

ARGEM: A Dynamic and Stochastic General Equilibrium Model for Argentina

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1. Introduction

The last few years have seen an explosion of Dynamic and Stochastic General Equilibrium (DSGE) models built for policy analysis and forecasting in industrialized countries. The set of papers presented to the recent joint U.S. Federal Reserve Board-European Central Bank-IMF conference: "DSGE Modeling at Policymaking Institutions: Progress & Prospects" is a significant sample. The need for better microfounded models that can contribute to policy analysis is also experienced by developing country Central Banks, Argentina being no exception. On top of the many difficulties encountered in developed countries in building, calibrating and/or estimating these models, those who seek to obtain models that can be relevant in the developing country context find various additional difficulties. One of these stems from the fact that the models built for industrialized countries typically assume a freely floating exchange rate and hence can avoid modeling exchange rate policy. Most developing countries do not have a pure exchange rate float and their Central Banks regularly intervene in the foreign exchange market with varying degrees of intensity and frequency. While the opposite "corner" of a pure interest float with a monetary policy based on determining a path for the nominal exchange rate is not difficult to model, one of the challenges faced by developing country modelers is to incorporate exchange rate intervention as an additional tool available for a Central Bank that also intervenes in the money market (typically by determining a short run interest rate). This is one of the main objectives of this paper.

The paper builds upon various recent developments in monetary macroeconomic modeling, including Christiano, Eichenbaum and Evans (2001) (CEE), Smets and Wouters (2003), Woodford (2003), and Adolfson, Laséen, Lindé and Villani (2005), to mention but a few. The model is perhaps closest in structure to Adolfson et al (2005), with a number of significant differences that include the following: 1) The Central Bank can use alternative monetary policies within the same overall framework, including a fixed exchange rate policy, a crawling peg policy, inflation targeting with a pure float and inflation targeting with a managed float. In the latter case, the Central Bank simultaneously intervenes in the foreign exchange and money markets with two parallel feedback policy rules. 2) Instead of postulating an "asymmetric productivity shock" we assume that there is cointegration between the small domestic economy's (SDE) unit root technology shock and the large rest of the world's (LRW). 3) The financial closure of the SDE is different in that households do not engage in external debt nor save in foreign assets. It is the government and banks that rely on foreign funding, the cost of which is increasing in their (detrended) level of net debt. A risk-adjusted uncovered interest parity condition naturally stems from Banks' profit maximization. 4) We have a full fledged banking system. Banks have a cost function that is quadratic and dependent on their loan and deposit stocks, with economies of scope between lending and deposit taking activities. They have a technical demand for cash, which is a (possibly sto-

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chastic and time-varying) fraction of deposits and must keep a regulatory fraction of their deposits in non-interest bearing reserves in the Central Bank. Also, they use the remaining fraction of their deposits as well as foreign funds to finance firms' demand for loans and the Government's exogenous demand for loans, to purchase Central Bank bonds, and to lend (or borrow) in the interbank market. To add inertia in the "uncovered interest parity condition" we assume that a fraction of the banks, instead of forming expectations rationally, have static expectations with respect to nominal currency depreciation. 5) The government tax structure is minimal (just lump sum taxes), but it can also finance its expenditures by issuing debt abroad, by obtaining bank loans, and by using the Central Bank's quasi-fiscal surplus. 6) The production of intermediate domestic goods requires imported goods as inputs (in addition to labor and physical capital services) and the firms that engage in this production obtain bank loans to finance a stochastic fraction of the capital rental bill and the imported inputs bill (in addition to a stochastic fraction of the wage bill). 7) Households use cash for consumption and investment using a stylized transactions technology that requires the use of domestic goods. Hence, cash is not in the utility function, and the resulting household demand for cash is dependent on private absorption (and the deposit interest rate). 8) We assume constant (instead of stochastic and time-varying) elasticities of substitution.

The rest of this paper has the following structure. Section 2 presents the household optimization problem, which determines their consumption and investment demands, the rate of utilization of physical capital, the dynamics of the stock of physical capital, their cash and bank deposit demands, and their nominal wage setting. Section 3 presents the decisions of domestic goods producers, including their demand for labor and physical capital services, their demand for imported inputs and bank funding, their supply of goods and their nominal price setting. Section 4 has the decisions of importing and exporting firms, all of which have sticky local currency pricing. Section 5 summarizes the main relative prices in the paper and makes explicit what we mean by a Small Domestic Economy (SDE) in a Large Rest of the World (LRW). Section 6 models Banks' decision problem, which determines their demand for cash and required reserves, their demand for foreign funds and for Central Bank bonds, and their supply of deposits and loans. Section 7 introduces the public sector, composed of the Government and the Central Bank. The Central Bank balance sheet plays a significant role in the modeling of the simultaneous intervention in the money and foreign exchange markets. Section 8 puts together the market clearing equations, the balance of payments equation, and the relation between the domestic sector output and GDP. Section 9 addresses a meaningful sample of the alternative monetary policies that may be accommodated into the overall structure. Section 10 lists the non-policy equations of the non-linear system so far encountered. Section 11 transforms this set of equations so that the variables are in stationary format, and adds the alternative sets of policy equations. Section 12 displays possible functional forms for the various auxiliary functions used, such as the investment adjustment cost function, the function that reflects the costs due to non-normal intensity of utilization of physical capital, the transactions cost function, and banks' risk premium function. Section 13 states our assumptions on the stochastic shocks that impinge on the economy, with emphasis on those pertaining to the introduction of (exogenous) growth producing techno-

logical progress. Section 14 presents the complete log-linearized system and puts it in a matrix form suitable for numerical solution. Finally, Section 15 concludes. The paper has three Appendixes. The first contains a lengthy analysis of the systems' non-stochastic steady states, which should be of help in the calibration process. The second contains the details of the more cumbersome log-linearizations: the Phillips equations for domestic goods inflation and wage inflation. The third lists the definitions of the compound parameters that result from the log-linearization of the model equations.

2. Households

Infinitely lived households are monopolistic competitors in the supply of differentiated labor. There is a domestic market for state-contingent securities that are held by households, insuring them against profit and wage idiosyncratic risks (see Woodford (2003)). This makes households essentially the same in equilibrium, and allows us to maintain the representative household fiction (i.e. dispense with the complexities that stem from household heterogeneity). Aside from these state-contingent securities, they hold financial net wealth in the form of domestic currency ($M_t^{0,H}$), and peso denominated one period nominal deposits issued by domestic commercial banks (D_t) that pay a nominal interest rate i_t^D . We assume that the Central Bank fully and credibly insures depositors, so the deposit rate is considered riskless. Households also invest a real amount V_t to expand the stock of final goods (capital goods) that they own and rent to firms, earning each period a real rental price i_t^K .

2.1. Physical capital, investment, and the rate of capital utilization

The household decides at t the rate of gross investment $V_t(h)$, which contributes to the determination of the quantity of physical capital K_{t+1} in period $t+1$ through the following law of motion for the stock of physical capital:

$$K_{t+1}(h) = (1 - \delta^K) K_t(h) + z_t^V V_t(h) \left[1 - \tau_V \left(\frac{V_t(h)}{V_{t-1}(h)} \right) \right], \quad (1)$$

where δ^K is the (constant) rate of capital depreciation, and z_t^V is an economy wide stationary investment efficiency shock. As in Christiano, Eichenbaum and Evans (2001), the second term on the right hand side is a representation of the technology that transforms investment goods into capital goods. These capital goods are rented by households to firms. We have no market for capital goods in the model and hence no explicit price for these goods. As we see below, we do have a shadow price for installed physical capital (as well as a rental rate).

The function $\tau_V(\cdot)$ represents investment adjustment costs, and is such that in the steady state rate of growth of V_t (which is μ^z),

$$\tau_V(\mu^z) = \tau_V'(\mu^z) = 0, \quad \tau_V''(\mu^z) > 0.$$

The household decision process includes establishing the rate of capital utilization intensity that the firm will use (and pay for) in period t for the stock of physical capital it rents. As Christiano et al (2001) argue, allowing for elastic capital utilization has the beneficial properties of 1) dampening movements in marginal cost

by reducing fluctuations in the rental rate of physical capital and also 2) reducing the fluctuations in labor productivity after monetary policy shocks (see also Smets and Wouters (2002)). Let u_t represent the rate of capital utilization. Hence, the flow of physical capital services that the firm uses as input is:

$$u_t K_t \equiv K_t^F.$$

Using a rate of utilization of capital that exceeds the normal (steady state) level, however, is costly (whereas a lower than normal utilization actually implies a savings in total cost) and impinges in the net return from renting. Let $\tau_u(u_t)$ be the amount of real resources (domestic goods) used up (or saved) when the rate of utilization is u_t . We assume that this function is increasing and convex and we normalize units so that the steady state rate of utilization is unity, at which there are no costs (or savings):

$$\tau'_u(u_t) > 0, \tau''_u(u_t) > 0 \text{ and } \tau_u(1) = 0.$$

Hence, taking utilization adjustment costs into account, the net return from renting $K_t(h)$ units of capital is:

$$[i_t^K u_t(h) - \tau_u(u_t(h))] K_t(h) \quad (2)$$

2.2. Currency and transaction costs

The household holds currency $M_t^{0,H}$ because doing so it economizes on transaction costs. We assume that transactions involve the use of real resources (domestic goods) and that these transaction costs per unit of expenditure in consumption and investment goods (private absorption) are a convex function τ_M of the currency/absorption ratio ϖ_t (see Feenstra (1986)):

$$\tau_M(\varpi_t) \quad \tau'_M < 0, \tau''_M > 0,$$

$$\varpi_t \equiv \frac{M_t^{0,H}(h)}{P_t^C C_t(h) + P_t^V V_t(h)} = \frac{M_t^{0,H}(h)/P_t}{p_t^C C_t(h) + p_t^V V_t(h)}$$

where C_t is consumption (of private goods), P_t , P_t^C and P_t^V are the price indexes of domestic, consumption, and investment goods, respectively. All price indexes are in monetary units. The two basic price indexes in the domestic economy are those of domestically produced ('domestic') goods, P_t , and imported goods P_t^N . The consumption and investment price indexes are both CES composites of these basic price indexes, as we see below. For convenience, we define the relative prices of consumption and investment goods in terms of domestic goods:

$$p_t^C \equiv \frac{P_t^C}{P_t}, \quad p_t^V \equiv \frac{P_t^V}{P_t}.$$

When the currency/absorption ratio increases, transaction costs per unit of absorption decrease at a decreasing rate, reflecting a diminishing marginal productivity of currency in reducing transaction costs.

2.3. Sticky nominal wage setting

We model nominal stickiness as in Calvo (1983), adapted to discrete time (Rotemberg (1987)) and extended to (full) indexation (Yun (1996) and Christiano, Eichenbaum and Evans (2001)). Household $h \in [0, 1]$ supplies labor of type h , and makes the wage setting decision taking the aggregate wage index and labor supply as parametric. Every period, each household has a probability $1 - \alpha_W$ of being able to set the optimum wage for its specific labor type. This probability is independent of when it last set the optimal wage. When it can't optimize, the household adjusts its wage rate by fully indexing to last period's overall rate of wage inflation. Hence, when it can set the optimal wage rate it must take into account that in any future period j there is a probability α_W^j that its wage will be the one it sets today plus full indexation. Hence, the household faces a wage survival constraint, according to which the wage rate it sets at t , $W_t(h)$ has a probability α_W^j of surviving (indexed) until period $t + j$:

$$\begin{aligned} W_{t+j}(h) &= W_t(h) \frac{W_t}{W_{t-1}} \frac{W_{t+1}}{W_t} \cdots \frac{W_{t+j-1}}{W_{t+j-2}} \\ &\equiv W_t(h) \pi_t^w \pi_{t+1}^w \cdots \pi_{t+j-1}^w \equiv W_t(h) \Psi_{t,j}^w, \end{aligned} \quad (3)$$

where we define the rate of wage inflation $\pi_t^w \equiv W_t/W_{t-1}$, and the cumulative wage inflation between $t + j - 2$ and t , $\Psi_{t,j}^w$, with $\Psi_{t,0}^w \equiv 1$. In deriving the first order condition for $W_t(h)$ below we will use the following identity:

$$\frac{W_{t+j}(h) \Psi_{t,j}^w}{W_{t+j}} = \frac{W_t(h)}{W_t} \frac{\pi_t^w \pi_{t+1}^w \cdots \pi_{t+j-1}^w}{\pi_{t+j}^w \pi_{t+j-1}^w \cdots \pi_{t+1}^w} = \frac{W_t(h)}{W_t} \frac{\pi_t^w}{\pi_{t+j}^w}. \quad (4)$$

Another constraint the household faces is its labor demand function:

$$h_t(h) = h_t \left(\frac{W_t(h)}{W_t} \right)^{-\psi}, \quad (5)$$

where W_t is the aggregate wage index, defined as:

$$W_t = \left\{ \int_0^1 W_t(h)^{1-\psi} dh \right\}^{1/(1-\psi)}, \quad (6)$$

and where ψ is the elasticity of substitution between differentiated labor services². When h sets the optimal wage, it must take into account that there is a probability α_W^j that at time $t + j$ its wage will be the $W_t(h) \Psi_{t,j}^w$, and that hence the labor demand it faces is:

$$h_{t+j}(h) = h_{t+j} \left(\frac{W_t(h) \Psi_{t,j}^w}{W_{t+j}} \right)^{-\psi}. \quad (7)$$

2.4. The household optimization problem

The household receives income from profits, wage, rent, and interest, and spends on consumption, investment, taxes, and transaction costs. It's real budget constraint

²We derive these equations from domestic intermediate firms' cost minimization in section 3.2 below.

in period t is:

$$\begin{aligned} \frac{M_t^{0,H}(h)}{P_t} + \frac{D_t(h)}{P_t} &= \frac{\Pi_t(h)}{P_t} + \frac{W_t(h)}{P_t} h_t(h) - \frac{T_t(h)}{P_t} + \frac{\Upsilon_t(h)}{P_t} \\ + [i^K u_t(h) - \tau_u(u_t(h))] K_t(h) &+ \frac{M_{t-1}^{0,H}(h)}{P_t} + (1 + i_{t-1}^D) \frac{D_{t-1}(h)}{P_t} \\ - \left[1 + \tau_M \left(\frac{M_t^{0,H}(h)/P_t}{p_t^C C_t(h) + p_t^V V_t(h)} \right) \right] &(p_t^C C_t(h) + p_t^V V_t(h)) \end{aligned} \quad (8)$$

where $\Pi_t(h)$ is pre-tax nominal profits, $h_t(h)$ is hours of labor exertion, $T_t(h)$ is lump sum taxes net of transfers, and $\Upsilon_t(h)$ is the income obtained in t from holding state-contingent securities.

Household h maximizes an inter-temporal utility function which is additively separable in the consumption of private goods C_t , public goods C_t^G , and leisure:

$$\begin{aligned} E_t \sum_{j=0}^{\infty} \beta^j \{ z_{t+j}^C \log [C_{t+j}(h) - \xi C_{t+j-1}(h)] + \\ + \eta_G \log [C_{t+j}^G(h) - \xi_G C_{t+j-1}^G(h)] + [\bar{h} - \frac{\eta_H z_{t+j}^H}{1+\chi} h_{t+j}(h)^{1+\chi}] \}, \end{aligned} \quad (9)$$

where β is the intertemporal discount factor, \bar{h} is the maximum labor time available (and hence the last term in square brackets is "leisure"), and z_t^C and z_t^H are consumption demand and labor supply shocks that are common to all households. Consumption nests habit formation, where ξ and ξ_G are less than unity (see Fuhrer (2000) and Christiano, Eichenbaum and Evans (2001)) into a log utility function. Consumers hence care about both their level of consumption and their rate of consumption growth. Since the consumption of public goods is not a decision variable for the household, the term that includes it is only relevant for the evaluation of the welfare effects of alternative fiscal policies. We drop it below for simplicity.

The household's inter-temporal solvency is guaranteed by its inability to incur in debt, which we assume does not bind in any finite time:

$$D_{t+T} \geq 0, \quad \forall T \geq 0. \quad (10)$$

Household h chooses $C_{t+j}(h)$, $V_{t+j}(h)$, $K_{t+1+j}(h)$, $u_{t+j}(h)$, $D_{t+j}(h)$, $M_{t+j}^{0,H}(h)$, ($j=1,2,\dots$) and $W_t(h)$, to maximize (9) subject to its sequence of budget constraints (8), physical capital accumulation constraints (1), its combined labor demands and wage survival constraints (7), and its "no debt" constraints (10). The Lagrangian

is hence:

$$\begin{aligned}
& E_t \sum_{j=0}^{\infty} (\beta \alpha_W)^j \{ z_{t+j}^C \log [C_{t+j}(h) - \xi C_{t+j-1}(h)] + \bar{h} \} \\
& - \frac{\eta_H z_{t+j}^H}{1 + \chi} \left(h_{t+j} \left(\frac{W_t(h) \Psi_{t,j}^w}{W_{t+j}} \right)^{-\psi} \right)^{1+\chi} + \lambda_{t+j}(h) \left\{ \frac{\Pi_{t+j}(h)}{P_{t+j}} - \frac{T_{t+j}(h)}{P_{t+j}} \right. \\
& + \frac{W_t(h) \Psi_{t,j}^w}{P_{t+j}} h_{t+j} \left(\frac{W_t(h) \Psi_{t,j}^w}{W_{t+j}} \right)^{-\psi} + [i_{t+j}^K u_{t+j}(h) - \tau_u(u_{t+j}(h))] K_{t+j}(h) \\
& - \left[1 + \tau_M \left(\frac{M_{t+j}^{0,H}(h)/P_{t+j}}{p_{t+j}^C C_{t+j}(h) + p_{t+j}^V V_{t+j}(h)} \right) \right] (p_{t+j}^C C_{t+j}(h) + p_{t+j}^V V_{t+j}(h)) \\
& + \frac{M_{t+j-1}^{0,H}(h)}{P_{t+j}} + (1 + i_{t+j-1}^D) \frac{D_{t+j-1}(h)}{P_{t+j}} - \frac{M_{t+j}^{0,H}(h)}{P_{t+j}} - \frac{D_{t+j}(h)}{P_{t+j}} + \frac{\Upsilon_{t+j}(h)}{P_{t+j}} \} \\
& + \zeta_{t+j}(h) \left\{ (1 - \delta^K) K_{t+j}(h) + z_{t+j}^V V_{t+j}(h) \left[1 - \tau_V \left(\frac{V_{t+j}(h)}{V_{t-1+j}(h)} \right) \right] \right. \\
& \left. - K_{t+1+j}(h) \right\}.
\end{aligned} \tag{11}$$

where $\beta^j \lambda_{t+j}(h)$ and $\beta^j \zeta_{t+j}(h)$ are the Lagrange multipliers (for the budget constraints and the capital accumulation constraints), which can be interpreted as the marginal utility of real income, and the shadow price of installed physical capital, respectively. We will refer to λ_t and ζ_t as the undiscounted Lagrange multipliers.

