\right)^{\theta} \left(h_t^k\right)^{1-\theta} - r_t k_t^k - r_t^f w_t h_t^k,$$

where $\varphi_t^k$ is an idiosyncratic shock coming from a uniform distribution from $\left[\varphi^l, \varphi^u\right]$ and $r_t^f$ is the gross interest rate paid by the firms for borrowing working capital to pay labor. The aggregate technology shock $\lambda_t$ is known at the beginning of period $t$ but the idiosyncratic shock , $\varphi_t^k$, is known only after production has occurred. Note that the managers of firms don't receive the profits from their company, but instead are paid in some utility enhancing (or reducing) way based on the performance of their firm. The function $G(\pi_t^k)$ is concave and increasing in time $t$ profits. Since production decisions are separable from the other decisions and are independent of the individuals income from labor or capital, firms want to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t G(\pi_t^k),$$

subject to

$$\pi_t^k = \lambda_t \varphi_t^k \left(k_t^k\right)^{\theta} \left(h_t^k\right)^{1-\theta} - r_t k_t^k - r_t^f w_t h_t^k.$$

Firms are owned by the families who receive dividends that are the sum of the profits of the firms that make profits. Firms borrow from the financial intermediary to finance their labor bill and pay off capital rentals with earnings before they pay back the loan for working capital. The idiosyncratic shocks are independent across periods for each firm and there is no memory of the managers previous performance. For these reasons, in each period a firm maximizes

$$E_t G(\lambda_t \varphi_t^k \left(k_t^k\right)^{\theta} \left(h_t^k\right)^{1-\theta} - r_t k_t^k - r_t^f w_t h_t^k),$$

where the expectation is over the idiosyncratic shock $\varphi_t^k$.

When the distribution of idiosyncratic shocks are uniform, assuming that $\varphi_t^j$ is distributed uniformly over $\left[\varphi^l, \varphi^u\right]$, this problem can be written as

$$\max_{k_t^k, h_t^k} \frac{1}{\varphi^u - \varphi^l} \int_{\varphi^l}^{\varphi^u} G(\lambda_t \varphi_t^j \left(k_t^k\right)^{\theta} \left(h_t^k\right)^{1-\theta} - r_t k_t^k - r_t^f w_t h_t^k) dj.$$

The first order conditions for this problem (after removing the constant term $\frac{1}{\varphi^u - \varphi^l}$) are, for capital,

$$\begin{aligned} & \int_{\varphi^l}^{\varphi^u} G'(\lambda_t \varphi_t^j \left(k_t^k\right)^{\theta} \left(h_t^k\right)^{1-\theta} - r_t k_t^k - r_t^f w_t h_t^k) \theta \lambda_t \varphi_t^j \left(k_t^k\right)^{\theta-1} \left(h_t^k\right)^{1-\theta} dj \\ = \quad & r_t \int_{\varphi^l}^{\varphi^u} G'(\lambda_t \varphi_t^j \left(k_t^k\right)^{\theta} \left(h_t^k\right)^{1-\theta} - r_t k_t^k - r_t^f w_t h_t^k) dj, \end{aligned}$$

and, for labor,

$$\begin{aligned}
& \int_{\varphi^l}^{\varphi^u} G'(\lambda_t \varphi_t^j \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta} - r_t k_t^k - r_t^f w_t h_t^k) \left(1-\theta\right) \lambda_t \varphi_t^j \left(k_t^k\right)^\theta \left(h_t^k\right)^{-\theta} dj \\
= \; & r_t^f w_t \int_{\varphi^l}^{\varphi^u} G'(\lambda_t \varphi_t^j \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta} - r_t k_t^k - r_t^f w_t h_t^k) dj.
\end{aligned}$$

A $G(\cdot)$ function that has characteristics that we might want is

$$G(x) = \eta \left(1 - \exp\left(-\alpha x\right)\right).$$

Using this function, $G'(\cdot)$ is

$$G'(x) = \alpha \eta \exp\left(-\alpha x\right)$$

and the explicit version of the first order conditions can be written as

$$\begin{aligned}
& \exp\left(-r_t k_t^k - r_t^f w_t h_t^k\right) \times \\
& \int_{\varphi^l}^{\varphi^u} \exp\left(\varphi_t^j \left(-\alpha \lambda_t \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)\right) \theta \lambda_t \varphi_t^j \left(k_t^k\right)^{\theta-1} \left(h_t^k\right)^{1-\theta} dj \\
= \; & r_t \exp\left(-r_t k_t^k - r_t^f w_t h_t^k\right) \int_{\varphi^l}^{\varphi^u} \exp\left(\varphi_t^j \left(-\alpha \lambda_t \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)\right) dj,
\end{aligned}$$

and

$$\begin{aligned}
& \exp\left(-r_t k_t^k - r_t^f w_t h_t^k\right) \times \\
& \int_{\varphi^l}^{\varphi^u} \exp\left(\varphi_t^j \left(-\alpha \lambda_t \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)\right) \left(1-\theta\right) \lambda_t \varphi_t^j \left(k_t^k\right)^{\theta} \left(h_t^k\right)^{-\theta} dj \\
= \; & r_t^f w_t \exp\left(-r_t k_t^k - r_t^f w_t h_t^k\right) \int_{\varphi^l}^{\varphi^u} \exp\left(\varphi_t^j \left(-\alpha \lambda_t \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)\right) dj.
\end{aligned}$$

These simplify to

$$\begin{aligned}
& \theta \lambda_t \left(k_t^k\right)^{\theta-1} \left(h_t^k\right)^{1-\theta} \int_{\varphi^l}^{\varphi^u} \varphi_t^j \exp\left(\varphi_t^j \left(-\alpha \lambda_t \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)\right) dj \\
= \; & r_t \int_{\varphi^l}^{\varphi^u} \exp\left(\varphi_t^j \left(-\alpha \lambda_t \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)\right) dj,
\end{aligned}$$

and

$$\begin{aligned}
& \left(1-\theta\right) \lambda_t \left(k_t^k\right)^{\theta} \left(h_t^k\right)^{-\theta} \int_{\varphi^l}^{\varphi^u} \varphi_t^j \exp\left(\varphi_t^j \left(-\alpha \lambda_t \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)\right) dj \\
= \; & r_t^f w_t \int_{\varphi^l}^{\varphi^u} \exp\left(\varphi_t^j \left(-\alpha \lambda_t \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)\right) dj.
\end{aligned}$$

Taking the integrals gives

$$\frac{\left.\frac{\lambda_t \varphi_t^j \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}}{\exp\left(\alpha\lambda_t \varphi_t^j \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)}\right|_{\varphi^l}^{\varphi^u}}{\left.\frac{1}{\exp\left(\alpha\lambda_t \varphi_t^j \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)}\right|_{\varphi^l}^{\varphi^u}} + \frac{1}{\alpha} = \frac{r_t k_t^k}{\theta},$$

and

$$\frac{\left.\frac{\lambda_t \varphi_t^j \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}}{\exp\left(\alpha\lambda_t \varphi_t^j \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)}\right|_{\varphi^l}^{\varphi^u}}{\left.\frac{1}{\exp\left(\alpha\lambda_t \varphi_t^j \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}\right)}\right|_{\varphi^l}^{\varphi^u}} + \frac{1}{\alpha} = \frac{r_t^f w_t h_t^k}{(1-\theta)}.$$

This implies that in equilibrium, one of the usual cost minimization conditions applies,

$$\frac{r_t k_t^k}{\theta} = \frac{r_t^f w_t h_t^k}{(1-\theta)}.$$

Define $y_t^j = \lambda_t \varphi^j \left(k_t^k\right)^\theta \left(h_t^k\right)^{1-\theta}$, for $j = l, u$. Then the above condition is simply

$$\frac{y_t^u e^{\alpha y_t^l} - y_t^l e^{\alpha y_t^u}}{e^{\alpha y_t^l} - e^{\alpha y_t^u}} + \frac{1}{\alpha} = \frac{r_t k_t^k}{\theta} = \frac{r_t^f w_t h_t^k}{(1-\theta)}.$$

The left hand side of this equation can be further simplified to

$$\begin{aligned}
\frac{y_t^u e^{\alpha y_t^l} - y_t^l e^{\alpha y_t^u}}{e^{\alpha y_t^l} - e^{\alpha y_t^u}} + \frac{1}{\alpha} &= y_t^l \frac{\frac{\varphi^u}{\varphi^l} - e^{\alpha\left(y_t^u - y_t^l\right)}}{1 - e^{\alpha\left(y_t^u - y_t^l\right)}} + \frac{1}{\alpha} \\
&= y_t^l \frac{\frac{\varphi^u}{\varphi^l} - e^{\alpha\left(\varphi^u - \varphi^l\right) E y_t}}{1 - e^{\alpha(\varphi^u - \varphi^l) E y_t}} + \frac{1}{\alpha} \\
&= y_t^l \frac{\frac{\varphi^u}{\varphi^l} - e^{\alpha\left(\varphi^u - \varphi^l\right) y_t}}{1 - e^{\alpha(\varphi^u - \varphi^l) y_t}} + \frac{1}{\alpha},
\end{aligned}$$

where, since the uniform distribution on the idiosyncratic shock is $E(\varphi_t) = 1$, then $E y_t = \lambda_t k_t^\theta h_t^{1-\theta} = y_t$, where $y_t$ is the aggregate output in period $t$.

One can interpret both $r_t k_t^k/\theta$ and $r_t^f w_t h_t^k/(1-\theta)$ as the total costs of production and, from the above calculations, this is equal to

$$TC_t = y_t^l \frac{\frac{\varphi^u}{\varphi^l} - e^{\alpha\left(\varphi^u - \varphi^l\right) y_t}}{1 - e^{\alpha(\varphi^u - \varphi^l) y_t}} + \frac{1}{\alpha}.$$

All firms, ex ante the realization of the idiosyncratic productivity shock, are identical and expect to produce the same amount and all have the same total costs. Because of the idiosyncratic shocks, a fraction

$$\int_{\varphi^l}^{\frac{TC_t}{y_t}} \frac{1}{(\varphi^u - \varphi^l)} di = \frac{TC_t - \varphi^l y_t}{(\varphi^u - \varphi^l)\, y_t}$$

of the firms will not be able to meet their costs and will be closed. The total losses that these firms will suffer in period $t$ are equal to

$$TL_t = \int_{\varphi^l}^{\frac{TC_t}{y_t}} \frac{TC_t - iy_t}{(\varphi^u - \varphi^l)} di = \frac{\left(TC_t - \varphi^l y_t\right)^2}{2\,(\varphi^u - \varphi^l)\, y_t}.$$

The profits that the successful firms make are distributed as lump sum payments to the households as dividends, $D_t$. These dividends are equal to

$$D_t = \int_{\frac{TC_t}{y_t}}^{\varphi^u} \frac{iy_t - TC_t}{(\varphi^u - \varphi^l)} di = \frac{(\varphi^u y_t - TC_t)^2}{2\,(\varphi^u - \varphi^l)\, y_t}.$$

# 3 Financial intermediaries

At the beginning of each period, financial intermediaries receive deposits from households and lend these to the firms for working capital. At the end of the period, the firms that are successful pay back the loan and financial intermediary takes over the remaining assets of the unsuccessful firms. These assets are equal to the value of the goods that were sold minus the rentals paid for the use of capital. From the section on **Firms**, we know that these real losses total

$$TL_t = \frac{\left(TC_t - \varphi^l y_t\right)^2}{2\,(\varphi^u - \varphi^l)\, y_t}.$$

Since the firms only lend working capital to cover the wage bill, the amount that they will lend to the firms is equal to $P_t w_t K_t$. The financial intermediaries borrow this amount from the households at a gross interest rate equal to $r_t^d$. The rate that they lend to the firms, $r_t^f$, must be just enough so that, with the aggregate technology shock known and given the distribution of the idiosyncratic shocks, they will end up with zero profits. This zero profit restriction results in a nominal budget constraint of

$$r_t^d N_t = r_t^f P_t w_t H_t - P_t T L_t.$$

The interest rate paid by firms needs to be high enough to cover the expected (and realized) losses so, from the budget constraint of the financial intermediary and the equilibrium condition in the financial market,

$$N_t + (1-\rho)\,(g_t - 1)\, M_{t-1} = P_t w_t H_t,$$

we get

$$r_t^f = r_t^d \frac{N_t}{P_t w_t H_t} + \frac{TL_t}{w_t H_t}.$$

The term $(1-\rho)\,(g_t - 1)\, M_{t-1}$ is the portion of new money issue that goes directly to the financial intermediary. The importance of the choice of $\rho$ for the effect of inflation in a cash in advance economy is studied in McCandless [3].