2.5. First order conditions

Since households only differ on whether they can choose the optimal wage, we eliminate the household index, and use \bar{W}_t to distinguish the newly optimal wage from the aggregate wage index W_t (which includes both optimal and indexed wages). The first order conditions for an optimum (including the transversality condition) are the following:

$$C_t : \quad \frac{z_t^C}{C_t - \xi C_{t-1}} - \beta \xi E_t \left(\frac{z_{t+1}^C}{C_{t+1} - \xi C_t} \right) = \lambda_t \varphi_M \left(\frac{M_t^{0,H}/P_t}{p_t^C C_t + p_t^V V_t} \right) \tag{12}$$

$$\begin{aligned}
V_t : \quad & \zeta_t z_t^V \varphi_V \left(\frac{V_t}{V_{t-1}} \right) + \beta E_t \left\{ \zeta_{t+1} z_{t+1}^V \tau'_V \left(\frac{V_{t+1}}{V_t} \right) \left(\frac{V_{t+1}}{V_t} \right)^2 \right\} \\
& = \lambda_t \varphi_M \left(\frac{M_t^{0,H}/P_t}{p_t^C C_t + p_t^V V_t} \right)
\end{aligned} \tag{13}$$

$$K_{t+1} : \quad \zeta_t = \beta E_t \left\{ \zeta_{t+1} (1 - \delta^K) + \lambda_{t+1} [i_{t+1}^K u_{t+1} - \tau_u(u_{t+1})] \right\} \tag{14}$$

$$u_t : \quad \lambda_t K_t [\tau'_u(u_t) - i_t^K] = 0 \tag{15}$$

$$D_t : \quad \lambda_t = \beta (1 + i_t^D) E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) \tag{16}$$

$$M_t^{0,H} : \quad \lambda_t \left[1 + \tau'_M \left(\frac{M_t^{0,H}/P_t}{p_t^C C_t + p_t^V V_t} \right) \right] = \beta E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) \tag{17}$$

$$W_t : \quad 0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_W)^j \lambda_{t+j} h_{t+j} \frac{W_{t+j}}{P_{t+j}} (\pi_{t+j}^w)^\psi \quad (18)$$

$$\left\{ \left(\frac{\widetilde{W}_t \pi_t^w}{W_t \pi_{t+j}^w} \right) - \frac{\psi}{\psi - 1} \frac{\eta_H z_{t+j}^H h_{t+j}^\chi}{\lambda_{t+j} W_{t+j} / P_{t+j}} \left(\frac{\widetilde{W}_t \pi_t^w}{W_t \pi_{t+j}^w} \right)^{-\psi \chi} \right\}.$$

$$\lim_{t \rightarrow \infty} \beta^t D_t = 0. \quad (19)$$

Several comments are in order on these first order conditions.

First, we have used some auxiliary functions to alleviate notation. In (12) and (13) we have defined the function φ_M that gives the total effect on expenditure (i.e., including transaction cost related expenditures) of a marginal increase in absorption :³

$$\begin{aligned} \varphi_M(\varpi_t) &\equiv 1 + \tau_M(\varpi_t) - \varpi_t \tau'_M(\varpi_t), \\ \varphi'_M(\varpi_t) &= -\varpi_t \tau''_M(\varpi_t) < 0. \end{aligned} \quad (20)$$

Observe that φ_M is decreasing in the money to absorption ratio ϖ_t and that the effect on expenditure generated by a marginal increase in ϖ_t is given by the increase in expenditure with the initial money/absorption ratio, $1 + \tau_M$, plus the increase due to the reduction in the money/absorption ratio, $\varpi_t (-\tau'_M(\varpi_t))$.

In analogous fashion, in (13) we have used the function φ_V defined as:

$$\varphi_V(\mu_t^V) \equiv 1 - \tau_V(\mu_t^V) - \mu_t^V \tau'_V(\mu_t^V),$$

(where μ_t^V is the gross growth rate of V_t) which gives the increase in gross investment net of adjustment costs (but not of capital stock depreciation) resulting from a marginal increase in the rate of gross investment growth.⁴

(12) shows that in equilibrium the utility gain from a marginal increase in consumption, corrected for the habit related reduction in utility it is expected to generate next period (left side of the equality), equals the foregone marginal utility of real income it generates, including that which is related to transaction costs (given by $\varphi_M(\cdot)$).

(13) shows that the loss in utility from marginally increasing gross investment (measured through the undiscounted shadow price of installed physical capital ζ_t and including investment adjustment costs) minus the discounted increase in utility it is expected to generate next period, equals the foregone marginal utility of real income it generates (including that which is related to transaction costs).

(14) states that the shadow value of a marginal addition to installed capital equals its discounted expected shadow value next period corrected for capital depreciation plus the discounted net addition to rental income it is expected to generate.

(15) states that whenever the marginal utility of real income and the stock of physical capital are different from zero (which we assume is the case for all t), the equilibrium rate of utilization of physical capital is such that the marginal cost of having it different from the normal level equals its rental rate. Hence, this

³ $\varphi_M(m/a)$ is the partial derivative of $[1 + \tau_M(m/a)] a$ with respect to a .

⁴ $\varphi_V(V/V_{-1})$ is the partial derivative of $[1 - \tau_V(V/V_{-1})] V$ with respect to V .

condition directly determines the optimal intensity of utilization of physical capital as a function of the rental rate

$$u_t = (\tau'_u)^{-1} (i_t^K). \quad (21)$$

Inserting this expression in (2) gives the following auxiliary function for the real return from renting one unit of capital after taking utilization adjustment costs into account:

$$\Gamma^K (i_t^K) \equiv i_t^K (\tau'_u)^{-1} (i_t^K) - \tau_u \left((\tau'_u)^{-1} (i_t^K) \right). \quad (22)$$

(16) states that the loss in utility from marginally increasing the holding of deposits equals the discounted expected utility of the addition to real interest income it generates next period. And (17) states that the net loss of utility from marginally increasing the holding of currency after taking into account the reduction in transaction costs it generates, is equal to the discounted expected marginal utility of having it available tomorrow with its purchasing power corrected for inflation. Combining (16) and (17) yields:

$$-\tau'_M \left(\frac{M_t^{0,H}/P_t}{p_t^C C_t + p_t^V V_t} \right) = 1 - \frac{1}{1 + i_t^D}, \quad (23)$$

which shows that the optimum stock of currency as a fraction of expenditure in consumption and investment is such that the reduction in transaction costs generated by a marginal increase in this ratio equals the opportunity cost of holding cash. Inverting $-\tau'_M$ gives the following demand function for cash as a vehicle for transactions (sometimes called "liquidity preference" function):

$$\frac{M_t^{0,H}}{P_t} = \mathcal{L} (1 + i_t^D) [p_t^C C_t + p_t^V V_t], \quad (24)$$

where:

$$\begin{aligned} \mathcal{L} (1 + i_t^D) &\equiv (-\tau'_M)^{-1} \left(1 - \frac{1}{1 + i_t^D} \right) \\ \mathcal{L}' (1 + i_t^D) &= \left[-\tau''_M(\cdot) (1 + i_t^D)^2 \right]^{-1} < 0. \end{aligned}$$

From here on we replace the first order condition (17) by (24) and also use it to eliminate the household currency to absorption ratio wherever it appears through the use of the following auxiliary functions:

$$\tilde{\varphi}_M (\cdot) \equiv \varphi_M (\mathcal{L} (\cdot)), \quad \tilde{\tau}_M (\cdot) \equiv \tau_M (\mathcal{L} (\cdot)). \quad (25)$$

Note in (18) that since all households that can set their optimal wage in t make the same decision we have denoted the optimum wage rate \widetilde{W}_t . Hence, (6) and (3) imply the following law of motion for the aggregate wage rate (after assuming that the average wage rate of non-optimizers is the average overall wage level in $t - 1$ indexed by wage inflation no matter when they optimized for the last time):

$$W_t^{1-\theta} = \alpha_W (W_{t-1} \pi_{t-1}^w)^{1-\theta} + (1 - \alpha_W) \widetilde{W}_t^{1-\theta}. \quad (26)$$

Defining the real wage in terms of domestic goods and the relative wage between the optimizers and the general level:

$$w_t = \frac{W_t}{P_t}, \quad \tilde{w}_t = \frac{\widetilde{W}_t}{W_t},$$

the first order condition for W_t becomes:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_W)^j \lambda_{t+j} h_{t+j} w_{t+j} (\pi_{t+j}^w)^\psi \left\{ \left(\frac{\tilde{w}_t \pi_t^w}{\pi_{t+j}^w} \right) - \frac{\psi}{\psi - 1} \frac{\eta_H z_{t+j}^H h_{t+j}^\chi}{\lambda_{t+j} w_{t+j}} \left(\frac{\tilde{w}_t \pi_t^w}{\pi_{t+j}^w} \right)^{-\psi \chi} \right\}. \quad (27)$$

And dividing through (26) by $W_{t-1}^{1-\theta}$ we get:

$$(\pi_t^w)^{1-\theta} = \alpha_W (\pi_{t-1}^w)^{1-\theta} + (1 - \alpha_W) (\tilde{w}_t \pi_t^w)^{1-\theta}, \quad (28)$$

which can be used to eliminate \tilde{w}_t from (27), leaving a dynamic equation in π_t^w . We will refrain from doing so in the non-linear model, maintaining two dynamic equations for each inflation rate (wage and domestic, imported and exported goods) for the sake of clarity in the analysis of the steady state, but we will eliminate this relative wage (and the corresponding relative prices for different types of goods) when we log-linearize the model.⁵

2.6. Domestic and imported consumption and investment goods

So far we have ignored the open economy attributes of consumption and investment, as well as the product differentiation within these classes. We now consider the household allocation of consumption and investment expenditures across these product classes and varieties. First we distinguish between domestic and imported consumption and investment goods. The consumption index we used in the household optimization problem is actually a constant elasticity of substitution (CES) aggregate consumption index of domestic and imported consumption goods:

$$C_t = \left(a_D \frac{1}{\theta_C} (C_t^D)^{\frac{\theta_C-1}{\theta_C}} + a_N \frac{1}{\theta_C} (C_t^N)^{\frac{\theta_C-1}{\theta_C}} \right)^{\frac{\theta_C}{\theta_C-1}}, \quad a_D + a_N = 1. \quad (29)$$

θ_C is the elasticity of substitution between domestic and imported consumption goods. And C_t^D and C_t^N are themselves CES aggregates of the domestic and imported varieties of goods available:

$$C_t^D = \left(\int_0^1 C_t^D(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \quad (30)$$

$$C_t^N = \left(\int_0^1 C_t^N(i)^{\frac{\theta_N-1}{\theta_N}} di \right)^{\frac{\theta_N}{\theta_N-1}}, \quad \theta_N > 1. \quad (31)$$

⁵The detailed log-linearization of (27) and (28) is in Appendix 2.

θ and θ_N are the elasticities of substitution between varieties of domestic and imported goods in household expenditure, respectively. We assume that these elasticities hold for household expenditures in these goods whether they are for consumption or investment purposes. Total consumption expenditure is:

$$P_t^C C_t = P_t C_t^D + P_t^N C_t^N. \quad (32)$$

Then minimization of (32) subject to (29) for a given C_t , yields the following relations:

$$P_t = a_D^{\frac{1}{\theta_C}} P_t^C \left(\frac{C_t^D}{C_t} \right)^{-\frac{1}{\theta_C}} \quad (33)$$

$$P_t^N = a_N^{\frac{1}{\theta_C}} P_t^C \left(\frac{C_t^N}{C_t} \right)^{-\frac{1}{\theta_C}}. \quad (34)$$

Introducing these in (29) yields the consumption price index:

$$P_t^C = \left(a_D (P_t)^{1-\theta_C} + a_N (P_t^N)^{1-\theta_C} \right)^{\frac{1}{1-\theta_C}}. \quad (35)$$

Furthermore, it is readily seen that a_D and a_N in (29) are the shares of domestic and imported consumption in total consumption expenditures:

$$a_D = \frac{P_t C_t^D}{P_t^C C_t}, \quad a_N = \frac{P_t^N C_t^N}{P_t^C C_t}. \quad (36)$$

With investment demand we proceed in exactly the same way. V_t is a CES aggregate investment index of domestic and imported investment goods:

$$V_t = \left(b_D^{\frac{1}{\theta_V}} (V_t^D)^{\frac{\theta_V-1}{\theta_V}} + b_N^{\frac{1}{\theta_V}} (V_t^N)^{\frac{\theta_V-1}{\theta_V}} \right)^{\frac{\theta_V}{\theta_V-1}}, \quad b_D + b_N = 1, \quad (37)$$

where θ_V is the elasticity of substitution between domestic and imported investment goods, and V_t^D and V_t^N are CES aggregates of domestic and imported goods:

$$V_t^D = \left(\int_0^1 V_t^D(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \quad (38)$$

$$V_t^N = \left(V_t^N(i)^{\frac{\theta_N-1}{\theta_N}} di \right)^{\frac{\theta_N}{\theta_N-1}}, \quad \theta_N > 1. \quad (39)$$

Then it follows that the investment price index is:

$$P_t^V = \left(b_D (P_t)^{1-\theta_V} + b_N (P_t^N)^{1-\theta_V} \right)^{\frac{1}{1-\theta_V}}. \quad (40)$$

and that the following relations hold:

$$P_t^V V_t = P_t V_t^D + P_t^N V_t^N.$$

$$P_t = b_D^{\frac{1}{\theta_V}} P_t^V \left(\frac{V_t^D}{V_t} \right)^{-\frac{1}{\theta_V}} \quad (41)$$

$$P_t^N = b_N^{\frac{1}{\theta_V}} P_t^V \left(\frac{V_t^N}{V_t} \right)^{-\frac{1}{\theta_V}}. \quad (42)$$

$$b_D = \frac{P_t V_t^D}{P_t^V V_t}, \quad b_N = \frac{P_t^N V_t^N}{P_t^V V_t}. \quad (43)$$

Conditions (33), (34), (41), and (42) are necessary for the optimal allocation of household expenditures across domestic and imported goods in consumption and investment, respectively. Similarly, for the optimal allocation across varieties of domestic and imported goods within these classes, and using (30), (31), (38), and (39), the following conditions must hold:

$$P_t(i) = P_t \left(\frac{C_t^D(i)}{C_t^D} \right)^{-\frac{1}{\theta_C}}$$

$$P_t^N(i) = P_t^N \left(\frac{C_t^N(i)}{C_t^N} \right)^{-\frac{1}{\theta_C}}.$$

$$P_t(i) = P_t \left(\frac{V_t^D(i)}{V_t^D} \right)^{-\frac{1}{\theta_V}}$$

$$P_t^N(i) = P_t^N \left(\frac{V_t^N(i)}{V_t^N} \right)^{-\frac{1}{\theta_V}}.$$

3. Domestic goods firms

3.1. Final domestic goods

There is perfect competition in the production (or bundling) of final domestic output Q_t , with the output of intermediate firms as inputs. A representative final domestic output firm uses the following CES technology:

$$Q_t = \left(\int_0^1 Q_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \quad (44)$$

where θ is the elasticity of substitution between any two varieties of domestic goods and $Q_t(i)$ is the output of the intermediate domestic good i . Then the final domestic output representative firm solves the following problem each period:

$$\max_{Q_t(i)} P_t \left(\int_0^1 Q_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i) Q_t(i) di, \quad (45)$$

the solution of which is:

$$Q_t(i) = Q_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta}. \quad (46)$$

Introducing (46) in (44) and simplifying, it is readily seen that the domestic goods price index is:

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \quad (47)$$

Also, introducing (46) into the cost part of (45) yields:

$$\int_0^1 P_t(i)Q_t(i)di = P_tQ_t.$$

3.2. Intermediate domestic goods

A continuum of monopolistically competitive firms produce intermediate domestic goods using labor, capital, and imported inputs, with no entry or exit. They face a perfectly competitive physical capital rental market and perfectly competitive bundlers of import goods and labor types. The production function of firm i is:

$$Q_t(i) = \begin{cases} \epsilon_t [K_t^F(i)^a (z_t h_t(i))^{1-a}]^b N_t^F(i)^{1-b} - z_t F^D & \text{if this is positive} \\ 0 & \text{otherwise.} \end{cases} \quad (48)$$

ϵ_t and z_t are industry-wide productivity shocks. K_t^F is the flow of services rendered by the (hired) stock of capital when used at the intensity determined by the households that own them, N_t^F is the use of imported inputs. $z_t F^D$ is a fixed cost that grows along with the economy and can be used to calibrate profits in the steady state⁶. $h_t(i)$ is a CES index of all the labor types:

$$h_t(i) = \left(\int_0^1 h_t(h, i)^{\frac{\psi-1}{\psi}} dh \right)^{\frac{\psi}{\psi-1}}, \quad (49)$$

where $h_t(h, i)$ is the amount of labor type h used by the domestic firm i . The production decision of i is subject to the demand function of final goods producers (46) and the price survival constraint, whereby the price it sets at t , $P_t(i)$ has a probability α of surviving (indexed) until period $t + j$.