# 4 Households

The household side of the economy is pretty standard. Households maximize discounted utility subject to a flow budget constraint and a cash-in-advance constraint. In addition, we assume that labor is indivisible along the lines of Hansen [2]. This means that a representative household maximizes

$$\sum_{t=0}^{\infty} \beta^t \left[ \ln(c_t) + \frac{h_t}{h_0} A \ln(1-h_0) \right],$$

where $h_t/h_0$ is the probability that this family will be required to supply $h_0$ units of labor to the market. The budget constraint, measured in real terms, is

$$\frac{m_t}{P_t} + k_{t+1} = w_t h_t + r_t k_t + (1-\delta)k_t + d_t + \frac{r_t^d n_t}{P_t},$$

and the cash-in-advance constraint, written in nominal terms, is

$$P_t c_t = m_{t-1} + \rho(g_t - 1) M_{t-1} - n_t,$$

where $m_{t-1}$ is money carried over from the previous period, $n_t$ are the deposits in the financial intermediary, $d_t$ is the family's share of profits from the firms which are paid as lump sum transfers, and the rest of the terms are standard. The term $\rho(g_t - 1) M_{t-1}$ is the portion of new money issue that goes directly to households and can be used immediately for consumption or for savings in the financial intermediary.

The first order conditions that come from the family's decision problem are

$$\begin{aligned} \frac{1}{w_t} &= E_t \frac{\beta}{w_{t+1}} \left( r_{t+1} + (1-\delta) \right), \\ w_t &= -B r_t^d c_t, \\ \frac{1}{r_t^d} &= \beta E_t \frac{P_t c_t}{P_{t+1} c_{t+1}}, \end{aligned}$$

where we define

$$B \equiv \frac{A \ln(1-h_0)}{h_0}.$$

# 5 Equilibrium conditions

Given that all families are alike and are of unit mass, the aggregate variables (expressed in upper case letters) are equal to

$$\begin{aligned} H_t &= h_t, \\ M_t &= m_t, \\ K_t &= k_t, \\ D_t &= d_t, \\ C_t &= c_t, \quad \text{and} \\ N_t &= n_t. \end{aligned}$$

From the side of the firm, we add

$$Y_t = y_t.$$

Market clearing in the working capital markets imply that

$$N_t + (1-\rho)(g_t - 1) M_{t-1} = P_t w_t H_t.$$

# 6 The full model

The full model contains the aggregate variables $Y_t$, $C_t$, $K_{t+1}$, $H_t$, $D_t$, $w_t$, $r_t$, $r_t^d$, $r_t^f$, $M_t$, $N_t$, $P_t$, $TC_t$, $TL_t$, the technology shock, $\lambda_t$, and the money growth shock, $g_t$. The household side of the full model written in terms of the aggregate variables is given by the first order conditions,

$$\begin{aligned}
\frac{1}{w_t} &= E_t \frac{\beta}{w_{t+1}} \left( r_{t+1} + (1-\delta) \right), \\
w_t &= -B r_t^d C_t, \\
\frac{1}{r_t^d} &= \beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}},
\end{aligned}$$

the flow budget constraint,

$$\frac{M_t}{P_t} + K_{t+1} = w_t H_t + r_t K_t + (1-\delta) K_t + D_t + \frac{r_t^d N_t}{P_t},$$

and the cash-in-advance constraint,

$$P_t C_t = M_{t-1} + \rho (g_t - 1) M_{t-1} - N_t.$$

The equations that come from the firm's decisions are the total cost equations,

$$TC_t = \varphi^l Y_t \frac{\frac{\varphi^u}{\varphi^l} - e^{\alpha\left(\varphi^u - \varphi^l\right) Y_t}}{1 - e^{\alpha\left(\varphi^u - \varphi^l\right) Y_t}} + \frac{1}{\alpha},$$

the equations for the factor market demands,

$$TC_t = \frac{r_t K_t}{\theta} = \frac{r_t^f w_t H_t}{(1-\theta)},$$

the total loss equations,

$$TL_t = \frac{\left(TC_t - \varphi^l Y_t\right)^2}{2\left(\varphi^u - \varphi^l\right) Y_t},$$

and the equation for dividends,

$$D_t = \frac{\left(\varphi^u Y_t - TC_t\right)^2}{2\left(\varphi^u - \varphi^l\right) Y_t}.$$

The aggregate production function is

$$Y_t = \lambda_t K_t^{\theta} H_t^{1-\theta}.$$

From the financial intermediary, we get the equation that determines the spread on interest rates,

$$r_t^f = r_t^d \frac{N_t}{P_t w_t H_t} + \frac{TL_t}{w_t H_t},$$

and the equilibrium conditions for the credit market,

$$N_t + (1-\rho)(g_t - 1) M_{t-1} = P_t w_t H_t.$$

Money grows by the rule,

$$M_t = g_t M_{t-1}.$$

The stochastic processes for technology and money growth are, respectively,

$$\ln(\lambda_{t+1}) = \gamma^{\lambda} \ln(\lambda_t) + \varepsilon_{t+1}^{\lambda},$$

and

$$\ln(g_{t+1}) = \gamma^{g} \ln(g_t) + \varepsilon_{t+1}^{g},$$

where the error terms $\varepsilon_{t+1}^i$, $i = \lambda, g$, are independent and each is distributed $\varepsilon_{t+1}^i \sim N\left(0, \sigma_i^2\right)$.

# 7 The stationary state

A stationary state is defined as a constant set of values for the real variables, $\overline{r}$, $\overline{r}^d$, $\overline{r}^f$, $\overline{Y}$, $\overline{C}$, $\overline{K}$, $\overline{H}$, $\overline{w}$, $\overline{TC}$, $\overline{TL}$, $\overline{D}$, constant values for nominal variables divided by the price level, $M_t/P_t = \overline{M/P}$, $N_t/P_t = \overline{N/P}$, for the inflation rate, $\overline{\pi} = P_{t+1}/P_t$, and where the stochastic variables, $\overline{\lambda} = 1$ and $\overline{g}$, are constants. The stationary state values for a variable $X_t$ are designated by $\overline{X}$. The stationary state version of the fourteen equations of the model is

$$\overline{r} = \frac{1}{\beta} - 1 + \delta, \tag{1}$$

$$\overline{w} = -B\overline{r}^d\overline{C}, \tag{2}$$

$$\overline{r}^d = \frac{\overline{\pi}}{\beta}, \tag{3}$$

$$\overline{M/P} = \overline{w}\overline{H} + (\overline{r} - \delta)\overline{K} + \overline{D} + \overline{r}^d\overline{N/P}, \tag{4}$$

$$\overline{C} = \frac{\overline{M/P}}{\overline{\pi}} + \rho(\overline{g} - 1)\frac{\overline{M/P}}{\overline{\pi}} - \overline{N/P}, \tag{5}$$

$$\overline{TC} = \varphi^l \overline{Y} \frac{\frac{\varphi^u}{\varphi^l} - e^{\alpha\left(\varphi^u - \varphi^l\right)\overline{Y}}}{1 - e^{\alpha(\varphi^u - \varphi^l)\overline{Y}}} + \frac{1}{\alpha}, \tag{6}$$