3.3. Marginal cost and input demands

Extending the assumptions in Christiano, Eichenbaum and Evans (2001) and in Adolfson et al (2005) to the use of physical capital and imported goods, and allowing for randomness in the fractions of the different input costs that are bank financed, we assume that stochastic fractions v_t^W of the labor bill, v_t^K of the capital rental bill, and v_t^N of the imported input bill are financed by the domestic banking system. Let i_t^L be the nominal bank loan rate. Then we may write total variable cost as:

$$\Omega_t^K P_t i_t^K K_t^F(i) + \Omega_t^W W_t h_t(i) + \Omega_t^N P_t^N N_t^F(i)$$

where⁷

$$\Omega_t^q = 1 + v_t^q i_t^L = [1 - v_t^q + v_t^q (1 + i_t^L)], \quad q = K, W, N. \quad (50)$$

To maximize profits, the firm must minimize costs. Consider first the minimization of total labor cost:

$$\int_0^1 W_t(h) h_t(h, i) dh \quad (51)$$

⁶Christiano, Eichenbaum and Evans (2001), for example, calibrate profits to zero.

⁷The last expression is convenient for log-linearizing.

subject to a constant aggregate index or labor types (49). We call the Lagrange multiplier W_t . It does not depend on i since the problem is the same for all firms. Then the minimization results in i 's inverse demand function for labor type h :

$$W_t(h) = W_t \left(\frac{h_t(h, i)}{h_t(i)} \right)^{-\frac{1}{\psi}}. \quad (52)$$

Defining the aggregate demand for labor of type h :

$$h_t(h) = \int_0^1 h_t(h, i) di,$$

and the aggregate demand for the labor bundle:

$$h_t = \int_0^1 h_t(i) di,$$

(52) implies the household labor demand (5) we used for the household problem. Furthermore, introducing (52) in (49) yields:

$$W_t = \left(\int_0^1 W_t(h)^{1-\psi} di \right)^{\frac{1}{1-\psi}},$$

confirming that the Lagrange multiplier is indeed the wage index. And introducing (52) in (51) yields a more convenient expression for the wage bill of firm i :

$$\int_0^1 W_t(h) h_t(h, i) dh = W_t h_t(i).$$

We now obtain factor and bank loan demands by solving the following cost minimization problem:

$$\min_{K_t^F(i), h_t(i), N_t^F(i)} \{ \Omega_t^K P_t i_t^K K_t^F(i) + \Omega_t^W W_t h_t(i) + \Omega_t^N P_t^N N_t^F(i) \}$$

subject to (48), where $Q_t(i)$ is given. The problem is the same for all firms, so we eliminate the firm index. The first order conditions are:

$$\Omega_t^K P_t i_t^K K_t^F = ab MC_t [Q_t + z_t F^D] \quad (53)$$

$$\Omega_t^W W_t h_t = (1-a)b MC_t [Q_t + z_t F^D] \quad (54)$$

$$\Omega_t^N P_t^N N_t^F = (1-b) MC_t [Q_t + z_t F^D], \quad (55)$$

where MC_t is the Lagrange multiplier. Adding these equations term by term shows that total variable cost is:

$$\Omega_t^K P_t i_t^K K_t^F + \Omega_t^W W_t h_t + \Omega_t^N P_t^N N_t^F = MC_t [Q_t + z_t F^D],$$

and that MC_t is indeed the nominal marginal cost. Furthermore, introducing the first order conditions and (50) in the production function (48) yields the following expressions for the nominal marginal cost:

$$\begin{aligned} MC_t &= \frac{1}{\kappa \epsilon_t z_t^{(1-a)b}} \left[(\Omega_t^K P_t i_t^K)^a (\Omega_t^W W_t)^{1-a} \right]^b (\Omega_t^N P_t^N)^{1-b}, \quad (56) \\ &= \frac{1}{\kappa \epsilon_t z_t^{(1-a)b}} f_{MC} (1 + i_t^L) (P_t i_t^K)^{ab} W_t^{(1-a)b} (P_t^N)^{1-b} \end{aligned}$$

where we defined:

$$\kappa \equiv [a^a (1-a)^{1-a}]^b b^b (1-b)^{1-b},$$

and the auxiliary function:

$$f_{MC} (1 + i_t^L) \equiv [1 - v_t^K + v_t^K (1 + i_t^L)]^{ab} [1 - v_t^W + v_t^W (1 + i_t^L)]^{(1-a)b} \\ [1 - v_t^N + v_t^N (1 + i_t^L)]^{1-b}, \quad f'_{MC} (1 + i_t^L) > 0.$$

Hence, the (own) real marginal cost is

$$m_{C_t} \equiv \frac{MC_t}{P_t} = \frac{1}{\kappa \epsilon_t} f_{MC} (1 + i_t^L) (i_t^K)^{ab} \left(\frac{w_t}{z_t} \right)^{(1-a)b} (p_t^N)^{1-b}, \quad (57)$$

where

$$p_t^N \equiv \frac{P_t^N}{P_t}$$

is the relative (domestic currency) price between imported and domestic goods. We refer to this relative price as the internal terms of trade.

Aggregate demand functions for h_t , K_t^F , and N_t^F are obtained directly from (53)-(55) and (56). Note that they all depend on the loan rate, through the Ω_t^q ($q = W, K, N$). Also, the resulting aggregate nominal demand for bank loans by firms is:

$$L_t^F = f_L (1 + i_t^L) MC_t [Q_t + z_t F^D], \quad (58)$$

where we defined the auxiliary function:

$$f_L (1 + i_t^L) \equiv \frac{abv_t^K}{1 + v_t^K i_t^L} + \frac{(1-a)bv_t^W}{1 + v_t^W i_t^L} + \frac{(1-b)v_t^N}{1 + v_t^N i_t^L} \quad (59) \\ = \frac{ab}{(1/v_t^K - 1) + (1 + i_t^L)} + \frac{(1-a)b}{(1/v_t^W - 1) + (1 + i_t^L)} \\ + \frac{1-b}{(1/v_t^N - 1) + (1 + i_t^L)}, \quad f'_L (1 + i_t^L) < 0.$$

3.4. Sticky nominal price setting

As in the case of households, firms make pricing decisions taking the aggregate price and quantity indexes as parametric. Every period, each firm has a probability $1 - \alpha$ of being able to set the optimum price for its specific type of good and whenever it can't optimize it adjusts its price by fully indexing to last period's overall rate of domestic inflation. Hence, when it can set its optimal price it must take into account that in any future period j there is a probability α^j that its price will be the one it sets today plus full indexation. Hence, the firm's price survival constraint states that the price it sets at t , $P_t(i)$ has a probability α^j of surviving (indexed) until period $t + j$:

$$P_{t+j}(i) = P_t(i) \pi_t \pi_{t+1} \dots \pi_{t+j-1} \equiv P_t(i) \Psi_{t,j}^q. \quad (60)$$

where $\Psi_{t,0}^q \equiv 1$. As in the case of wages (see (4)), we make use of the following identity:

$$\frac{P_{t+j}(i)}{P_{t+j}} \Psi_{t,j}^q = \frac{P_t(i)}{P_t} \frac{\pi_t}{\pi_{t+j}}. \quad (61)$$

Hence, we can express the firm's pricing problem as:

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \alpha^j \Lambda_{t,t+j} \left\{ \frac{P_t(i) \Psi_{t,j}^q}{P_{t+j}} Q_{t+j}(i) - mc_{t+j}(i) [Q_{t+j}(i) + z_{t+j} F^D] \right\}$$

subject to

$$Q_{t+j}(i) = Q_{t+j} \left(\frac{P_t(i) \Psi_{t,j}^q}{P_{t+j}} \right)^{-\theta}.$$

$\Lambda_{t,t+j}$ is the pricing kernel used by firms for discounting, which is equal to households' intertemporal marginal rate of substitution in consumption between periods $t+j$ and t :

$$\Lambda_{t,t+j} = \beta^j \frac{U_{C,t+j}}{U_{C,t}} = \beta^j \frac{\lambda_{t+j} \tilde{\varphi}_M (1 + i_{t+j}^D)}{\lambda_t \tilde{\varphi}_M (1 + i_t^D)} \equiv \beta^j \frac{\bar{\Lambda}_{t+j}}{\bar{\Lambda}_t},$$

where $U_{C,t}$ is the household's marginal utility of consumption in t corrected for habit, and the second equality derives from (12) and (25).

The first order condition is the following (after dropping the firm index):

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \bar{\Lambda}_{t+j} Q_{t+j} \pi_{t+j}^{\theta} \left\{ \frac{\tilde{P}_t \pi_t}{P_t \pi_{t+j}} - \frac{\theta}{\theta-1} mc_{t+j} \right\}. \quad (62)$$

Since all optimizing firms make the same decision we call the optimum price \tilde{P}_t . Hence, (47) and (60) imply the following law of motion for the aggregate domestic goods price index:

$$P_t^{1-\theta} = \alpha (P_{t-1} \pi_{t-1})^{1-\theta} + (1-\alpha) \tilde{P}_t^{1-\theta}. \quad (63)$$

Proceeding as we did with the wage inflation Phillips equation, we define the relative optimal to average domestic price:

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t},$$

and express the preceding equations as:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \bar{\Lambda}_{t+j} Q_{t+j} (\pi_{t+j})^{\theta} \left\{ \frac{\tilde{p}_t \pi_t}{\pi_{t+j}} - \frac{\theta}{\theta-1} mc_{t+j} \right\},$$

$$\pi_t^{1-\theta} = \alpha \pi_{t-1}^{1-\theta} + (1-\alpha) (\tilde{p}_t \pi_t)^{1-\theta}.$$

4. Foreign trade firms

We follow Adolfson et al (2005) in allowing for an imperfect pass-through of exchange rate fluctuations by recurring to monopolistically competitive import and export firms that set prices with stickiness and local currency pricing. Because the "small open economy" concept is not always used with the same meaning, we refer to the domestic economy as a "small domestic economy" (SDE) and explain what we mean by this below. An asterisk * as a superscript that is not followed by

a letter (though it may be preceded by one) means that the variable is exogenous in the model and refers to the "large rest of the world" (LRW). Hence, P_t^* and Q_t^* are the LRW's "domestic" price and quantity indexes and P_t^{N*} is its import price index. And an asterisk in a superscript that is followed by a letter in a price index means that it refers to prices in foreign currency and may or may not be an exogenous variable in the model. For example, the SDE's export firms set export prices P_t^{*X} in the foreign currency (local currency pricing) which are endogenous variables, while P_t^{*N} refers to the SDE's import price index in foreign currency and is exogenous.

4.1. Imported goods firms

Final imported goods

Perfectly competitive (trade) firms produce (or bundle) final imported goods using the output of monopolistically competitive intermediate imported goods producers. The representative firm in this sector uses the following CES technology:

$$N_t = \left(\int_0^1 N_t(i)^{\frac{\theta_N-1}{\theta_N}} di \right)^{\frac{\theta_N}{\theta_N-1}}, \quad \theta_N > 1,$$

where θ_N is the elasticity of substitution between varieties of imported goods in consumption and investment as well as in their use as inputs for domestic goods firms. Maximizing profits (as in (45) for final domestic output firms) gives the demand function that the intermediate importer of good i faces:

$$N_t(i) = N_t \left(\frac{P_t^N(i)}{P_t^N} \right)^{-\theta_N},$$

where both price indexes are in the domestic currency. The resulting (domestic currency) price index for imported goods is:

$$P_t^N = \left(\int_0^1 P_t^N(i)^{1-\theta_N} di \right)^{\frac{1}{1-\theta_N}}, \quad (64)$$

and the import cost bill is:

$$\int_0^1 P_t^N(i) N_t(i) di = P_t^N N_t.$$

Intermediate imported goods

A continuum of monopolistically competitive firms generate intermediate imported goods. They buy a bundled final good abroad at the foreign price and turn it into differentiated goods to be sold in the domestic market in domestic currency (see Adolfson et al (2005)). They purchase the bundled final good at the price $S_t P_t^{*N}$, where P_t^{*N} is the foreign currency price index of the imported bundle (which we assume differs from the LRW's "domestic" price index P_t^*) and S_t is the nominal exchange rate (pesos per unit of foreign currency). Note that $S_t P_t^{*N}$ is thus the marginal cost for these firms. Their pricing (in the domestic currency) follows

the same setup we used for firms producing domestic intermediate goods, with a probability $1 - \alpha_N$ of optimal price setting and full indexation when they can't optimize price. According to the price survival constraint, the price $P_t^N(i)$ the firm sets at t has a probability α_N^j of surviving (indexed) until $t + j$:

$$P_{t+j}^N(i) = P_t^N(i)\pi_t^N\pi_{t+1}^N\cdots\pi_{t+j-1}^N \equiv P_t^N(i)\Psi_{t,j}^N, \quad (\Psi_{t,0}^N \equiv 1). \quad (65)$$

Due to this, when the firm optimizes it takes into account that there is a probability α_N^j that the demand for its good in $t + j$ will be:

$$N_{t+j}(i) = N_{t+j} \left(\frac{P_t^N(i)\Psi_{t,j}^N}{P_{t+j}^N} \right)^{-\theta_N}. \quad (66)$$

Hence, they solve:

$$\max_{P_t^N(i)} E_t \sum_{j=0}^{\infty} \alpha_N^j \Lambda_{t,t+j} N_{t+j}(i) \left\{ \frac{P_t^N(i)\Psi_{t,j}^N}{P_{t+j}^N} - \frac{S_{t+j}P_{t+j}^{*N}}{P_{t+j}^N} \right\}$$

subject to (66). After eliminating the firm index, the resulting first order condition is:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha_N)^j \bar{\Lambda}_{t+j} N_{t+j} (\pi_{t+j}^N)^{\theta_N} \left\{ \frac{\tilde{P}_t^N \pi_t^N}{P_t^N \pi_{t+j}^N} - \frac{\theta_N}{\theta_N - 1} \frac{S_{t+j}P_{t+j}^{*N}}{P_{t+j}^N} \right\}. \quad (67)$$

Since all optimizing firms make the same decision, we call the optimal import price \tilde{P}_t^N . Hence (64) and (65) imply the following law of motion for the aggregate domestic currency import price index:

$$(P_t^N)^{1-\theta_N} = \alpha_N (P_{t-1}^N \pi_{t-1}^N)^{1-\theta_N} + (1 - \alpha_N) (\tilde{P}_t^N)^{1-\theta_N}. \quad (68)$$

We now define the real exchange rate and the relative price between optimized and overall imported goods:

$$e_t \equiv \frac{S_t P_t^{*N}}{P_t}, \quad \tilde{p}_t^N \equiv \frac{\tilde{P}_t^N}{P_t^N}.$$

Hence, using our definition of the internal terms of trade p_t^N , we can express the preceding equations as:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha_N)^j \bar{\Lambda}_{t+j} N_{t+j} (\pi_{t+j}^N)^{\theta_N} \left\{ \frac{\tilde{p}_t^N \pi_t^N}{\pi_{t+j}^N} - \frac{\theta_N}{\theta_N - 1} \frac{e_{t+j}}{p_{t+j}^N} \right\}.$$

$$(\pi_t^N)^{1-\theta_N} = \alpha_N (\pi_{t-1}^N)^{1-\theta_N} + (1 - \alpha_N) (\tilde{p}_t^N \pi_t^N)^{1-\theta_N}.$$

4.2. Exported goods firms

Each of a continuum of intermediate exporting firms purchases the final domestic good at its price P_t (which is hence its marginal cost) and differentiates it to sell in different foreign markets with local currency pricing.

Final exported goods

The goods are purchased by a representative perfectly competitive final export firm that has a CES technology:

$$X_t = \left(\int_0^1 X_t(i)^{\frac{\theta^*-1}{\theta^*}} di \right)^{\frac{\theta^*}{\theta^*-1}}, \quad \theta^* > 1,$$

where θ^* is the elasticity of substitution in the rest of the world for the imported goods that originate in the SDE. Maximizing profit, as in the previous cases, gives the demand function each intermediate exporting firm faces from the final exporters:

$$X_t(i) = X_t \left(\frac{P_t^{*X}(i)}{P_t^{*X}} \right)^{-\theta^*}. \quad (69)$$

Note that the price $P_t^{*X}(i)$ is in foreign currency. The resulting foreign currency price index for exported goods is:

$$P_t^{*X} = \left(\int_0^1 P_t^{*X}(i)^{1-\theta^*} di \right)^{\frac{1}{1-\theta^*}}, \quad (70)$$

and the foreign currency cost bill for the representative final exporting (bundling) firm is:

$$\int_0^1 P_t^{*X}(i) X_t(i) di = P_t^{*X} X_t.$$

Note that in the demand function for exports (69), X_t is the rest of the world's imports from the SDE (which we can alternatively write as N_t^*) and P_t^{*X} is the rest of the world's aggregate import price from the SDE (not to be confused with the SDE's aggregate import price from the LRW P_t^{*N}). Hence, we can alternatively write (69) as:

$$X_t(i) = N_t^* \left(\frac{P_t^{*X}(i)}{P_t^{*N}} \right)^{-\theta^*}.$$

We further assume that the rest of the world's aggregate imports from the SDE X_t is related to its output (Q_t^*) and its output price index (P_t^*) by:

$$N_t^* \equiv X_t = x_t^* Q_t^* \left(\frac{P_t^{*X}}{P_t^*} \right)^{-\theta^*},$$

where x_t^* is an export demand shock. Note that the relative price in the last expression can be written as:

$$\frac{P_t^{*X}}{P_t^*} = \frac{P_t^{*X}}{P_t^{*N}} \frac{P_t^{*N}}{P_t^*} = p_t^* p_t^{N*},$$

where we defined

$$p_t^* \equiv \frac{P_t^{*X}}{P_t^{*N}}, \quad p_t^{N*} \equiv \frac{P_t^{*N}}{P_t^*}$$

as the SDE's external terms of trade and the LRW's internal terms of trade. The first of these relative prices is endogenous in our model due to exporters' price

setting, as we further elaborate below. The second is clearly exogenous in our model. Note that we do not assume that the law of one price prevails in the long run (non-stochastic steady state). In the context of monopolistic competition, any good produced by a firm in the domestic economy is not produced by any other firm in the world. Hence, the law of one price only means that any domestic good i must be sold in the rest of the world at the same price it sells domestically after expressing it in foreign currency: $P_t^{*X}(i) = P_t(i)/S_t$, and that any good i produced in the LRW must be sold domestically at the price $P_t^N(i) = S_t P_t^{*N}(i)$. We see no reason to assume such lack of market segmentation, even in the model's long run.