$$\theta\overline{TC} = \overline{r}\overline{K}, \tag{7}$$

$$(1-\theta)\overline{TC} = \overline{r}^f\overline{w}\overline{H}, \tag{8}$$

$$
\begin{aligned}
\overline{TL} &= \frac{\left(\overline{TC} - \varphi^l \overline{Y}\right)^2}{2\left(\varphi^u - \varphi^l\right)\overline{Y}}, & (9)\\
\overline{D} &= \frac{\left(\varphi^u \overline{Y} - \overline{TC}\right)^2}{2\left(\varphi^u - \varphi^l\right)\overline{Y}}, & (10)\\
\overline{Y} &= \overline{K}^{\theta}\overline{H}^{1-\theta} & (11)\\
\overline{r}^f &= \overline{r}^d \frac{\overline{N/P}}{\overline{w}\overline{H}} + \frac{\overline{TL}}{\overline{w}\overline{H}}, & (12)\\
\overline{w}\overline{H} &= \overline{N/P} + (1-\rho)\left(\overline{g}-1\right)\frac{\overline{M/P}}{\overline{\pi}} & (13)\\
\overline{\pi} &= \overline{g}. & (14)
\end{aligned}
$$

The values for $\overline{r}$, $\overline{r}^d$, and $\overline{\pi}$ are know instantly from the parameters. For a range of $\overline{Y}$, use equation 6 to find $\overline{TC}$. From this, $\overline{K}$ can be determined using equation 7. Then the production function, equation 11, can be used to determine $\overline{H}$. Using equation 12 written as

$$\overline{r}^f \overline{w}\overline{H} = \overline{r}^d \overline{N/P} + \overline{TL},$$

and equation 8, $\overline{N/P}$ can be found. Putting equation 13 into equation 4, gives

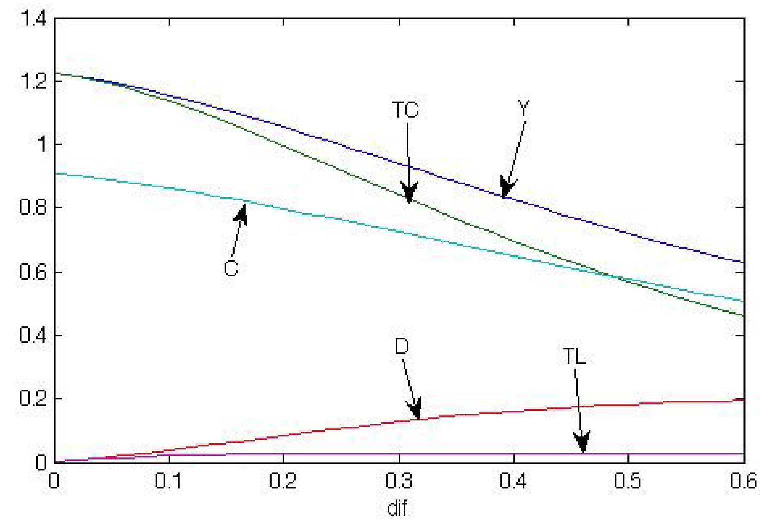
$$\overline{M/P}\left(1 - \frac{(1-\rho)\left(\overline{g}-1\right)}{\overline{\pi}}\right) = (\overline{r} - \delta)\overline{K} + \overline{D} + \left(\overline{r}^d + 1\right)\overline{N/P},$$

from which we can find $\overline{M/P}$. With this known, equation 13 can be used to find $\overline{w}$, and, using equation 2, we get the values for $\overline{C}$. A second way of calculating $\overline{C}$ comes from using equation 5. Where these two $\overline{C}$'s are equal is the equilibrium $\overline{Y}$ (and all the corresponding values). With these values determined, $\overline{r}^f$ can be found using equation 8.

Given the complexity of equation 6, a stationary state cannot be found analytically but must be calculated. For an economy where $\alpha = 4$, $\beta = .99$, $\delta = .025$, $\theta = .36$, $A = 1.72$, and $h_0 = .572$, (so $B = -2.5805$), the values of the variables in the stationary states for values of $\varphi^u = 1 + dif$ and $\varphi^l = 1 - dif$ for $dif \in [0, .6]$ are shown in Figures 1 and 2, for $\overline{g} = 1$. Figures 3 and 4 show the stationary state values for $\rho = 1$ and $\overline{g} = 1.19$, representing an annual inflation rate of 100%. Recall that when $\rho = 1$, the new money issue goes directly to the households. In this case, the new money issue operates as a tax, reducing production and other values measured in terms of goods and raising the interest rate paid by the firms. When $\rho = 0$, the new money issue goes to the financial intermediary. The values for output, consumption, total costs, total losses, and dividends are similar to those in the model without inflation but slightly higher. Figure 5 shows the stationary state values for the other variables with an annual inflation rate of 100% where the money issue goes to the financial intermediaries. Notice that the interest rate paid by the firms first rises and then declines with the increase variance of the idiosyncratic production shock.

wc y tc c d tl

g0

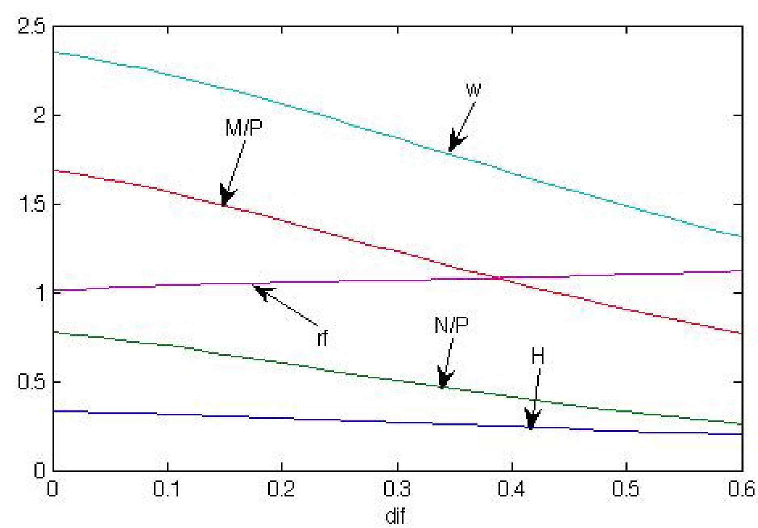



1.jpg

Figure 1: Stationary state values for $\overline{g} = 1$

wc h np mp w

rf

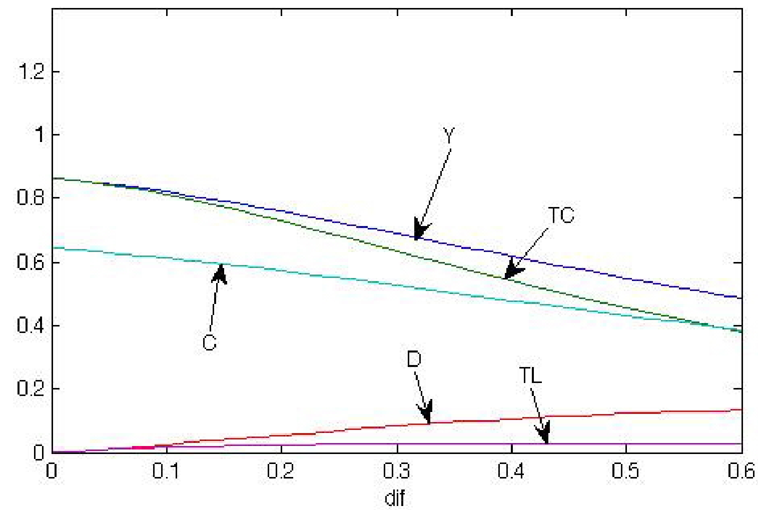



2.jpg

Figure 2: More stationary state values for $\overline{g} = 1$

wc y tc c d tl

g119

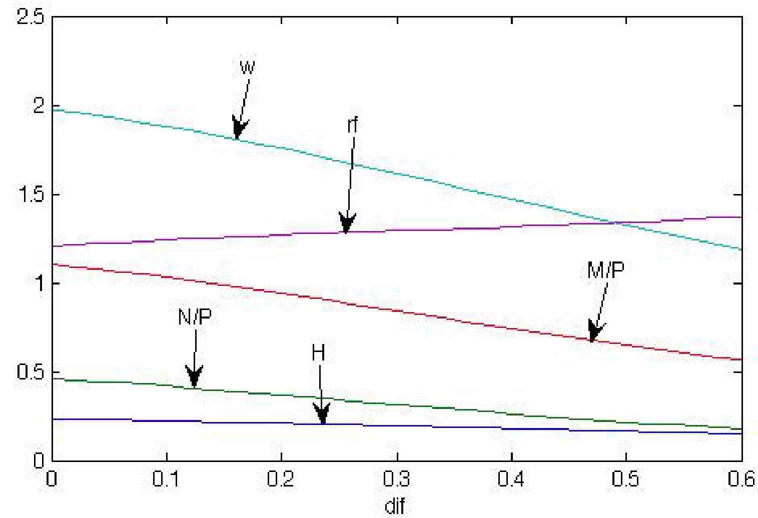



$_3.jpg$

Figure 3: Stationary state values for $\overline{g} = 1.19$

wc h np mp w rf

g119

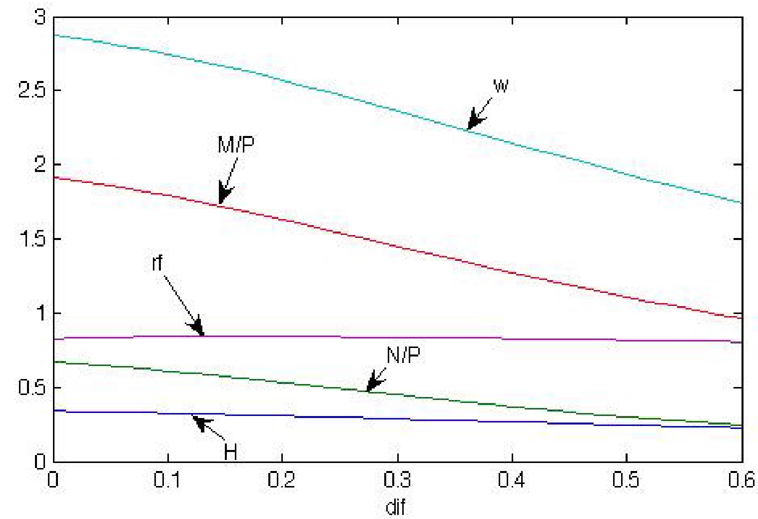



$_4.jpg$

Figure 4: More stationary state values for $\overline{g} = 1.19$

wc h np mp w rf g119

roe0




5.jpg

Figure 5: More stationary state values for g=1.19 and $\rho = 0$

# 8 Log-linearization of the model

The log-linear version of the model is written in terms of the variables $\widetilde{X}_t$, where

$$\widetilde{X}_t = \ln(X_t) - \ln(\overline{X}),$$

or

$$X_t = \overline{X} e^{\widetilde{X}_t},$$

and $\overline{X}$ is the stationary state value of the variable. The fourteen variables of the model are $r_t$, $\widetilde{r}_t^d$, $\widetilde{r}_t^f$, $\widetilde{Y}_t$, $\widetilde{C}_t$, $\widetilde{K}_t$, $\widetilde{H}_t$, $\widetilde{w}_t$, $\widetilde{M}_t$, $\widetilde{N}_t$, $\widetilde{P}_t$, $\widetilde{TC}_t$, $\widetilde{TL}_t$, and $\widetilde{D}_t$. The two stochastic variables are $\widetilde{\lambda}_t$ and $\widetilde{g}_t$. Using the method of Uhlig [4], we find that the log-linear version of the model is

$$\begin{aligned}
0 &= \widetilde{w}_t - E_t \widetilde{w}_{t+1} + \beta \overline{r} E_t \widetilde{r}_{t+1}, \\
0 &= \widetilde{r}_t^d - \widetilde{w}_t + \widetilde{C}_t, \\
0 &= \widetilde{w}_t + \widetilde{P}_t - E_t \widetilde{P}_{t+1} - E_t \widetilde{C}_{t+1}, \\
0 &= \overline{M/P}\widetilde{M}_t + \left[\overline{r}^n \overline{N/P} - \overline{M/P}\right] \widetilde{P}_t + \overline{K}\widetilde{K}_{t+1} - \overline{w}\overline{H}(\widetilde{w}_t + \widetilde{H}_t) \\
&\quad -\overline{r}\overline{K}\widetilde{r}_t - (\overline{r} + 1 - \delta)\overline{K}\widetilde{K}_t - \overline{D}\widetilde{D}_t - \overline{r}^d \overline{N/P}\widetilde{N}_t - \overline{r}^d \overline{N/P}\widetilde{r}_t^d,
\end{aligned}$$