Intermediate exported goods

Intermediate export firms set prices in foreign currency taking the foreign price and quantity indexes P_t^* , N_t^* , as parameters. The local (foreign) currency pricing of intermediate exporting firms follows the same setup we used previously, with a probability $1 - \alpha_X$ of optimal price setting and full indexation when they can't change price. Hence, according to their price survival constraint they face a probability α_X^j of having the price they set at t survive (indexed) until $\alpha_X t + j$:

$$P_{t+j}^{*X}(i) = P_t^{*X}(i) \pi_t^{*X} \pi_{t+1}^{*X} \dots \pi_{t+j-1}^{*X} \equiv P_t^{*X}(i) \Psi_{t,j}^{*X}. \quad (71)$$

Hence, when taking (69) as a constraint, they must consider that there is a probability α_X^j that their demand in $t + j$ will be:

$$X_{t+j}(i) = X_{t+j} \left(\frac{P_t^{*X}(i) \Psi_{t,j}^{*X}}{P_{t+j}^{*X}} \right)^{-\theta^*}, \quad (72)$$

When they can set their optimal price they solve:

$$\max_{P_t^{*X}(i)} E_t \sum_{j=0}^{\infty} \alpha_X^j \Lambda_{t,t+j} X_{t+j}(i) \left\{ \frac{P_t^{*X}(i) \Psi_{t,j}^{*X}}{P_{t+j}^{*X}} - \frac{P_{t+j}}{S_{t+j} P_{t+j}^{*X}} \right\}$$

subject to (72). The first order condition is:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_X)^j \bar{\Lambda}_{t,t+j} X_{t+j} (\pi_{t+j}^{*X})^{\theta^*} \left\{ \frac{\tilde{P}_t^{*X} \pi_t^{*X}}{P_t^{*X} \pi_{t+j}^{*X}} - \frac{\theta^*}{\theta^* - 1} \frac{P_{t+j}}{S_{t+j} P_{t+j}^{*X}} \right\}.$$

Since all optimizing firms make the same decision we call the optimal foreign currency export price \tilde{P}_t^{*X} , and (70) and (71) imply the following law of motion for the aggregate price level of exports:

$$(P_t^{*X})^{1-\theta^*} = \alpha_X (P_{t-1}^{*X} \pi_{t-1}^{*X})^{1-\theta^*} + (1 - \alpha_X) \left(\tilde{P}_t^{*X} \right)^{1-\theta^*}. \quad (73)$$

To simplify these expressions as we did previously, note first that the own marginal cost of intermediate export firms is the inverse of the product of the SDE's RER and its external terms of trade:

$$\frac{P_t}{S_t P_t^{*X}} = \frac{1}{e_t p_t^*}.$$

Next, we define the relative price between optimizing and overall export prices:

$$\tilde{p}_t^{*X} \equiv \frac{\tilde{P}_t^{*X}}{P_t^{*X}},$$

and express the dynamic equations for export prices as:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_X)^j \bar{\Lambda}_{t+j} X_{t+j} (\pi_{t+j}^{*X})^{\theta^*} \left\{ \frac{\tilde{p}_t^{*X} \pi_t^{*X}}{\pi_{t+j}^{*X}} - \frac{\theta^*}{\theta^* - 1} \frac{1}{e_{t+j} p_{t+j}^*} \right\}.$$

$$(\pi_t^{*X})^{1-\theta^*} = \alpha_X (\pi_{t-1}^{*X})^{1-\theta^*} + (1 - \alpha_X) (\tilde{p}_t^{*X} \pi_t^{*X})^{1-\theta^*}.$$

5. A review of some important relative prices

Chart I highlights the international pricing of the model. The SDE's and the LRW's main monetary price indexes P_t , P_t^N , and P_t^* , P_t^{*X} , P_t^{*N} , respectively, are shown in the two central columns. For each there is a domestic price index and an imported price index, each in terms of its own currency. The two outer columns show the main relative prices. In each economy, the relative price between imported and domestic price indexes defines the domestic terms of trade (DTT): p_t^N and p_t^{N*} , respectively. In the LRW, however, we also distinguish an export price index P_t^{*N} , different from its domestic price index P_t^* . Hence, there is an additional relative price p_t^* between its import and export goods, both in its "domestic" currency (i.e. foreign currency), which is the SDE's external terms of trade. Also, in each economy a certain price index is converted into the corresponding export price index through local currency pricing (i.e. pricing in the partner's currency) and this is the trade partner's import price index. However, in the case of the SDE we do not distinguish between its domestic and export goods, so it is the domestic goods that are exported to the LRW. The solid arrows indicate the local currency pricing of exporters. Also, for each economy, the domestic price index is converted into the partner's currency through the exchange rate: P_t/S_t and $S_t P_t^{*N}$, respectively. Finally, in each economy the the RER is defined as the relative price between the partner's export bundle, converted to the domestic (or "domestic") currency through the nominal exchange rate, and the domestic bundle.

The SDE's real exchange rate (RER) is the relative price between imported goods as they are purchased in the LRW by importers and domestic goods, both expressed in a common currency:

$$e_t \equiv \frac{S_t P_t^{*N}}{P_t}. \quad (74)$$

Here, P_t^{*N} is the rest of the world's export price index and, hence, is an exogenous variable in our model. With the same definition, the LRW's RER turns out to be its export to domestic relative price divided by the SDE's RER:

$$e_t^* \equiv \frac{P_t/S_t}{P_t^*} = \frac{P_t}{S_t P_t^{*N}} \frac{P_t^{*N}}{P_t^*} = \frac{p_t^{*X}}{e_t}.$$

Chart I				
	SDE		LRW	
	Domestic Currency		Foreign Currency	
	Relative Prices	Monetary Prices	Monetary Prices	Relative Prices
		SP^{*N}	P/S	
RERs	$e=SP^{*N}/P$			$e^* = P/SP^* = p^{*X}/e$
		P	P^*	
Internal TT	$p^N = P^N/P$			$p^{N*} = P^{*X}/P^* = p^* p^{*X}$
		P^N	P^{*X}	
SDE's External TT				$p^* = P^{*X}/P^{*N}$
			P^{*N}	

The numerator is obviously exogenous in our model, but the denominator is clearly endogenous. Since the SDE is insignificant in size in relation to the LRW, its actions have no influence in the LRW's allocation of resources.

The SDE's internal terms of trade (ITT) is the relative price between imported and domestic goods as faced by households and domestic firms:

$$p_t^N \equiv \frac{P_t^N}{P_t}. \quad (75)$$

It is a ratio between two domestic currency prices. With the same definition, the LRW's ITT is a ratio between its imported and "domestic" goods prices (both in foreign currency):

$$p_t^{N*} \equiv \frac{P_t^{*X}}{P_t^*} = \frac{P_t^{*X}}{P_t^{*N}} \frac{P_t^{*N}}{P_t^*} = p_t^* p_t^{*X}, \quad (76)$$

and it is equal to the product of the SDE's XTT and the LRW's export to domestic relative price index.

6. Banks

We assume that there is a perfectly competitive banking industry. Banks, like firms, are owned by households, and are price takers in financial markets. They obtain funds in the international market B_t^{*B} , supply one period deposit facilities to households D_t , and use the proceeds to supply one period loans to firms and the government ($L_t = L_t^F + L_t^G$), lend (or borrow) in the interbank market, purchase (or sell) Central Bank bonds B_t^{CB} , and hold vault cash $M_t^{0,B}$ as well as regulatory reserves R_t^B in the Central Bank. Any interbank loans cancel out and profits are distributed to owners period by period, so the aggregate balance sheet constraint for the representative bank is:

$$L_t + B_t^{CB} + M_t^{0,B} + R_t^B = D_t + S_t B_t^{*B}. \quad (77)$$

We assume that vault cash is a (technical) fraction γ_t^B of deposits, and that interbank deposits are perfect substitutes for Central Bank bonds (so they earn the same interest rate i_t). Since we also assume that the Central Bank does not pay

interest on regulatory reserves, banks keep these at the minimum, which is assumed to be a proportion γ_t^R of deposits. Hence, (77) is equivalent to:

$$L_t + B_t^{CB} = (1 - \gamma_t^B - \gamma_t^R)D_t + S_t B_t^{*B} \quad (78)$$

We assume that the interest rate on banks' foreign debt is paid out in the following period. Since banks' business is (assumed to be) in domestic currency, they face exchange rate uncertainty. For every unit of foreign currency they repay they must expect to have pesos in the amount of

$$E_t \delta_{t+1}^e (1 + i_t^B),$$

where

$$\delta_t = \frac{S_t}{S_{t-1}}$$

and δ_t^e , are the rate and the expected rate of nominal peso depreciation. To add some additional inertia, we assume that a fraction β^B of banks has rational expectations and that the remaining fraction has simple static expectations by which

$$\delta_{t+1}^e = \delta_t.$$

Except for this heterogeneity in expectations, all banks are the same. Hence, on average the expected rate of nominal depreciation is:

$$\delta_{t+1}^e = \beta^B E_t \delta_{t+1} + (1 - \beta^B) \delta_t.$$

We also assume that must pay a premium on the international riskless rate i_t^* . Since we do not model the rest of the world, the risk premium (function) is exogenously given. It has an exogenous component ϕ_t^{*B} (a risk premium shock) as well as an endogenous component $p_*(\cdot)$ that is an increasing function of the trend adjusted (individual) bank foreign debt (see Turnovsky (2000) and Schmitt-Grohé and Uribe (2003)). Individual banks thus fully internalize the fact that their individual foreign debt decision determines the foreign currency interest rate they face, which is:

$$1 + i_t^B = (1 + i_t^*) \left[1 + \phi_t^{*B} + p_* \left(\frac{S_t B_t^{*B}}{P_t z_t} \right) \right]. \quad (79)$$

where z_t is the stochastic trend in productivity, and we assume $p_*' > 0$ and $p_*'' > 0$.

Banks have a real cost function that depends on the real deposit and loan creating activities of the bank. We assume this cost function is quadratic and implies that there are economies of scope between lending and deposit taking activities (see Freixas and Rochet (1997), chapter 3). Specifically, we assume the following real cost function:

$$\begin{aligned} C_t^B &= C^B(L_t, D_t, z_t P_t) = & (80) \\ &= \frac{1}{2} \left[a_L^B \left(\frac{L_t}{z_t P_t} \right)^2 + a_D^B \left(\frac{D_t}{z_t P_t} \right)^2 - 2a_0^B \left(\frac{L_t}{z_t P_t} \right) \left(\frac{D_t}{z_t P_t} \right) \right] \\ &= \frac{1}{2} \left[\frac{a_L^B L_t^2 + a_D^B D_t^2 - 2a_0^B L_t D_t}{(z_t P_t)^2} \right], & (a_L^B > a_0^B > 0, a_D^B > a_0^B). \end{aligned}$$

We make the assumption that

$$a^B \equiv a_L^B a_D^B - (a_0^B)^2 > 0.$$

The representative bank maximizes profit each period:

$$\begin{aligned} \Pi^B &= (1 + i_t^L) L_t + (1 + i_t) B_t^{CB} - (1 + i_t^D) D_t \\ &\quad - \delta_{t+1}^e (1 + i_t^B) S_t B_t^{*B} - C^B(L_t, D_t, z_t P_t) z_t P_t \end{aligned}$$

subject to its balance sheet constraint (78) and its supply of foreign funds constraint (79). The solution to this program gives the supply of loans and deposits in terms of the loan margin $i_t^L - i_t$ and the deposit margin $(1 - \gamma_t^B - \gamma_t^R)(1 + i_t) - (1 + i_t^D)$, and the optimal amount of foreign funding in the form of a "risk-adjusted uncovered interest parity" relation:

$$L_t^S = \frac{z_t P_t}{a^B} \{ a_D^B [(1 + i_t^L) - (1 + i_t)] + \tag{81}$$

$$+ a_0^B [(1 - \gamma_t^B - \gamma_t^R)(1 + i_t) - (1 + i_t^D)] \}$$

$$D_t^S = \frac{z_t P_t}{a^B} \{ a_L^B [(1 - \gamma_t^B - \gamma_t^R)(1 + i_t) - (1 + i_t^D)] + \tag{82}$$

$$+ a_0^B [(1 + i_t^L) - (1 + i_t)] \}$$

$$1 + i_t = [\beta^B E_t \delta_{t+1} + (1 - \beta^B) \delta_t] (1 + i_t^*) \varphi_B \left(\frac{S_t B_t^{*B}}{P_t z_t} \right), \tag{83}$$

where we defined following auxiliary function for the multiplicative gross risk adjustment to the uncovered interest parity:

$$\varphi_B \left(\frac{S_t B_t^{*B}}{P_t z_t} \right) \equiv 1 + \phi_t^{*B} + p_* \left(\frac{S_t B_t^{*B}}{P_t z_t} \right) + \left(\frac{S_t B_t^{*B}}{P_t z_t} \right) p'_* \left(\frac{S_t B_t^{*B}}{P_t z_t} \right). \tag{84}$$

Given our assumptions on $p_*(\cdot)$, the condition $a^B > 0$ is necessary and sufficient to ensure that the first order conditions yield maximum profits. The resulting optimal bank cost and (pre-tax) profit are:

$$\begin{aligned} C_t^B &= \frac{1}{2a^B} \{ a_D^B [i_t^L - i_t]^2 + a_L^B [(1 - \gamma_t^B - \gamma_t^R)(1 + i_t) - (1 + i_t^D)]^2 \\ &\quad + 2a_0^B [i_t^L - i_t] [(1 - \gamma_t^B - \gamma_t^R)(1 + i_t) - (1 + i_t^D)] \}. \end{aligned}$$

$$\frac{\Pi^B}{z_t P_t} = C_t^B + \left(\frac{S_t B_t^{*B}}{P_t z_t} \right)^2 p'_* \left(\frac{S_t B_t^{*B}}{P_t z_t} \right).$$

Given L_t^S , D_t^S , and B_t^{*B} , the aggregate bank demand for Central Bank bonds is given by the aggregate bank balance sheet constraint:

$$B_t^{C,D} = (1 - \gamma_t^B - \gamma_t^R) D_t^S + S_t B_t^{*B} - L_t^S. \tag{85}$$

7. The public sector

The public sector is made up of the Government and the Central Bank.

7.1. The Government

The Government issues foreign currency denominated bonds in the international markets, obtains loans from banks and pays interest on these loans, spends on goods, and collects taxes. We assume that fiscal policy consists of exogenous paths for nominal lump-sum tax collection (T_t), nominal bank loans (L_t^G), and real expenditures (G_t). It finances any resulting deficit by issuing foreign currency denominated bonds (B_t^{*G}). The exogenous paths are assumed to be compatible with a finite non-stochastic steady state for government debt. To hold foreign currency denominated government bonds, foreign investors charge a risk premium over the risk-free foreign interest rate (i_t^*). As in the case of banks, the risk premium (function) is exogenously given and is assumed to have an exogenous stochastic component (an external financing shock) and an endogenous component which is an increasing function of the trend adjusted public sector net foreign liability. Hence the gross interest rate on the government's foreign debt is:

$$1 + i_t^G = (1 + i_t^*) \left[1 + \phi_t^{*G} + p_G \left(\frac{S_t (B_t^{*G} - R_t^{*CB})}{P_t z_t} \right) \right]. \quad (86)$$

where $p'_G > 0$, and R_t^{*CB} is the Central Bank's international reserves.

The Government flow budget constraint is:

$$S_t B_t^{*G} = P_t G_t + i_t^L L_t^G - T_t + (1 + i_{t-1}^G) S_t B_{t-1}^{*G}. \quad (87)$$

To simplify, we assume the interest on bank loans is paid by the government within the period.