$$
\begin{aligned}
0 &= \overline{C}\left(\widetilde{P}_t + \widetilde{C}_t\right) - (1 + \rho\overline{g} - \rho)\frac{\overline{M/P}}{\overline{g}}\widetilde{M}_{t-1} - \rho\overline{M/P}\widetilde{g}_t + \overline{N/P}\widetilde{N}_t, \\
0 &= \overline{TC}\widetilde{TC}_t - \left(\frac{\varphi^l\overline{Y}\left(\frac{\varphi^u}{\varphi^l} - e^{\alpha\left(\varphi^u-\varphi^l\right)\overline{Y}}\right)}{\left(1 - e^{\alpha\left(\varphi^u-\varphi^l\right)\overline{Y}}\right)} + \frac{e^{\alpha\left(\varphi^u-\varphi^l\right)\overline{Y}}\alpha\left(\varphi^u - \varphi^l\right)^2\overline{Y}^2}{\left(1 - e^{\alpha\left(\varphi^u-\varphi^l\right)\overline{Y}}\right)^2}\right)\widetilde{Y}_t, \\
0 &= \widetilde{TC}_t - \widetilde{r}_t - \widetilde{K}_t,
\end{aligned}
$$

$$
\begin{aligned}
0 &= \widetilde{TC}_t - \widetilde{r}^f_t - \widetilde{w}_t - \widetilde{H}_t, \\
0 &= \overline{TL}\widetilde{TL}_t - \frac{\overline{TC}\left(\overline{TC} - \varphi^l\overline{Y}\right)}{\left(\varphi^u - \varphi^l\right)\overline{Y}}\widetilde{TC}_t + \frac{\overline{TC}^2 - \left(\varphi^l\overline{Y}\right)^2}{2\left(\varphi^u - \varphi^l\right)\overline{Y}}\widetilde{Y}_t, \\
0 &= \overline{D}\widetilde{D}_t + \frac{\overline{TC}\left(\varphi^u\overline{Y} - \overline{TC}\right)}{\left(\varphi^u - \varphi^l\right)\overline{Y}}\widetilde{TC}_t - \frac{\left(\varphi^u\overline{Y}\right)^2 - \overline{TC}^2}{2\left(\varphi^u - \varphi^l\right)\overline{Y}}\widetilde{Y}_t \\
0 &= \widetilde{Y}_t - \widetilde{\lambda}_t - \theta\widetilde{K}_t - (1-\theta)\widetilde{H}_t, \\
0 &= \overline{r}^f\widetilde{r}^f_t - \frac{\overline{r}^d\overline{N/P}}{\overline{w}\overline{H}}\left(\widetilde{r}^d_t + \widetilde{N}_t - \widetilde{P}_t\right) - \frac{\overline{TL}}{\overline{w}\overline{H}}\widetilde{TL}_t + \overline{r}^f\left(\widetilde{w}_t + \widetilde{H}_t\right), \\
0 &= \overline{N/P}\widetilde{N}_t - \overline{w}\overline{H}\left(\widetilde{w}_t + \widetilde{H}_t\right) - \left[(1-\rho)\left(1 - \frac{1}{\overline{g}}\right)\overline{M/P} + \overline{N/P}\right]\widetilde{P}_t \\
&\quad + (1-\rho)\left(1 - \frac{1}{\overline{g}}\right)\overline{M/P}\widetilde{M}_{t-1} + (1-\rho)\overline{M/P}\widetilde{g}_t, \\
0 &= \widetilde{M}_t - \widetilde{g}_t - \widetilde{M}_{t-1}.
\end{aligned}
$$

The log-linearized equations for two stochastic variables are

$$\widetilde{\lambda}_{t+1} = \gamma^\lambda\widetilde{\lambda}_t + \varepsilon^\lambda_{t+1},$$

and

$$\widetilde{g}_{t+1} = \gamma^g\widetilde{g}_t + \varepsilon^g_{t+1}.$$

We remove $\widetilde{D}_t$ and $\widetilde{TL}_t$ from the system by substituting them out. This reduces the dimension of the problem and makes the $C$ matrix (below) invertible in those economies where these are equal to zero. Defining the state variables as $x_t = \left[\widetilde{K}_{t+1}, \widetilde{M}_t, \widetilde{P}_t\right]'$, $y_t = \left[\widetilde{r}_t, \widetilde{w}_t, \widetilde{Y}_t, \widetilde{C}_t, \widetilde{H}_t, \widetilde{N}_t, , \widetilde{TC}_t, \widetilde{r}^n_t, \widetilde{r}^f_t\right]'$ as the jump variables, and $z_t = \left[\widetilde{\lambda}_t, \widetilde{g}_t\right]'$ as the stochastic variables, the system can be written as

$$
\begin{aligned}
0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t, \\
0 &= E_t\left[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\right], \\
z_{t+1} &= Nz_t + \varepsilon_{t+1},
\end{aligned}
$$

where

$$A=\begin{bmatrix} 0 & 0 & 0 \\ \overline{K} & \overline{M/P} & \overline{r}^{n}\overline{N/P}-\overline{M/P} \\ 0 & 0 & \overline{C} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\overline{r}^{d}\overline{N/P}}{\overline{w}\overline{H}} \\ 0 & 0 & -\overline{w}\overline{H} \end{bmatrix},$$

$$B=\begin{bmatrix} 0 & 0 & 0 \\ -\left(\overline{r}+1-\delta\right)\overline{K} & 0 & 0 \\ 0 & -\left(1+\rho\overline{g}-\rho\right)\frac{\overline{M/P}}{\overline{g}} & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ -\theta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \overline{w}\overline{H}-\overline{N/P} & 0 \end{bmatrix},$$

defining

$$Z=\frac{\varphi^{l}\overline{Y}\left(\frac{\varphi^{u}}{\varphi^{l}}-e^{\alpha\left(\varphi^{u}-\varphi^{l}\right)\overline{Y}}\right)}{\left(1-e^{\alpha\left(\varphi^{u}-\varphi^{l}\right)\overline{Y}}\right)}+\frac{e^{\alpha\left(\varphi^{u}-\varphi^{l}\right)\overline{Y}}\alpha\left(\varphi^{u}-\varphi^{l}\right)^{2}\overline{Y}^{2}}{\left(1-e^{\alpha\left(\varphi^{u}-\varphi^{l}\right)\overline{Y}}\right)^{2}},$$

$$C=\begin{bmatrix} 0 & -1 & 0 & 1 & 0 \\ -\overline{r}\overline{K} & -\overline{w}\overline{H} & -\frac{\left(\varphi^{u}\overline{Y}\right)^{2}-\overline{TC}^{2}}{2\left(\varphi^{u}-\varphi^{l}\right)\overline{Y}} & 0 & -\overline{w}\overline{H} \\ 0 & 0 & 0 & \overline{C} & 0 \\ 0 & 0 & Z & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -\left(1-\theta\right) \\ 0 & \overline{r}^{f} & \frac{\overline{TC}^{2}-\left(\varphi^{l}\overline{Y}\right)^{2}}{2\overline{w}\overline{H}\left(\varphi^{u}-\varphi^{l}\right)\overline{Y}} & 0 & \overline{r}^{f} \\ 0 & -\overline{w}\overline{H} & 0 & 0 & -\overline{w}\overline{H} \end{bmatrix} \quad .....$$

$$
\left.\begin{matrix}
0 & 0 & 1 & 0 \\
-\overline{r}^{d}\overline{N/P} & \frac{\overline{TC}\left(\varphi^{u}\overline{Y}-\overline{TC}\right)}{(\varphi^{u}-\varphi^{l})\overline{Y}} & -\overline{r}^{d}\overline{N/P} & 0 \\
-\overline{N/P} & 0 & 0 & 0 \\
0 & \overline{TC} & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 \\
-\frac{\overline{r}^{d}\overline{N/P}}{\overline{w}\overline{H}} & -\frac{\overline{TC}\left(\overline{TC}-\varphi^{l}\overline{Y}\right)}{\overline{w}\overline{H}(\varphi^{u}-\varphi^{l})\overline{Y}} & -\frac{\overline{r}^{d}\overline{N/P}}{\overline{w}\overline{H}} & \overline{r}^{f} \\
\overline{N/P} & 0 & 0 & 0
\end{matrix}\right]
$$

$$
D = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & \rho\overline{M/P} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
-1 & 0 \\
0 & 0 \\
0 & (1-\rho)\,\overline{M/P}
\end{bmatrix},
$$

$$
F = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & -1 \\
0 & 0 & 0
\end{bmatrix},
$$

$$
G = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{bmatrix},
$$

$$
H = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & -1 & 0
\end{bmatrix},
$$

$$
J = \begin{bmatrix}
\beta\overline{r} & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
$$

$$
K = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
$$

$$
L = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix},
$$

$$
M = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & -1
\end{bmatrix},
$$

and where

$$N = \begin{bmatrix} \gamma^{\lambda} & 0 \\ 0 & \gamma^{g} \end{bmatrix}.$$

# 9 Some results

## 9.1 Technology shocks

First, we study how this economy responds to aggregate technology shocks. Technology evolves according to

$$\widetilde{\lambda}_{t+1} = \gamma^{\lambda}\widetilde{\lambda}_t + \varepsilon^{\lambda}_{t+1},$$

where, for simulations, we choose a process for $\varepsilon^{\lambda}_{t+1}$ with mean zero and a standard error of .0178. Runs of 50 simulations of 115 periods each were used to calculate the average standard error of each variable and its correlation coefficient with output. The model used for these simulations has the same coefficients as those given for finding the stationary state. The standard errors of the variables and their correlation coefficient with respect to output for these simulations are shown in Table 1.

The coefficient of absolute risk aversion, $\alpha = 4$, is so large that the firm managers behave in a very conservative manner and the standard error of most variables actually declines when we allow the idiosyncratic error to grow. Interesting is to note the correlation with output of the interest rate paid by the firms. In the model with almost no idiosyncratic error, the correlation is positive and close to one. When the idiosyncratic error is large, this term is negative and very close to minus one.

| | std error dif = .001 | corr - output dif = .001 | std error dif = .6 $\alpha = 4$ | corr - output dif = .6 $\alpha = 4$ |
|---|---|---|---|---|
| $Y$ | 0.0828 | 100% | 0.0596 | 100% |
| $C$ | 0.0459 | 80.87% | 0.0391 | 91.70% |
| $I$ | 0.2280 | 94.53% | 0.1694 | 92.52% |
| $r^f$ | 0.0024 | 95.38% | 0.0028 | −99.16% |
| $r^d$ | 0.0024 | 95.62% | 0.0009 | 91.35% |
| $r$ | 0.0689 | 70.24% | 0.0384 | 44.72% |
| $w$ | 0.0474 | 83.38% | 0.0397 | 92.36% |
| $H$ | 0.0480 | 86.24% | 0.0175 | 56.85% |
| $K$ | 0.0623 | 62.21% | 0.0450 | 66.07% |
| $N$ | 0.0272 | 86.79% | 0.0120 | 58.83% |
| $TC$ | 0.0828 | 100% | 0.0433 | 100% |
| $P$ | 0.0591 | −96.97% | 0.0407 | −96.77% |

Table 1

For versions of the model where the coefficient of absolute risk aversion is positive but smaller than one, the results are substantially different and are shown in Table 2. In general, the standard errors of all the variables are much larger, around two to three times as large. Output variance has grown substantially and is now larger than it was in the case with almost no idiosyncratic risk. The standard error of the interest rate for lending to the firms, $r^f$, is more than an order of magnitude larger. The standard errors of both hours worked and deposits in the financial intermediary is also much larger and both are much more, positively, correlated with output.

| | std error<br>dif = .6<br>$\alpha = .5$ | corr - output<br>dif = .6<br>$\alpha = .5$ |
|---|---|---|
| $Y$ | 0.1146 | 100% |
| $C$ | 0.0862 | 88.65% |
| $I$ | 0.3512 | 94.07% |
| $r^f$ | 0.0493 | −99.98% |
| $r^d$ | 0.0031 | 95.89% |
| $r$ | 0.0929 | 56.96% |
| $w$ | 0.0885 | 89.82% |
| $H$ | 0.0868 | 90.72% |
| $K$ | 0.0983 | 63.68% |
| $N$ | 0.0551 | 90.93% |
| $TC$ | 0.1084 | 100% |
| $P$ | 0.1107 | −97.57% |

Table 2

When the coefficient of absolute risk aversion is not too large, the addition of idiosyncratic shocks to a working capital model has the effect of increasing the variance of output in the economy. This increase in variance comes from the increase in firm failures and the subsequent increased spread between the lending and borrowing rates that the financial intermediaries offer. This increased spread causes hours worked to become more variable and this feeds back onto output variance.