7.2. The Central Bank

The Central Bank issues currency (M_t^0), domestic currency bonds (B_t^{CB}), and debt certificates to banks for non-remunerated reserves (R_t^B), and holds international reserves (R_t^{*CB}) in the form of foreign currency denominated riskless bonds issued abroad. We assume that Central Bank bonds are only held by domestic banks. The (flow) budget constraint of the Central Bank is:

$$\begin{aligned} M_t^0 + B_t^{CB} + R_t^B - S_t R_t^{*CB} &= M_{t-1}^0 + (1 + i_{t-1}) B_{t-1}^{CB} - R_{t-1}^{*B} - (1 + i_{t-1}^*) S_t R_{t-1}^{*CB} = \\ & \quad (88) \\ & [M_{t-1}^0 + B_{t-1}^{CB} + R_{t-1}^B - S_{t-1} R_{t-1}^{*CB}] + [i_{t-1} B_{t-1}^{CB} - i_{t-1}^* S_t R_{t-1}^{*CB} - (S_t - S_{t-1}) R_{t-1}^{*CB}] \end{aligned}$$

We assume that the Central Bank transfers its real quasi-fiscal surplus or deficit to the Government every period. This includes all the factors that would otherwise change the net worth of the Central Bank: interest earned and capital gains on its international reserves net of interest paid on its bonds, i.e. the second term in square brackets in (88). Hence, the Central Bank's balance sheet constraint is always preserved:

$$M_t^0 + R_t^B = S_t R_t^{*CB} - B_t^{CB}. \quad (89)$$

In our model, this equation implicitly defines the Central Bank's backing of its monetary base ($M_t^0 + R_t^B$) with its international reserves net of its bond liabilities. The Central Bank supplies whatever monetary base is demanded by households

and banks, and can influence these supplies by changing R_t^{*CB} or B_t^{CB} (intervene in the foreign exchange or interbank markets).

Adding (87) and (88) gives the consolidated public sector budget constraint:

$$M_t^0 + B_t^{CB} + R_t^B + S_t (B_t^{*G} - R_t^{*CB}) = P_t G_t - T_t + i_t^L L_t^G + M_{t-1}^0 + \quad (90)$$

$$+ (1 + i_{t-1}) B_{t-1}^{CB} + R_{t-1}^B + (1 + i_{t-1}^G) S_t B_{t-1}^{*G} - (1 + i_{t-1}^*) S_t R_{t-1}^{*CB}.$$

8. Market clearing equations, the balance payments and GDP

8.1. Market clearing

In the physical capital rental market, market clearing implies that the household supply at the optimal intensity level equals domestic firms' demand:

$$(\tau'_u)^{-1} (i_t^K) K_t = ab \frac{mc_t}{(1 + v_t^W i_t^L) i_t^K} [Q_t + z_t F^D]. \quad (91)$$

In the labor market, the household supply h_t must equal domestic firms' demand:

$$h_t = (1 - a)b \frac{mc_t}{(1 + v_t^W i_t^L) w_t} [Q_t + z_t F^D]. \quad (92)$$

In the loan market we have $L_t^S = L_t$, where the latter is loan demand by firms and the government. Hence, from (58) we obtain:

$$\frac{L_t}{P_t} = f_L (1 + i_t^L) mc_t [Q_t + z_t F^D] + \frac{L_t^G}{P_t}. \quad (93)$$

Note that in the last three equations mc_t is given by (57).

In the deposit market we have $D_t^S = D_t$, where the latter is deposit demand by households. Hence, combining this with (82) yields:

$$\frac{D_t}{P_t} = z_t \frac{a_L^B [(1 - \gamma_t^B - \gamma_t^R) (1 + i_t) - (1 + i_t^D)] + a_0^B [(1 + i_t^L) - (1 + i_t)]}{a_L^B a_D^B - (a_0^B)^2} \quad (94)$$

In the interbank cum Central Bank bond market, interbank loans cancel out and Central Bank supply B_t^{CB} must equal aggregate bank demand $B_t^{CB,D}$ as given by (85):

$$B_t^{CB} = (1 - \gamma_t^B - \gamma_t^R) D_t + S_t B_t^{*B} - L_t, \quad (95)$$

where Central Bank supply is derived from its balance sheet constraint (89).

In the currency market, the supply of currency must equal household and bank demand:

$$M_t^0 = \mathcal{L} (1 + i_t^D) [P_t^C C_t + P_t^V V_t] + \gamma_t^B D_t, \quad (96)$$

where the Central Bank supply is again derived from (89).

In the domestic goods market, the output of domestic firms Q_t must satisfy final demand from households, the government, and the LRW, as well as intermediate demand for abnormal capital utilization costs, transaction costs, and bank costs:

$$Q_t = a_D p_t^C C_t + b_D p_t^V V_t + G_t + X_t + \tau_u ((\tau'_u)^{-1} (i_t^K)) K_t \quad (97)$$

$$+ \tilde{\tau}_M (1 + i_t^D) (p_t^C C_t + p_t^V V_t) + z_t C_t^B,$$

where C_t^B are the real resources used up by the banking sector, as given by (80).

8.2. The balance of payments

Total imports N_t , is the sum of household and firm demand:

$$P_t^N N_t = (1 - a_D) P_t^C C_t + (1 - b_D) P_t^V V_t + P_t^N N_t^F. \quad (98)$$

The nominal aggregate household budget constraint (where the Υ_t cancel out) can be written as:

$$\begin{aligned} (M_t^{0,H} - M_{t-1}^{0,H}) + (D_t - D_{t-1}) &= \Pi_t + W_t h_t + [I_t^K u_t - \tau_u(u_t)] P_t K_t \\ + i_{t-1}^D D_{t-1} - [1 + \tilde{\tau}_M ((1 + i_t^D))] & (P_t^C C_t + P_t^V V_t) - T_t. \end{aligned} \quad (99)$$

Here Π_t is the sum of profits from all three types of firms (domestic, export, and import) as well as banks:

$$\begin{aligned} \Pi_t &= \Pi_t^D + \Pi_t^X + \Pi_t^N + \Pi_t^B = [P_t Q_t - W_t h_t - P_t i_t^K u_t K_t - P_t^N N_t^F - i_t^L L_t^F] \\ &+ \{P_t^N [(1 - a_D) p_t^C C_t + (1 - b_D) p_t^V V_t + N_t^F] - S_t P_t^{*N} N_t\} \\ &+ [S_t P_t^{*X} X_t - P_t X_t] \\ &+ \{(i_t^L - i_t) L_t + [(1 - \gamma_t^B - \gamma_t^R) (1 + i_t) - (1 + i_t^D)] D_t \\ &- (1 + i_t^B) S_t B_t^{*B} - P_t C_t^B\}. \end{aligned} \quad (100)$$

Consolidating (90), (99) and (100), taking into account (97) and the consolidated balance sheet constraint of banks and firms yields the balance of payments constraint:

$$\begin{aligned} (R_t^{*CB} - R_{t-1}^{*CB}) - (B_t^{*G} - B_{t-1}^{*G}) - (B_t^{*B} - B_{t-1}^{*B}) &= P_t^{*X} X_t - P_t^{*N} N_t \\ + i_{t-1}^* R_{t-1}^{*CB} - i_{t-1}^G B_{t-1}^{*G} - i_{t-1}^B B_{t-1}^{*B}. \end{aligned} \quad (101)$$

8.3. GDP

Using (36), (43), and (98), we can express domestic output (97) as:

$$\begin{aligned} Q_t &= p_t^C C_t + p_t^V V_t + X_t + G_t - p_t^N (C_t^N + V_t^N - Q_t^N) + Q_t^D + p_t^N Q_t^N \\ &= (p_t^C C_t + p_t^V V_t + X_t + G_t - p_t^N N_t) + Q_t^D + p_t^N Q_t^N \\ &= Y_t + Q_t^D + p_t^N Q_t^N. \end{aligned}$$

where we defined intermediate output of domestic and imported origin and real GDP in terms of domestic goods as:

$$\begin{aligned} Q_t^D &= \tau_u((\tau'_u)^{-1}(i_t^K)) K_t + \tilde{\tau}_M (1 + i_t^D) (p_t^C C_t + p_t^V V_t) + z_t C_t^B \\ Q_t^N &= N_t^F. \end{aligned}$$

$$Y_t = p_t^C C_t + p_t^V V_t + X_t + G_t - p_t^N N_t. \quad (102)$$

9. Monetary Policy

We have endeavored to include banks and the central bank with some detail in order to be able to consider alternative monetary (including exchange rate) policies within a unified framework. In the model, the Central Bank, through its regular interventions in the interbank and foreign exchange markets, is able to aim for

the achievement of two operational targets: the interbank interest rate i_t , and the nominal rate of currency depreciation δ_t , through two corresponding policy feedback rules. We define alternative monetary policies according to the nominal anchor that prevails and how the operational targets and feedback rules are defined. In particular, we consider two broad classes of monetary policies: crawling pegs, in which the nominal anchor is the nominal exchange rate, and inflation targeting, in which the nominal anchor is the target rate of inflation. In the case of crawling pegs, the Central Bank mainly intervenes in the foreign exchange market, aiming to achieve a certain rate of nominal depreciation of the domestic currency. In the case of inflation targeting, the Central Bank mainly intervenes in the interbank market, aiming to achieve a certain operational target for the (short run) nominal interest rate that it considers appropriate for reaching a target inflation rate. There are consequently two *pure* monetary policies. In the case of a Crawling Peg with Pure Interest Rate Float (CP-PIF) policy, the Central Bank does not actively intervene at all in the interbank market. By this we mean that the Central Bank's international reserves grow at the economy's trend growth rate. In the particular case of a fixed exchange rate, we can think of this regime as one in which there is a Currency Board, which is a very restricted kind of Central Bank. And in the case of an Inflation Targeting with Pure Exchange Rate Float (IT-PEF), the Central Bank does not actively intervene at all in the foreign exchange market, by which we mean that its peso bond liability grows with the economy's trend.

The latter is the case that draws the greatest attention in the literature, due perhaps to the much higher degree of exchange rate flexibility that exists in developed countries and their high and increasing use of an inflation targeting monetary policy. In a few developed countries, such as the U.S.A. and Japan, foreign exchange market interventions are sporadic and mainly have a signaling purpose. However, this is not the typical situation, and in most developed countries foreign exchange market intervention with the purpose of influencing the nominal exchange rate is quite common (see Bofinger and Wollmerhäuser (2001) and Wollmerhäuser (2003)). In developing countries daily exchange market intervention is even more frequent and is often the most important policy action that the Central Bank exerts.

Since our model is mainly intended to be used in developing countries (though not necessarily exclusively), we construct it so as to allow for a wide range of alternative monetary policies. In the following we consider the two "pure" (atypical) extremes just mentioned, but also some of the "mixed" monetary policies. In particular, we consider inflation targeting with a managed exchange rate float (IT-MEF), in which the Central Bank pursues an inflation rate target through a feedback rule for the operational target for the interest rate and simultaneously has an active intervention policy in the foreign exchange market, with a feedback rule on its international reserves that tends to "lean against the wind" of movements in the nominal exchange rate that exceed (or fall short of) an operational rate of depreciation which is subordinate to the inflation rate target.

Below we consider these alternative monetary policies more explicitly. The focus is more on obtaining a consistent framework that can deal with the actual complexities of monetary policies in developing countries than on the proposal or analysis of a particular "mixed" monetary policy in which the Central Bank

simultaneously actively intervenes in both the interbank (or money) market and the foreign exchange market.

9.1. Pure Exchange rate Crawl regimes (PEC)

We define a pure exchange rate crawl regime as one where the Central Bank abstains from actively intervening in the money market. Hence, it maintains its real liabilities in domestic currency bonds growing along with the economy's trend growth:

$$\frac{B_t^{CB}}{P_t} = b_0^{CB} z_t \quad \forall t.$$

Also, the Central Bank pegs the nominal exchange rate to the foreign currency by intervening in the foreign exchange market so as to ensure that the rate of nominal depreciation follows a predetermined target path $\{\delta_t^T\}$ such that $S_t/S_{t-1} = \delta_t^T$, for all t . We restrict attention to paths that converge to a constant δ^T in a finite time. This implies that the Central Bank purchases any excess supply or satisfies any excess demand of foreign exchange that the private sector may have at the nominal exchange rate $S_t = S_{t-1}\delta_t^T$. We could formalize this as an infinitely fast feedback rule in which the Central Bank counteracts (excessive) nominal appreciations (depreciations) by purchasing (selling) international reserves (thus "leaning against the wind"). In the case considered here the Central Bank counteracts any deviation whatsoever of the rate of nominal depreciation from its target path. Hence, the stock of international reserves is endogenous and the following equation must be included in the system:

$$\delta_t = \delta_t^T \quad \forall t. \quad (103)$$

In the particular case of a fixed crawling peg policy the nominal rate of depreciation is kept at a constant level δ^T , and in the particular case of a fixed exchange rate policy, that constant level is unity.

9.2. Inflation Targeting regimes

Under Inflation Targeting there are various possibilities for monetary policy feedback rules that can define the Central Bank's operational target for the nominal (domestic currency) interest rate i_t . A fairly general one is one where the Central Bank simultaneously responds to deviations of the gross domestic inflation rate from a target path $\{\pi_t^T\}$ that converges to a constant π^T , to deviations of the trend adjusted output level from a target path $\{(Y_t/z_t)^T\}$ (that converges to the long run average (or non-stochastic steady state) detrended output y), and possibly also to deviations of the gross wage inflation from a target path $\{\pi_t^{wT}\}$ (that converges to the same constant π^T as the domestic inflation target). All these target paths, if they are time varying, are assumed to converge to a constant in finite time. We also introduce a preference for slow changes in the nominal interest rate through

the presence of the lagged interest rate:

$$1 + i_t = \left[\frac{\pi^T}{\pi^{*N}} (1 + i^*) \varphi_B \left(\frac{B^{*B} e}{z P^{*N}} \right) \right]^{1-h_0} (1 + i_{t-1})^{h_0} \quad (104)$$

$$\left(\frac{\pi_t}{\pi_t^T} \right)^{h_1} \left(\frac{\pi_t^w}{\pi_t^{wT} \mu_t^z} \right)^{h_2} \left(\frac{Y_t/z_t}{(Y_t/z_t)^T} \right)^{h_3},$$

$$h_0 \in [0, 1], h_1 > 1, h_2 \geq 0, h_3 \geq 0.$$

Several comments are in order. The first (rather awkward) multiplicative term in the feedback rule is specially designed to have a consistent non-stochastic steady state for the model. We deal with the steady state at length in an Appendix. Second, we assume that this rule has the so-called "Taylor property" ($h_1 > 1$) whereby the Central Bank responds to excess expected goods inflation by increasing the expected real interest rate (and not merely the nominal interest rate). Third, we assume that the interest rate smoothing coefficient h_0 is not greater than one, but note that we could have greater generality by allowing for "superinertial" policy rules for the nominal interest rate ($h_0 > 1$) (see Woodford (2003), chapter 2). Fourth, a variant of the interest rate feedback rule makes the Central Bank respond to consumer inflation. In that case π_t must be replaced by π_t^C in (104), which, according to (35) is:

$$\pi_t^C = \left[\frac{1}{1 + \frac{a_N}{a_D} (p_{t-1}^N)^{1-\theta_C}} (\pi_t)^{1-\theta_C} + \left(1 - \frac{1}{1 + \frac{a_N}{a_D} (p_{t-1}^N)^{1-\theta_C}} \right) (\pi_t^N)^{1-\theta_C} \right]^{\frac{1}{1-\theta_C}}.$$

Fifth, another variant for the feedback rule has a forward looking reaction function, replacing the deviation of inflation from target by the expected deviation for next period.

Inflation Targeting under a Pure Exchange rate Float regime (IT-PEF)

We define an Inflation Targeting under a Pure Exchange Rate Float (IT-PEF), as one where additionally the Central Bank abstains from actively intervening in the foreign exchange market. By this we mean that it maintains the real value of its international reserves growing along with the economy's trend growth:

$$\frac{R_t^{*CB}}{P_t^{*N}} = r_0 z_t \quad \forall t.$$

Inflation Targeting under a Managed Exchange rate Float regime (IT-MEF)

Alternatively, we define an inflation targeting under a managed exchange rate float (IT-MEF) as one in which the Central Bank actively intervenes in the foreign exchange market. We assume that aside from its operational target for the nominal interest rate, the Central Bank also has an operational target for the level of international reserves. The following is one possible feedback rule for the international reserves, in which the Central Bank tends to "lean against" real appreciations or

depreciations (last multiplicative term), has a preference for smoothing the variations in the level of international reserves, and where it also has a long run target (γ^T) for the fraction of total financial system liabilities that are backed by Central Bank international reserves:

$$\frac{R_t^{*CB}}{P_t^{*N} z_t} = \left(\gamma^T \frac{(M^{0,H} + D)}{Pze} + \frac{B^{*B}}{zP^{*N}} \right)^{1-k_0} \left(\frac{R_{t-1}^{*CB}}{P_{t-1}^{*N} z_{t-1}} \right)^{k_0} \left(\frac{\delta_t \pi_t^{*N}}{\pi_t} \right)^{-k_1}$$

$$k_0 \in (0, 1), k_1 > 0.$$

The first multiplicative term is designed so as to have a consistent steady state in which the reserves target is satisfied. Note that under this policy feedback rule the Central Bank does not aim at any specific level of the nominal exchange rate. However, it does have a policy of "leaning against the wind" by increasing the purchase of reserves whenever there is real peso appreciation ($\delta_t \pi_t^{*N} < \pi_t$). The nominal anchor is still clearly the target inflation rate, as when there is a pure float.

10. Putting (most of) the non-linear system together

In this section we put together the non-linear equations thus far encountered that are common to all the monetary regime models. For clarity, we gather the equations in a few categories and give each a distinctive name that characterizes it:

Dynamic equations:

Consumption:

$$\frac{z_t^C}{C_t - \xi C_{t-1}} - \beta \xi E_t \left(\frac{z_{t+1}^C}{C_{t+1} - \xi C_t} \right) = \lambda_t \tilde{\varphi}_M (1 + i_t^D)$$

Investment:

$$\zeta_t z_t^V \varphi_V \left(\frac{V_t}{V_{t-1}} \right) + \beta E_t \left\{ \zeta_{t+1} z_{t+1}^V \tau'_V \left(\frac{V_{t+1}}{V_t} \right) \left(\frac{V_{t+1}}{V_t} \right)^2 \right\}$$

$$= \lambda_t \tilde{\varphi}_M (1 + i_t^D)$$

Marginal utility of installed physical capital:

$$\zeta_t = \beta E_t \left\{ (1 - \delta^K) \zeta_{t+1} + \lambda_{t+1} \Gamma^K (i_{t+1}^K) \right\}$$

Marginal utility of real income:

$$\lambda_t = \beta (1 + i_t^D) E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right)$$

Physical capital accumulation:

$$K_{t+1} = (1 - \delta^K) K_t + z_t^V V_t \left[1 - \tau_V \left(\frac{V_t}{V_{t-1}} \right) \right],$$

Wage Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_W)^j \lambda_{t+j} w_{t+j} h_{t+j} (\pi_{t+j}^w)^\psi \left\{ \left(\frac{\tilde{w}_t \pi_t^w}{\pi_{t+j}^w} \right) - \frac{\psi}{\psi-1} \frac{\eta_H z_{t+j}^H h_{t+j}^X}{\lambda_{t+j} w_{t+j}} \left(\frac{\tilde{w}_t \pi_t^w}{\pi_{t+j}^w} \right)^{-\psi \chi} \right\}$$

$$(\pi_t^w)^{1-\theta} = \alpha_W (\pi_{t-1}^w)^{1-\theta} + (1 - \alpha_W) (\tilde{w}_t \pi_t^w)^{1-\theta}$$

Domestic price Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \bar{\Lambda}_{t+j} Q_{t+j} (\pi_{t+j})^\theta \left\{ \frac{\tilde{p}_t \pi_t}{\pi_{t+j}} - \frac{\theta}{\theta-1} m c_{t+j} \right\},$$

$$\pi_t^{1-\theta} = \alpha \pi_{t-1}^{1-\theta} + (1 - \alpha) (\tilde{p}_t \pi_t)^{1-\theta}.$$

Imported goods price Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_N)^j \bar{\Lambda}_{t+j} N_{t+j} (\pi_{t+j}^N)^{\theta_N} \left\{ \frac{\tilde{p}_t^N \pi_t^N}{\pi_{t+j}^N} - \frac{\theta_N}{\theta_N-1} \frac{e_{t+j}}{p_{t+j}^N} \right\}.$$

$$(\pi_t^N)^{1-\theta_N} = \alpha_N (\pi_{t-1}^N)^{1-\theta_N} + (1 - \alpha_N) (\tilde{p}_t^N \pi_t^N)^{1-\theta_N}.$$

Exported goods price Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_X)^j \bar{\Lambda}_{t+j} X_{t+j} (\pi_{t+j}^{*X})^{\theta^*} \left\{ \frac{\tilde{p}_t^{*X} \pi_t^{*X}}{\pi_{t+j}^{*X}} - \frac{\theta^*}{\theta^*-1} \frac{1}{e_{t+j} p_{t+j}^{*X}} \right\}.$$

$$(\pi_t^{*X})^{1-\theta^*} = \alpha_X (\pi_{t-1}^{*X})^{1-\theta^*} + (1 - \alpha_X) (\tilde{p}_t^{*X} \pi_t^{*X})^{1-\theta^*}.$$

Identities:

$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}$$

$$\frac{p_t^N}{p_{t-1}^N} = \frac{\pi_t^N}{\pi_t}$$

$$\frac{p_t^{*X}}{p_{t-1}^{*X}} = \frac{\pi_t^{*X}}{\pi_t^*}$$

$$\frac{e_t}{e_{t-1}} = \frac{\delta_t \pi_t^{*N}}{\pi_t}$$

Fiscal:

$$\frac{e_t B_t^{*G}}{P_t^{*N}} = G_t + \frac{i_t^L L_t^G}{P_t} - \frac{T_t}{P_t} \tag{106}$$

$$+ (1 + i_{t-1}^*) \left[1 + \phi_{t-1}^{*G} + p_G \left(\left(\frac{B_{t-1}^{*G}}{z_{t-1} P_{t-1}^{*N}} - \frac{R_{t-1}^{*CB}}{z_{t-1} P_{t-1}^{*N}} \right) e_{t-1} \right) \right] \frac{e_t B_{t-1}^{*G}}{\pi_t^{*N} P_{t-1}^{*N}}.$$

Balance of Payments:

$$\begin{aligned} \frac{R_t^{*CB}}{P_t^{*N}} - \frac{B_t^{*G}}{P_t^{*N}} - \frac{B_t^{*B}}{P_t^{*N}} &= p_t^{*X} X_t - N_t + (1 + i_{t-1}^*) \frac{R_{t-1}^{*CB}}{\pi_t^{*N} P_{t-1}^{*N}} \\ -(1 + i_{t-1}^*) \left[1 + \phi_{t-1}^{*G} + p_G \left(\left(\frac{B_{t-1}^{*G}}{z_{t-1} P_{t-1}^{*N}} - \frac{R_{t-1}^{*CB}}{z_{t-1} P_{t-1}^{*N}} \right) e_{t-1} \right) \right] &\frac{B_{t-1}^{*G}}{\pi_t^{*N} P_{t-1}^{*N}} \\ -(1 + i_{t-1}^*) \left[1 + \phi_{t-1}^{*B} + p_* \left(\frac{B_{t-1}^{*B} e_{t-1}}{z_{t-1} P_{t-1}^{*N}} \right) \right] &\frac{B_{t-1}^{*B}}{\pi_t^{*N} P_{t-1}^{*N}}. \end{aligned} \quad (107)$$

Bank arbitrage:

$$1 + i_t = [\beta^B E_t \delta_{t+1} + (1 - \beta^B) \delta_t] (1 + i_t^*) \varphi_B \left(\frac{S_t B_t^{*B}}{P_t z_t} \right)$$

Static equations:

Market balance equations: Physical capital rental market:

$$(\tau'_u)^{-1} (i_t^K) K_t = \frac{abmc_t}{(1 + v_t^K i_t^L) i_t^K} [Q_t + z_t F^D]$$

Labor market:

$$h_t = \frac{(1 - a) bmc_t}{(1 + v_t^W i_t^L) w_t} [Q_t + z_t F^D]$$

Loan market:

$$\frac{L_t}{P_t} = f_L (1 + i_t^L) mc_t [Q_t + z_t F^D] + \frac{L_t^G}{P_t}.$$

Deposit market:

$$\frac{D_t}{P_t} = \frac{z_t}{a^B} \left\{ a_L^B [(1 - \gamma_t^B - \gamma_t^R) (1 + i_t) - (1 + i_t^D)] + a_0^B [(1 + i_t^L) - (1 + i_t)] \right\}$$

Interbank cum Central Bank bond market:

$$\frac{B_t^{CB}}{P_t} = (1 - \gamma_t^B - \gamma_t^R) \frac{D_t}{P_t} + e_t \frac{B_t^{*B}}{P_t^{*N}} - \frac{L_t}{P_t}.$$

Cash market:

$$\frac{M_t^0}{P_t} = \mathcal{L} (1 + i_t^D) [p_t^C C_t + p_t^V V_t] + \gamma_t^B \frac{D_t}{P_t}$$

Domestic goods market:

$$Q_t = Y_t + Q_t^D + \frac{(1 - b) mc_t}{1 + v_t^N i_t^L} [Q_t + z_t F^D].$$

Other static equations: Real GDP

$$Y_t = p_t^C C_t + p_t^V V_t + X_t + G_t - p_t^N N_t.$$

Intermediate demand for domestic goods:

$$Q_t^D = \tilde{\tau}_M (1 + i_t^D) (p_t^C C_t + p_t^V V_t) + z_t C_t^B + \tau_u ((\tau_u')^{-1} (i_t^K)) K_t$$

Loan supply:

$$\frac{L_t}{P_t} = \frac{z_t}{a^B} \left\{ a_D^B [(1 + i_t^L) - (1 + i_t)] + a_0^B [(1 - \gamma_t^B - \gamma_t^R) (1 + i_t) - (1 + i_t^D)] \right\}$$

Bank real cost:

$$C_t^B = \frac{1}{2} \left[a_L^B \left(\frac{L_t}{z_t P_t} \right)^2 + a_D^B \left(\frac{D_t}{z_t P_t} \right)^2 - 2a_0^B \left(\frac{L_t}{z_t P_t} \right) \left(\frac{D_t}{z_t P_t} \right) \right].$$

Central Bank balance sheet:

$$\frac{B_t^{CB}}{P_t} = e_t \frac{R_t^{*CB}}{P_t^{*N}} - \frac{M_t^0}{P_t} - \gamma_t^R \frac{D_t}{P_t}.$$

Real marginal cost:

$$mc_t = \frac{1}{\kappa \epsilon_t} f_{MC} (1 + i_t^L) (i_t^K)^{ab} \left(\frac{w_t}{z_t} \right)^{(1-a)b} (p_t^N)^{1-b}.$$

Export demand:

$$X_t = x_t^* Q_t^* (p_t^{*X})^{-\theta^*}$$

Import demand:

$$p_t^N N_t = (1 - a^D) p_t^C C_t + (1 - b^D) p_t^V V_t + \frac{(1 - b) mc_t}{1 + v_t^N i_t^L} [Q_t + z_t F^D],$$

Consumption relative price:

$$p_t^C = \left[a^D + (1 - a^D) (p_t^N)^{1-\theta_C} \right]^{\frac{1}{1-\theta_C}}.$$

Investment relative price:

$$p_t^V = \left[b^D + (1 - b^D) (p_t^N)^{1-\theta_V} \right]^{\frac{1}{1-\theta_V}}.$$

In the preceding we have used (79) and (86) to substitute for the banks' and the government's external financing interest rate, respectively.

So far we have 37 equations to determine the following 39 endogenous variables:

Rates of return: i_t^K, i_t, i_t^L, i_t^D , (4)

Rates of inflation: $\pi_t^w, \pi_t, \pi_t^N, \pi_t^{*X}, \delta_t$, (5)

Relative prices: $p_t^N, p_t^{*X}, p_t^C, p_t^V, w_t, e_t, \tilde{w}_t, \tilde{p}_t, \tilde{p}_t^N, \tilde{p}_t^{*X}, mc_t$, (11)

Flows: $C_t, V_t, h_t, N_t, X_t, Q_t, Q_t^D, C_t^B, Y_t$ (9)

Stocks: $K_t, M_t^0/P_t, R_t^{*CB}/P_t^{*N}, B_t^{*B}/P_t^{*N}, B_t^{*G}/P_t^{*N}, B_t^{CB}/P_t, D_t/P_t, L_t/P_t,$
(8)

Lagrange multipliers: $\lambda_t, \zeta_t, (2)$.

Hence, we have room for the two monetary policy equations that define the alternative monetary regimes. Instead of listing them now again (see the section on the Central Bank), we will do so in the next section, where we put the model in terms of stationary variables. An additional system equation giving the dynamics of the rate of technological growth with the corresponding additional endogenous variable (the rate of technological growth) will be appended when we specify the assumptions on the relation between the SDE's and the LRW's growth rates below.

11. The non-linear equations in stationary format

In order to have a well defined steady state we need to express the system's equations in terms of stationary variables. The only source of growth in this model is technological progress, so we use lower case letters to express upper case letter variables when deflated by the permanent technology shock in the production of domestic goods z_t , and add a superscript \circ to the Lagrange multipliers to denote that they are inflated by the same factor:

$$\begin{aligned} \bar{w}_t &= \frac{w_t}{z_t} = \frac{W_t}{P_t z_t}, & c_t &= \frac{C_t}{z_t}, & v_t &= \frac{V_t}{z_t}, & q_t &= \frac{Q_t}{z_t}, & y_t &= \frac{Y_t}{z_t}, \\ q_t^D &= \frac{Q_t^D}{z_t^*}, & k_{t+1} &= \frac{K_{t+1}}{z_t}, & n_t &= \frac{N_t}{z_t}, & x_t &= \frac{X_t}{z_t}, & g_t &= \frac{G_t}{z_t}, \\ m_t^0 &= \frac{M_t^0}{P_t z_t}, & d_t &= \frac{D_t}{P_t z_t}, & b_t^{CB} &= \frac{B_t^{CB}}{P_t z_t}, & r_t^{*CB} &= \frac{R_t^{*CB}}{P_t^{*N} z_t}, \\ b_t^{*B} &= \frac{B_t^{*B}}{P_t^{*N} z_t}, & b_t^{*G} &= \frac{B_t^{*G}}{P_t^{*N} z_t}, & t_t &= \frac{T_t}{P_t z_t}, & \ell_t &= \frac{L_t}{P_t z_t}, & \ell_t^G &= \frac{L_t^G}{P_t z_t}, \\ \lambda_t^\circ &= \lambda_t z_t, & \zeta_t^\circ &= \zeta_t z_t, & \bar{\Lambda}_t^\circ &= \bar{\Lambda}_t z_t, & q_t^* &= \frac{Q_t^*}{z_t^*}, & z_t^{**} &= \frac{z_t^*}{z_t}. \end{aligned}$$

We also define the growth rate of the permanent technology shock:

$$\mu_t^z \equiv \frac{z_t}{z_{t-1}}. \quad (108)$$

Note that z_t^{**} is the LRW's permanent technology shock relative to the SDE's, which below we assume is stationary. Hence, we rewrite the equations of the nonlinear system in stationary format as:

Dynamic equations:

Consumption:

$$\mu_t^z \left(\frac{z_t^C}{c_t \mu_t^z - \xi c_{t-1}} \right) - \beta \xi E_t \left(\frac{z_{t+1}^C}{c_{t+1} \mu_{t+1}^z - \xi c_t} \right) = \lambda_t^\circ \tilde{\varphi}_M (1 + i_t^D)$$

Investment:

$$\begin{aligned} \zeta_t^\circ z_t^V \varphi_V \left(\frac{v_t}{v_{t-1}} \mu_t^z \right) + \beta E_t \left\{ \frac{\zeta_{t+1}^\circ z_{t+1}^V}{\mu_{t+1}^z} \tau_V' \left(\frac{v_{t+1}}{v_t} \mu_{t+1}^z \right) \left(\frac{v_{t+1}}{v_t} \mu_{t+1}^z \right)^2 \right\} \\ = \lambda_t^\circ \tilde{\varphi}_M (1 + i_t^D) \end{aligned}$$

Marginal utility of installed physical capital:

$$\zeta_t^\circ = \beta E_t \left\{ (1 - \delta^K) \left(\frac{\zeta_{t+1}^\circ}{\mu_{t+1}^z} \right) + \left(\frac{\lambda_{t+1}^\circ}{\mu_{t+1}^z} \right) \Gamma^K (i_{t+1}^K) \right\}$$

Marginal utility of real income:

$$\lambda_t^\circ = \beta (1 + i_t^D) E_t \left(\frac{\lambda_{t+1}^\circ}{\mu_{t+1}^z \pi_{t+1}} \right)$$

Physical capital accumulation:

$$k_{t+1} = (1 - \delta^K) \frac{k_t}{\mu_t^z} + z_t^V v_t \left[1 - \tau_V \left(\frac{v_t}{v_{t-1}} \mu_t^z \right) \right],$$

Wage Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_W)^j \lambda_{t+j}^\circ \bar{w}_{t+j} h_{t+j} (\pi_{t+j}^w)^\psi \left\{ \left(\frac{\tilde{w}_t \pi_t^w}{\pi_{t+j}^w} \right) - \frac{\psi}{\psi - 1} \frac{\eta_H z_{t+j}^H h_{t+j}^\chi}{\lambda_{t+j}^\circ \bar{w}_{t+j}} \left(\frac{\tilde{w}_t \pi_t^w}{\pi_{t+j}^w} \right)^{-\psi \chi} \right\}$$

$$(\pi_t^w)^{1-\theta} = \alpha_W (\pi_{t-1}^w)^{1-\theta} + (1 - \alpha_W) (\tilde{w}_t \pi_t^w)^{1-\theta}$$

Domestic goods price Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \bar{\Lambda}_{t+j}^\circ q_{t+j} (\pi_{t+j})^\theta \left\{ \frac{\tilde{p}_t \pi_t}{\pi_{t+j}} - \frac{\theta}{\theta - 1} m c_{t+j} \right\},$$

$$\pi_t^{1-\theta} = \alpha \pi_{t-1}^{1-\theta} + (1 - \alpha) (\tilde{p}_t \pi_t)^{1-\theta}.$$

Imported goods price Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_N)^j \bar{\Lambda}_{t+j}^\circ n_{t+j} (\pi_{t+j}^N)^{\theta_N} \left\{ \frac{\tilde{p}_t^N \pi_t^N}{\pi_{t+j}^N} - \frac{\theta_N}{\theta_N - 1} \frac{e_{t+j}}{p_{t+j}^N} \right\}.$$

$$(\pi_t^N)^{1-\theta_N} = \alpha_N (\pi_{t-1}^N)^{1-\theta_N} + (1 - \alpha_N) (\tilde{p}_t^N \pi_t^N)^{1-\theta_N}.$$

Exported goods price Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_X)^j \bar{\Lambda}_{t+j}^\circ x_{t+j} (\pi_{t+j}^{*X})^{\theta^*} \left\{ \frac{\tilde{p}_t^{*X} \pi_t^{*X}}{\pi_{t+j}^{*X}} - \frac{\theta^*}{\theta^* - 1} \frac{1}{e_{t+j} p_{t+j}^{*X}} \right\}.$$

$$(\pi_t^{*X})^{1-\theta^*} = \alpha_X (\pi_{t-1}^{*X})^{1-\theta^*} + (1 - \alpha_X) (\tilde{p}_t^{*X} \pi_t^{*X})^{1-\theta^*}.$$