## 9.2 Monetary shocks

In all the examples given below, the standard error for the error to the money growth process is .09. Tables 3, 4, and 5 show the results for the case where $\rho = 1$, so the money shocks occur directly in the cash in advance constraint of the households. Tables 3 and 4 show the results for the case where there is no stationary state money growth ($\overline{g} = 1$). For the economy with idiosyncratic risk and a high absolute risk aversion, the standard errors of most real variables declines, although that of consumption and the lending and deposit rates of the financial intermediary increase. Correlations with output are about the same as in the case with almost no idiosyncratic risk. For an identical economy with low absolute risk aversion (.5 instead of 4), the standard errors of most variables increase. One very notable feature is that the standard errors of the lending and deposit rates of the financial intermediary move apart with the deposit rate showing a much smaller standard error than the lending rate. Correlations with output are approximately the same as in the two other money shock economies except that the correlations of consumption and deposits with output decline and that of the wage rate increases.

| $g = 1$ $\rho = 1$ | std error dif = .001 | corr - output dif = .001 | std error dif = .6 $\alpha = 4$ | corr - output dif = .6 $\alpha = 4$ |
|---|---|---|---|---|
| $Y$ | 0.0438 | 100% | 0.0397 | 100% |
| $C$ | 0.0276 | 98.96% | 0.0327 | 99.77% |
| $I$ | 0.0926 | 99.20% | 0.0669 | 98.97% |
| $r^f$ | 0.0258 | −99.99% | 0.0339 | −99.99% |
| $r^d$ | 0.0258 | −99.99% | 0.0316 | −99.99% |
| $r$ | 0.0439 | 98.28% | 0.0288 | 98.03% |
| $w$ | 0.0046 | 33.73% | 0.0029 | 35.38% |
| $H$ | 0.0682 | 99.78% | 0.0618 | 99.87% |
| $K$ | 0.0082 | 36.68% | 0.0058 | 38.42% |
| $N$ | 0.5957 | −7.47% | 0.5955 | −8.23% |
| $TC$ | 0.0438 | 100% | 0.0288 | 100% |
| $P$ | 0.6038 | −20.92% | 0.6027 | −20.37% |

Table 3

| $g = 1$ $\rho = 1$ | std error dif = .6 $\alpha = .5$ | corr - output dif = .6 $\alpha = .5$ |
|---|---|---|
| $Y$ | 0.0684 | 100% |
| $C$ | 0.0259 | 88.12% |
| $I$ | 0.2708 | 98.96% |
| $r^f$ | 0.0472 | −99.99% |
| $r^d$ | 0.0161 | −99.94% |
| $r$ | 0.0666 | 93.13% |
| $w$ | 0.0147 | 45.27% |
| $H$ | 0.1062 | 99.18% |
| $K$ | 0.0245 | 38.19% |
| $N$ | 0.5974 | −3.18% |
| $TC$ | 0.0646 | 100% |
| $P$ | 0.6104 | −24.63% |

Table 4

For the same economy as above but with a stationary state annual money growth and inflation rate of 100% ($g = 1.19$), one observes a slight general increase in standard errors. The two most notable changes are the declines in correlation with output of consumption and deposits in the model with the idiosyncratic noise.[1]

[1] Here we show only the model with idiosyncratic risk where the coefficient of absolute risk aversion is small.

| $g = 1.19$<br>$\rho = 1$ | std error<br>$dif = .001$ | corr - output<br>$dif = .001$ | std error<br>$dif = .6$<br>$\alpha = .5$ | corr - output<br>$dif = .6$<br>$\alpha = .5$ |
|---|---|---|---|---|
| $Y$ | 0.0452 | 100% | 0.0786 | 100% |
| $C$ | 0.0284 | 98.96% | 0.0273 | 83.29% |
| $I$ | 0.0955 | 99.20% | 0.3260 | 98.95% |
| $r^f$ | 0.0266 | −99.99% | 0.0535 | −100% |
| $r^d$ | 0.0266 | −99.99% | 0.0144 | −99.94% |
| $r$ | 0.0452 | 98.28% | 0.0780 | 92.67% |
| $w$ | 0.0047 | 33.73% | 0.0179 | 45.64% |
| $H$ | 0.0703 | 99.78% | 0.1220 | 99.09% |
| $K$ | 0.0084 | 36.67% | 0.0297 | 38.22% |
| $N$ | 0.5958 | −6.93% | 0.5986 | −0.46% |
| $TC$ | 0.0452 | 100% | 0.0755 | 100% |
| $P$ | 0.6039 | −20.81% | 0.6126 | −25.17% |

Table 5

For an otherwise identical economy with inflation, but where the money supply shock and growth go directly to the financial intemediary instead of to the families, one important result is a general and realtively large increase in standard errors. A second important result are the changes in the signs of the correlations with output of consumption and the financial intermediary's deposit rate (both from positive to negative) and for deposits and prices (from negative to positive).

| $g = 1.19$<br>$\rho = 0$ | std error<br>$dif = .001$ | corr - output<br>$dif = .001$ | std error<br>$dif = .6$<br>$\alpha = .5$ | corr - output<br>$dif = .6$<br>$\alpha = .5$ |
|---|---|---|---|---|
| $Y$ | 0.0828 | 100% | 0.1313 | 100% |
| $C$ | 0.1047 | −91.65% | 0.1182 | −80.53% |
| $I$ | 0.6133 | 98.04% | 0.9714 | 97.42% |
| $r^f$ | 0.0594 | −99.95% | 0.1043 | −100% |
| $r^d$ | 0.1147 | 99.39% | 0.1296 | 98.76% |
| $r$ | 0.0919 | 80.9% | 0.1382 | 76.29% |
| $w$ | 0.0344 | 53.44% | 0.0601 | 56.00% |
| $H$ | 0.1279 | 97.03% | 0.2011 | 96.82% |
| $K$ | 0.0564 | 42.95% | 0.0926 | 45.33% |
| $N$ | 0.5851 | 7.46% | 0.5848 | 23.59% |
| $TC$ | 0.0828 | 100% | 0.1234 | 100% |
| $P$ | 0.5798 | 4.11% | 0.5625 | 13.56% |

Table 6

## 10 Conclusions

The addition of both idiosyncratic uncertainty and risk averse managers to the production process changes the way that the interest rate paid by borrowers responds to technology shocks and the correlation of these interest rates with output. Without idiosyncratic uncertainty, the interest rate for working capital borrowed by the firms is positively, and highly, correlated with output. Once idiosyncratic techonology shocks are added, the correlation between the interest rate for working capital borrowed by the firms and output becomes highly negatively correlated.

In stationary states, increases in the idiosyncratic shocks cause the risk averse firm managers to produce at levels where average total costs are well below average output, providing them with a partial cushion against a very low idiosyncratic shock.

When absolute risk aversion is high, the effect is to dampen the effects of monetary shocks in the economy with idiosyncratic risk, but when absolute risk aversion is low, the effects of monetary shocks on real variables is higher, the larger the idiosyncratic risk.

## 11 References

## References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[4] Uhlig, Harald, (1999) "A toolkit for analysing nonlinear dynamic stochastic models easily", in Ramon Marimon and Andrew Scott, Eds., *Computational Methods for the Study of Dynamic Economies*, Oxford University Press, Oxford, p.30-61.