Identities:

$$\frac{\bar{w}_t}{\bar{w}_{t-1}} = \frac{\pi_t^w}{\pi_t \mu_t^z}$$

$$\frac{p_t^N}{p_{t-1}^N} = \frac{\pi_t^N}{\pi_t}$$

$$\frac{p_t^{*X}}{p_{t-1}^{*X}} = \frac{\pi_t^{*X}}{\pi_t^*}$$

$$\frac{e_t}{e_{t-1}} = \frac{\delta_t \pi_t^{*N}}{\pi_t} \quad (110)$$

Fiscal:

$$b_t^{*G} = \frac{g_t + i_t^L \ell_t^G - t_t}{e_t} + (1 + i_{t-1}^*) [1 + \phi_{t-1}^{*G} + p_G ((b_{t-1}^{*G} - r_{t-1}^{*CB}) e_{t-1})] \frac{b_{t-1}^{*G}}{\mu_t^z \pi_t^{*N}}. \quad (111)$$

Balance of Payments:

$$r_t^{*CB} - b_t^{*G} - b_t^{*B} = p_t^{*X} x_t - n_t + (1 + i_{t-1}^*) \frac{r_{t-1}^{*CB}}{\mu_t^z \pi_t^{*N}} \quad (112)$$

$$-(1 + i_{t-1}^*) [1 + \phi_{t-1}^{*G} + p_G ((b_{t-1}^{*G} - r_{t-1}^{*CB}) e_{t-1})] \frac{b_{t-1}^{*G}}{\mu_t^z \pi_t^{*N}}$$

$$-(1 + i_{t-1}^*) [1 + \phi_{t-1}^{*B} + p_* (b_{t-1}^{*B} e_{t-1})] \frac{b_{t-1}^{*B}}{\mu_t^z \pi_t^{*N}}.$$

Bank arbitrage:

$$1 + i_t = [\beta^B E_t \delta_{t+1} + (1 - \beta^B) \delta_t] (1 + i_t^*) \varphi_B (e_t b_t^{*B}) \quad (113)$$

Static equations:

Market balance equations: Physical capital rental market:

$$(\tau'_u)^{-1} (i_t^K) \frac{k_t}{\mu_t^z} = \frac{abmc_t}{(1 + v_t^K i_t^L) i_t^K} [q_t + F^D]$$

Labor market:

$$h_t = \frac{(1 - a) bmc_t}{(1 + v_t^W i_t^L) \bar{w}_t} [q_t + F^D]$$

Loan market:

$$\ell_t = f_L (1 + i_t^L) mc_t [q_t + F^D] + \ell_t^G$$

Deposit market:

$$d_t = \frac{1}{a^B} \{ a_L^B [(1 - \gamma_t^B - \gamma_t^R) (1 + i_t) - (1 + i_t^D)] + a_0^B [(1 + i_t^L) - (1 + i_t)] \}$$

Interbank cum Central Bank bond market:

$$b_t^{CB} = (1 - \gamma_t^B - \gamma_t^R) d_t + e_t b_t^{*B} - \ell_t$$

Cash market:

$$m_t^0 = \mathcal{L} (1 + i_t^D) [p_t^C c_t + p_t^V v_t] + \gamma_t^B d_t.$$

Domestic goods market:

$$q_t = y_t + q_t^D + \frac{(1 - b) mc_t}{1 + v_t^N i_t^L} [q_t + F^D]$$

Other non-policy static equations: Real GDP

$$y_t = p_t^C c_t + p_t^V v_t + x_t + g_t - p_t^N n_t.$$

Intermediate demand for domestic goods:

$$q_t^D = \tilde{\tau}_M (1 + i_t^D) (p_t^C c_t + p_t^V v_t) + C_t^B + \tau_u ((\tau'_u)^{-1} (i_t^K)) k_t$$

Loan supply

$$\ell_t = \frac{1}{a^B} \{ a_D^B [(1 + i_t^L) - (1 + i_t)] + a_0^B [(1 - \gamma_t^B - \gamma_t^R)(1 + i_t) - (1 + i_t^D)] \}$$

Bank real cost:

$$C_t^B = \frac{1}{2} [a_L^B \ell_t^2 + a_D^B d_t^2 - 2a_0^B \ell_t d_t].$$

Central Bank balance sheet:

$$b_t^{CB} = e_t r_t^{*CB} - m_t^0 - \gamma_t^R d_t$$

Real marginal cost:

$$mc_t = \frac{1}{\kappa \epsilon_t} f_{MC} (1 + i_t^L) (i_t^K)^{ab} \bar{w}_t^{(1-a)b} (p_t^N)^{1-b}.$$

Export demand:

$$x_t = x_t^* z_t^{**} q_t^* (p_t^{*X})^{-\theta^*} \quad (114)$$

Import demand:

$$p_t^N n_t = (1 - a^D) p_t^C c_t + (1 - b^D) p_t^V v_t + \frac{(1 - b)mc_t}{1 + v_t^N i_t^L} [q_t + F^D]$$

Consumption relative price:

$$p_t^C = \left[a^D + (1 - a^D) (p_t^N)^{1-\theta_C} \right]^{\frac{1}{1-\theta_C}}.$$

Investment relative price:

$$p_t^V = \left[b^D + (1 - b^D) (p_t^N)^{1-\theta_V} \right]^{\frac{1}{1-\theta_V}}.$$

Policy equations: Pure Exchange rate Crawl regimes (PEC)

$$b_t^{CB} = b_0^{CB} \quad \forall t.$$

$$\delta_t = \delta_t^T \quad \forall t.$$

Inflation Targeting under a Pure Exchange rate Float regime (IT-PEF)

$$1 + i_t = \left(\frac{\pi^T}{\pi^{*N}} (1 + i^*) \varphi_B (eb^{*B}) \right)^{1-h_0} (1 + i_{t-1})^{h_0} \left(\frac{\pi_{t+1}}{\pi_{t+1}^T} \right)^{h_1} \left(\frac{\pi_t^w}{\pi_t^{wT} \mu_t^z} \right)^{h_2} \left(\frac{y_t}{y_t^T} \right)^{h_3}$$

$$r_t^{*CB} = r_0 \quad \forall t.$$

Inflation Targeting under a Managed Exchange rate Float regime (IT-MEF)

$$1 + i_t = \left(\frac{\pi^T}{\pi^{*N}} (1 + i^*) \varphi_B (eb^{*B}) \right)^{1-h_0} (1 + i_{t-1})^{h_0} \left(\frac{\pi_{t+1}}{\pi_{t+1}^T} \right)^{h_1} \left(\frac{\pi_t^w}{\pi_t^{wT}} \right)^{h_2} \left(\frac{y_t}{y_t^T} \right)^{h_3}$$

$$r_t^{*CB} = \left(\gamma^T \frac{m^{0,H} + d + eb^{*B}}{e} \right)^{1-k_0} (r_{t-1}^{*CB})^{k_0} \left(\frac{\delta_t \pi_t^{*N}}{\pi_t} \right)^{-k_1}$$

This system can be simplified considerably. First, note that we can eliminate the rate of nominal depreciation by leading (110) and using the resulting expression in (113):

$$1 + i_t = \left[\beta^B E_t \left(\frac{e_{t+1} \pi_{t+1}}{e_t \pi_{t+1}^{*N}} \right) + (1 - \beta^B) \frac{e_t \pi_t}{e_{t-1} \pi_t^{*N}} \right] (1 + i_t^*) \varphi_B (e_t b_t^{*B}). \quad (115)$$

In the case of PEC regimes, the exchange rate policy equation (103) becomes:

$$\frac{e_t \pi_t}{e_{t-1} \pi_t^{*N}} = \delta_t^T \quad \forall t. \quad (116)$$

And in the case of IT-MEF the Central Bank reserves rule becomes:

$$r_t^{*CB} = \left(\gamma^T \frac{m^{0,H} + d + eb^{*B}}{e} \right)^{1-k_0} (r_{t-1}^{*CB})^{k_0} \left(\frac{e_t}{e_{t-1}} \right)^{-k_1}.$$

Second, note that adding (111) and (112) term by term allows us to replace the balance of payments equation by the following simpler equation that combines the two:

$$r_t^{*CB} - b_t^{*B} = p_t^{*X} x_t - n_t + (1 + i_{t-1}^*) \frac{r_{t-1}^{*CB}}{\mu_t^z \pi_t^{*N}} \quad (117)$$

$$-(1 + i_{t-1}^*) [1 + \phi_{t-1}^{*B} + p_* (b_{t-1}^{*B} e_{t-1})] \frac{b_{t-1}^{*B}}{\mu_t^z \pi_t^{*N}} + \frac{g_t + i_t^L \ell_t^G - t_t}{e_t}.$$

Now the fiscal equation (111) is decomposable from the rest (since b_t^{*G} does not show up in any of the remaining equations), so we may leave it out of the system. Also, in any of the pure policies there is a variable we can convert to a constant. Obviously, many other variables may be substituted out of the system at the cost of having longer formulas.

12. Functional forms for auxiliary functions

The specific functional forms we use for the abnormal capital utilization cost, investment adjustment cost, and transaction cost functions are the following:

$$\tau_u(u_t) \equiv \frac{i^K}{a_u + 1} (u_t^{a_u+1} - 1), \quad a_u > 0 \quad (118)$$

$$\tau_V(\mu_t^V) \equiv \frac{a_V}{2} (\mu_t^V - \mu^z)^2, \quad a_V > 0 \quad (119)$$

$$\tau_M(\varpi_t) \equiv a_M \varpi_t + \frac{b_M}{\varpi_t}, \quad a_M, b_M > 0. \quad (120)$$

The utilization intensity of physical capital as a function of the rental rate is, according to (118) and (21):

$$u_t = \left(\frac{i_t^K}{i^K} \right)^{\frac{1}{a_u}}, \quad (121)$$

and hence the steady state utilization rate is unity ($u = 1$) and the real return from renting one unit of capital (gross of depreciation) is, according to (22):

$$\Gamma^K (i_t^K) \equiv \frac{i_t^K}{a_u + 1} \left\{ a_u \left(\frac{i_t^K}{i^K} \right)^{1 + \frac{1}{a_u}} + 1 \right\}.$$

In the steady state $\Gamma^K (i^K) = i^K$.

In the case of the investment adjustment cost we defined the rate of growth of real investment expenditure $\mu_t^V \equiv V_t/V_{t-1} = (v_t/v_{t-1})\mu_t^z$, which is μ^z in the steady state. Hence, in the steady state there are no investment adjustment costs ($\tau_V (\mu^V) = 0$), since investment grows along with the economy's trend. Furthermore, $\tau_V' (\mu^V) = 0$ and $\tau_V'' (\mu^V) = a_V > 0$.

In the case of transaction costs, we have two parameters for calibration using a simplification of the functional form used in Uribe and Schmitt-Grohé (2003). According to (23), the resulting liquidity preference function is:

$$\varpi_t = \frac{m_t^{0,H}}{p_t^C c_t + p_t^V v_t} = \mathcal{L} (1 + i_t^D) \equiv \left[\frac{b_M}{a_M + 1 - \frac{1}{1+i_t^D}} \right]^{\frac{1}{2}}, \quad (122)$$

where ϖ_t is the household money to absorption ratio. We prove in the Appendix that the steady state deposit rate is $1 + i^D = \pi^T \mu^z / \beta$ under inflation targeting and $1 + i^D = \delta^T \pi^{*N} \mu^z / \beta$ under a pure crawling peg. Hence, defining $\delta^T \equiv \pi^T / \pi^{*N}$ for inflation targeting, we can use the former formula for both regimes, and the steady state money to absorption ratio is:

$$\varpi = \mathcal{L} (\pi^T \mu^z / \beta) \equiv \left[\frac{b_M}{a_M + 1 - \frac{\beta}{\pi^T \mu^z}} \right]^{\frac{1}{2}}. \quad (123)$$

According to (122), household money demand decreases with the deposit rate and increases in proportion to private absorption. ϖ_t tends to infinity when the deposit rate approaches $(a_M + 1)^{-1} - 1$ from the right, is equal to $(b_M/a_M)^{0.5}$ when the deposit rate is zero, and decreases to $[b_M/(a_M + 1)]^{0.5}$ when the deposit rate tends to infinity. Also, the resulting auxiliary function for the total effect on expenditure of a marginal increase in absorption (20) is:

$$\varphi_M (\varpi_t) = 1 + \frac{2b_M}{\varpi_t}.$$

Hence, the formulas for $\tilde{\tau}_M (1 + i_t^D)$ and $\tilde{\varphi}_M (1 + i_t^D)$, are:

$$\begin{aligned} \tilde{\tau}_M (1 + i_t^D) &= a_M \varpi_t + \frac{b_M}{\varpi_t} = \\ &= a_M \left[\frac{b_M}{a_M + 1 - \frac{\beta}{\pi^T \mu^z}} \right]^{\frac{1}{2}} + b_M \left[\frac{b_M}{a_M + 1 - \frac{\beta}{\pi^T \mu^z}} \right]^{-\frac{1}{2}}. \end{aligned} \quad (124)$$

$$\tilde{\varphi}_M (1 + i_t^D) = 1 + 2b_M \left[\frac{b_M}{a_M + 1 - \frac{1}{1+i_t^D}} \right]^{-\frac{1}{2}}. \quad (125)$$

For the bank risk premium we use the following functional form:

$$p_* (e_t b_t^{*B}) \equiv \alpha_1^{RP} (e_t b_t^{*B})^{\alpha_2^{RP}}, \quad \alpha_1^{RP} > 0, \alpha_2^{RP} > 1.$$

Hence, in the risk adjusted uncovered interest parity (84) we have:

$$\varphi_B (e_t b_t^{*B}) = 1 + \phi_t^{B*} + (\alpha_2^{RP} + 1) \alpha_1^{RP} (e_t b_t^{*B})^{\alpha_2^{RP}}.$$

13. Stochastic shocks

In the shock specification for domestic output we follow Adolfson et al (2005) in having a permanent productivity shock z_t , and a transitory productivity shock ϵ_t . However, instead of postulating an AR(1) asymmetric technology shock (i.e. a law of motion for z_t^{**}), we assume a cointegrating relation between the logs of the technology shocks in the LRW and the SDE which includes a direct lagged influence of the LRW's rate of technological growth on that of the SDE. We assume, as in Adolfson et al (2005), that in the non-stochastic steady state total factor productivity levels and growth rates in the LRW and the SDE are equal ($z^{**} = 1$, and $\mu^{z*} = \mu^z$). Hence, $\hat{z}_{t-1}^{**} = \log z_{t-1}^{**}$. Also, we assume the following processes hold:

$$\hat{\mu}_t^{z*} = \rho^{z*} \hat{\mu}_{t-1}^{z*} + \varepsilon_t^{z*}, \quad (126)$$

$$\hat{\mu}_t^z = \rho^z \hat{\mu}_{t-1}^z + a_z \hat{\mu}_{t-1}^{z*} - \alpha_z \hat{z}_{t-1}^{**} + \varepsilon_t^z, \quad (127)$$

where ε_t^{z*} and ε_t^z are i.i.d. technology shocks. Putting these expressions in matrix form and including the stochastic trend through a constant perhaps reflects the cointegration assumption more transparently. We have:

$$\Delta \log \tilde{z}_t = \alpha_z A \log \tilde{z}_{t-1} + B (\Delta \log \tilde{z}_{t-1}) + C + \tilde{\varepsilon}_t,$$

where:

$$\tilde{z}_t \equiv \begin{bmatrix} z_t^* \\ z_t \end{bmatrix}, \quad \tilde{\varepsilon}_t \equiv \begin{bmatrix} \varepsilon_t^{z*} \\ \varepsilon_t^z \end{bmatrix}$$

$$A \equiv \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}, \quad B \equiv \begin{bmatrix} \rho^{z*} & 0 \\ a_z & \rho^z \end{bmatrix},$$

$$C \equiv \begin{bmatrix} (1 - \rho^{z*}) \log \mu^z \\ (1 - \rho^z - a_z) \log \mu^z \end{bmatrix}.$$

and $\rho^{z*}, a_z + \rho^z \in (0, 1)$. During the transition, the growth rate of the LRW influences the growth rate of the SDE through the coefficient a_z , while the growth rate of the SDE has no influence on the rate of growth of the LRW. Also, the persistence coefficients may be different. Furthermore, note that the law of motion for z_t^{**} can be obtained from the following identity:

$$\hat{\mu}_t^{z*} - \hat{\mu}_t^z = \hat{z}_t^{**} - \hat{z}_{t-1}^{**}.$$

We use this identity to eliminate \widehat{z}_t^{**} from the growth dynamics equation (127) and from the log-linear version of the export demand equation (114) by differencing, obtaining:

$$\begin{aligned}\widehat{\mu}_t^z - \widehat{\mu}_{t-1}^z &= \rho^z (\widehat{\mu}_{t-1}^z - \widehat{\mu}_{t-2}^z) + a_z (\widehat{\mu}_{t-1}^{z*} - \widehat{\mu}_{t-2}^{z*}) - \alpha_z (\widehat{\mu}_{t-1}^{z*} - \widehat{\mu}_{t-1}^z) + \varepsilon_t^z - \varepsilon_{t-1}^z, \\ \widehat{x}_t - \widehat{x}_{t-1} &= \widehat{q}_t^* - \widehat{q}_{t-1}^* - \theta^* (\widehat{p}_t^{*X} - \widehat{p}_{t-1}^{*X}) + \widehat{x}_t^* - \widehat{x}_{t-1}^* + \widehat{\mu}_t^{z*} - \widehat{\mu}_t^z.\end{aligned}$$

The first of these must be added on to the previous equations as one of the equations of the system(s), where $\widehat{\mu}_t^z$ is an additional endogenous variable. (For this, we lead the equation one period in order to eliminate the second lags). The second is the equation we use for exports (instead of the log-linearized version of (114)). Hence, the exports equation in our model becomes dynamic. On the other hand, $\widehat{\mu}_t^{z*}$ is given by the exogenous process (126) (which we also lead one period).

On the other hand, we assume that the stationary technology shock $\log(\epsilon_t)$ follows an AR(1) process:

$$\epsilon_t = (\epsilon_{t-1})^{\rho_\epsilon} \varepsilon_t^\epsilon$$

where $\rho_\epsilon \in (0, 1)$, and ε_t^ϵ is an i.i.d. shock.

We also assume that the stochastic process for the logs of z_t^H , z_t^C , z_t^V , x_t^* , and v_t^q are AR(1):

$$\begin{aligned}z_t^n &= (z_{t-1}^n)^{\rho_n} \varepsilon_t^n, & n &= H, C, V, \\ x_t^* &= (x_{t-1}^*)^{\rho_X} \varepsilon_t^X, \\ v_t^q &= (v_{t-1}^q)^{\rho_{v^q}} \varepsilon_t^{v^q}, & q &= W, K, N.\end{aligned}$$

where $\rho_V, \rho_X, \rho_{v^q} \in (0, 1)$, and $\varepsilon_t^V, \varepsilon_t^X, \varepsilon_t^{v^q}$ are i.i.d. shocks. We further assume that the steady state values ϵ , z^H , z^C , z^V are all unity. In the baseline model we take the bank technical demand for cash parameter γ_t^B and the regulatory parameter γ_t^R as constants, as well as the Central Bank target rates of inflation π_t^T , π_t^{wT} and output y_t^T . Therefore, $E_t \widehat{\pi}_{t+1}^T$, π_t^{wT} and \widehat{y}_t^T disappear from the interest rate feedback policy rules (129) when we implement the baseline model. We also treat g and t_t as constants, and the stock of bank loans to the government ℓ_t^G as an exogenous deterministic path that describes the gradual bank divestment of the large stock of government bonds in banks' assets that were converted to loans in the aftermath of the crisis.

14. The log-linear systems

In this section we list the log-linear system equations. The detailed log-linearization of the Phillips equations for domestic goods and wages can be found in Appendix 2. We change the order of the equations to facilitate the state space representation that we adopt for the model's numerical solution, listing the static equations first, the dynamic equations with no expectational terms second, and finally the dynamic equations with expectational terms.

14.1. The equations

The log-linear equations of our systems are the following:

Non-policy static equations:**Market balance equations:** Physical capital rental market:

$$(1 + 1/a_u)\widehat{i}_t^K = \widehat{\mu}_t^z - \widehat{k}_t + \widehat{m}c_t + \alpha_q\widehat{q}_t - (1 + 1/i^L)\alpha_K^{MC}\widehat{i}_t^L - \alpha_K^{MC}\widehat{v}_t^K$$

Labor market:

$$\widehat{h}_t = \widehat{m}c_t + \alpha_q\widehat{q}_t - \widehat{w}_t - (1 + 1/i^L)\alpha_W^{MC}\widehat{i}_t^L - \alpha_W^{MC}\widehat{v}_t^W$$

Loan market:

$$\widehat{\ell}_t = a^{LM} \left[\widehat{m}c_t + \alpha_q\widehat{q}_t - \gamma^{LM}\widehat{i}_t^L + \delta_K^{LM}\widehat{v}_t^K + \delta_W^{LM}\widehat{v}_t^W + \delta_N^{LM}\widehat{v}_t^N \right] + (1 - a^{LM})\widehat{\ell}_t^G$$

Deposit market:

$$\begin{aligned} \alpha_{DS}^B\widehat{d}_t + (1 - \alpha_{DS}^B) \left[(1 + \alpha^{MD})\widehat{i}_t^D - \alpha^{MD} \left(\widehat{i}_t - \alpha_B^{MD}\widehat{\gamma}_t^B - \alpha_R^{MD}\widehat{\gamma}_t^R \right) \right] \\ = (1 + \alpha^{ML})\widehat{i}_t^L - \alpha^{ML}\widehat{i}_t \end{aligned}$$

Interbank cum Central Bank bond market:

$$\alpha_A^B\widehat{\ell}_t + (1 - \alpha_A^B)\widehat{b}_t^{CB} = \alpha_L^B \left(\widehat{d}_t - \alpha_B^{MD}\widehat{\gamma}_t^B - \alpha_R^{MD}\widehat{\gamma}_t^R \right) + (1 - \alpha_L^B) \left(\widehat{e}_t + \widehat{b}_t^{*B} \right)$$

Cash market:

$$\widehat{m}_t^0 = a^{CM} \left[\alpha_A^{CM} (\widehat{c}_t + \widehat{p}_t^C) + (1 - \alpha_A^{CM}) (\widehat{v}_t + \widehat{p}_t^V) - \alpha_D^{CM}\widehat{i}_t^D \right] + (1 - a^{CM}) \left(\widehat{d}_t + \widehat{\gamma}_t^B \right)$$

Domestic goods market:

$$\widehat{q}_t = \alpha_y^Q\widehat{y}_t + \alpha_D^Q\widehat{q}_t^D + \left(1 - \alpha_y^Q - \alpha_D^Q \right) \left[\widehat{m}c_t + \alpha_q\widehat{q}_t - \delta_N^{LM}\widehat{i}_t^L - \alpha_N^{MC}\widehat{v}_t^N \right]$$

Other non-policy static equations: Real GDP

$$\begin{aligned} a^Y\widehat{y}_t = \alpha_c^Y (\widehat{c}_t + \widehat{p}_t^C) + \alpha_v^Y (\widehat{v}_t + \widehat{p}_t^V) + \alpha_x^Y\widehat{x}_t \\ + (1 - \alpha_c^Y - \alpha_v^Y - \alpha_x^Y)\widehat{g}_t - (1 - a^Y) (\widehat{p}_t^N + \widehat{n}_t). \end{aligned}$$

Intermediate demand for domestic goods:

$$\widehat{q}_t^D = a_1^{qD} \left[\alpha_A^{CM} (\widehat{c}_t + \widehat{p}_t^C) + (1 - \alpha_A^{CM}) (\widehat{v}_t + \widehat{p}_t^V) + b_\tau^Q\widehat{i}_t^D \right] + (1 - a_1^{qD})\widehat{C}_t^B + a_0^{qD}\widehat{i}_t^K$$

Loan supply:

$$\begin{aligned} \alpha_{LS}^B\widehat{\ell}_t + (1 - \alpha_{LS}^B) \left[(1 + \alpha^{MD})\widehat{i}_t^D - \alpha^{MD} \left(\widehat{i}_t - \alpha_B^{MD}\widehat{\gamma}_t^B - \alpha_R^{MD}\widehat{\gamma}_t^R \right) \right] \\ = (1 + \alpha^{ML})\widehat{i}_t^L - \alpha^{ML}\widehat{i}_t \end{aligned}$$

Central Bank balance sheet:

$$\widehat{r}_t^{*CB} = a_1^{CB}\widehat{m}_t^0 + a_2^{CB} \left(\widehat{d}_t + \widehat{\gamma}_t^R \right) + (1 - a_1^{CB} - a_2^{CB})\widehat{b}_t^{CB} - \widehat{e}_t$$

Real marginal cost:

$$\begin{aligned}\widehat{m}c_t &= a\widehat{b}i_t^K + (1-a)\widehat{b}\widehat{w}_t + (1-b)\widehat{p}_t^N + \alpha_L^{MC}\widehat{i}_t^L \\ &\quad + ab\alpha_K^{MC}\widehat{v}_t^K + (1-a)b\alpha_W^{MC}\widehat{v}_t^W + (1-b)\alpha_N^{MC}\widehat{v}_t^N - \widehat{e}_t.\end{aligned}$$

Import demand:

$$\begin{aligned}\widehat{n}_t &= a_1^N (\widehat{c}_t + \widehat{p}_t^C) + a_2^N (\widehat{v}_t + \widehat{p}_t^V) - \widehat{p}_t^N \\ &\quad + (1 - a_1^N - a_2^N) \left[\widehat{m}c_t + \alpha_q \widehat{q}_t - (1 + 1/i^L) \alpha_N^{MC} \widehat{i}_t^L - \alpha_N^{MC} \widehat{v}_t^N \right]\end{aligned}$$

Bank real cost:

$$\widehat{C}_t^B = a^{BC}\widehat{\ell}_t + (2 - a^{BC})\widehat{d}_t.$$

Consumption relative price:

$$\widehat{p}_t^C = a_{PC}\widehat{p}_t^N$$

Investment relative price:

$$\widehat{p}_t^V = a_{PV}\widehat{p}_t^N$$

Dynamic non-policy equations with no expectational terms:

Identities:

$$\begin{aligned}\widehat{w}_t - \widehat{w}_{t-1} &= \widehat{\pi}_t^w - \widehat{\pi}_t - \widehat{\mu}_t^z \\ \widehat{p}_t^N - \widehat{p}_{t-1}^N &= \widehat{\pi}_t^N - \widehat{\pi}_t \\ \widehat{p}_t^{*X} - \widehat{p}_{t-1}^{*X} &= \widehat{\pi}_t^{*X} - \widehat{\pi}_t^*\end{aligned}$$

Balance of Payments cum Fiscal:

$$\begin{aligned}&a_1^{BP}\widehat{r}_t^{*CB} + a_2^{BP}\widehat{n}_t + (1 - a_1^{BP} - a_2^{BP}) \\ &\quad \left[\widehat{i}_{t-1}^* + a^{RP}\widehat{\phi}_{t-1}^{*B} + (1 - a^{RP})\alpha_2^{RP}(\widehat{b}_{t-1}^{*B} + \widehat{e}_{t-1}) + \widehat{b}_{t-1}^{*B} - \widehat{\mu}_t^z - \widehat{\pi}_t^{*N} \right] \\ &\quad = a_3^{BP}\widehat{b}_t^{*B} + a_4^{BP}(\widehat{p}_t^{*X} + \widehat{x}_t) + a_5^{BP}(\widehat{i}_{t-1}^* + \widehat{r}_{t-1}^{*CB} - \widehat{\mu}_t^z - \widehat{\pi}_t^{*N}) \\ &+ (1 - a_3^{BP} - a_4^{BP} - a_5^{BP}) \left((1 + \alpha_1^{BP} - \alpha_2^{BP})\widehat{g}_t - \alpha_1^{BP}\widehat{t}_t + \alpha_2^{BP}\widehat{\ell}_t^G + \alpha_2^{BP}\widehat{v}_t^L - \widehat{e}_t \right)\end{aligned}$$

Exports:

$$\widehat{x}_t - \widehat{x}_{t-1} = \widehat{q}_t^* - \widehat{q}_{t-1}^* - \theta^*(\widehat{p}_t^{*X} - \widehat{p}_{t-1}^{*X}) + \widehat{x}_t^* - \widehat{x}_{t-1}^* + \widehat{\mu}_t^{z*} - \widehat{\mu}_t^z.$$

Physical capital accumulation:

$$\widehat{k}_{t+1} = a_K(\widehat{k}_t - \widehat{\mu}_t^z) + (1 - a_K)(\widehat{v}_t + \widehat{z}_t^V),$$

Growth:

$$\widehat{\mu}_t^z - \widehat{\mu}_{t-1}^z = \rho^z(\widehat{\mu}_{t-1}^z - \widehat{\mu}_{t-2}^z) + a_z(\widehat{\mu}_{t-1}^{z*} - \widehat{\mu}_{t-2}^{z*}) - \alpha_z(\widehat{\mu}_{t-1}^{z*} - \widehat{\mu}_{t-1}^z) + \varepsilon_t^z - \varepsilon_{t-1}^z,$$

Dynamic non-policy equations with expectational terms:

Consumption:

$$(1 + a_C) \{ \widehat{\mu}_t^z + \widehat{z}_t^C - [(1 + \alpha_C) (\widehat{c}_t + \widehat{\mu}_t^z) - \alpha_C \widehat{c}_{t-1}] \} \\ - a_C \{ E_t \widehat{z}_{t+1}^C - [(1 + \alpha_C) E_t (\widehat{c}_{t+1} + \widehat{\mu}_{t+1}^z) - \alpha_C \widehat{c}_t] \} = \widehat{\lambda}_t^\circ + \varepsilon_M \widehat{i}_t^D$$

Investment:

$$\widehat{\zeta}_t^\circ + \widehat{z}_t^V - a_V (\mu^z)^2 (\widehat{v}_t - \widehat{v}_{t-1} + \widehat{\mu}_t^z) + \beta a_V (\mu^z)^2 E_t (\widehat{v}_{t+1} - \widehat{v}_t + \widehat{\mu}_{t+1}^z) \\ = \widehat{\lambda}_t^\circ + \varepsilon_M \widehat{i}_t^D$$

Wage Phillips equation:

$$\widehat{\pi}_t^w - \widehat{\pi}_{t-1}^w = \beta E_t (\widehat{\pi}_{t+1}^w - \widehat{\pi}_t^w) + \frac{(1 - \alpha_W)(1 - \beta \alpha_W)}{\alpha_W (1 + \psi \chi)} (\chi \widehat{h}_t + \widehat{z}_t^H - \widehat{\lambda}_t^\circ - \widehat{w}_t).$$

Domestic price Phillips equation:

$$\widehat{\pi}_t - \widehat{\pi}_{t-1} = \beta (E_t \widehat{\pi}_{t+1} - \widehat{\pi}_t) + \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \widehat{m} \widehat{c}_t.$$

Imported goods price Phillips equation:

$$\widehat{\pi}_t^N - \widehat{\pi}_{t-1}^N = \beta (E_t \widehat{\pi}_{t+1}^N - \widehat{\pi}_t^N) + \frac{(1 - \alpha_N)(1 - \alpha_N \beta)}{\alpha_N} (\widehat{e}_t - \widehat{p}_t^N),$$

Exported goods price Phillips equation:

$$\widehat{\pi}_t^{*X} - \widehat{\pi}_{t-1}^{*X} = \beta (E_t \widehat{\pi}_{t+1}^{*X} - \widehat{\pi}_t^{*X}) - \frac{(1 - \alpha_X)(1 - \alpha_X \beta)}{\alpha_X} (\widehat{e}_t + \widehat{p}_t^{*X})$$

Shadow value of installed physical capital:

$$\zeta_t^\circ = E_t \left\{ \beta a_K \widehat{\zeta}_{t+1}^\circ + (1 - \beta a_K) \left[\left(\frac{2a_u + 1}{a_u + 1} \right) \widehat{i}_{t+1}^K + \widehat{\lambda}_{t+1}^\circ \right] - \widehat{\mu}_{t+1}^z \right\}$$

Marginal utility of real income:

$$\widehat{\lambda}_t^\circ = E_t \widehat{\lambda}_{t+1}^\circ + \widehat{i}_t^D - E_t \widehat{\pi}_{t+1} - E_t \widehat{\mu}_{t+1}^z$$

Bank arbitrage:

$$\widehat{i}_t = \widehat{i}_t^* + \beta^B E_t (\widehat{e}_{t+1} - \widehat{e}_t + \widehat{\pi}_{t+1} - \widehat{\pi}_{t+1}^{*N}) + (1 - \beta^B) (\widehat{e}_t - \widehat{e}_{t-1} + \widehat{\pi}_t - \widehat{\pi}_t^{*N}) \\ + \bar{a}^{RP} \widehat{\phi}_t^{*B} + (1 - \bar{a}^{RP}) \alpha_1^{RP} (\widehat{e}_t + \widehat{b}_t^{*B})$$

Policy equations:

Pure Exchange rate Crawl regimes (PEC)

$$\begin{aligned}\widehat{b}_t^{CB} &= 0 \\ \widehat{e}_t - \widehat{e}_{t-1} + \widehat{\pi}_t - \widehat{\pi}_t^{*N} &= \widehat{\delta}_t^T.\end{aligned}$$

Inflation Targeting under a Pure Exchange rate Float regime (IT-PEF)

$$\begin{aligned}\widehat{r}_t^{*CB} &= 0, \\ \widehat{i}_t &= h_0 \widehat{i}_{t-1} + h_1 (\widehat{\pi}_t - \widehat{\pi}_t^T) + h_2 (\widehat{\pi}_t^w - \widehat{\pi}_t^{wT} - \widehat{\mu}_t^z) + h_3 (\widehat{y}_t - \widehat{y}_t^T)\end{aligned}\quad (129)$$

Inflation Targeting under a Managed Exchange rate Float regime (IT-MEF)

$$\begin{aligned}\widehat{r}_t^{*CB} &= k_0 \widehat{r}_{t-1}^{*CB} - k_1 (\widehat{e}_t - \widehat{e}_{t-1}). \\ \widehat{i}_t &= h_0 \widehat{i}_{t-1} + h_1 (\widehat{\pi}_t - \widehat{\pi}_t^T) + h_2 (\widehat{\pi}_t^w - \widehat{\pi}_t^{wT} - \widehat{\mu}_t^z) + h_3 (\widehat{y}_t - \widehat{y}_t^T).\end{aligned}\quad (130)$$

When the consumption inflation is used instead of the domestic inflation, we can use (35) to obtain the rate of inflation of the consumption deflator (which is the model's approximation to the CPI) in deviation from its steady state value as a simple average of imported and domestic inflation:

$$\widehat{\pi}_t^C = a_{\Pi C} \widehat{\pi}_t^N + (1 - a_{\Pi C}) \widehat{\pi}_t.$$

Also, under alternative specifications, the expected deviation $E_t(\widehat{\pi}_{t+1} - \widehat{\pi}_{t+1}^T)$ of the inflation rate is used in (129) instead of the current deviation, and/or the year over year inflation rate $\widehat{\pi}_t^A = \widehat{\pi}_t + \widehat{\pi}_{t-1} + \widehat{\pi}_{t-2} + \widehat{\pi}_{t-3}$ (and similarly for the inflation target rate) is used instead of the quarter over quarter specification.

The definitions of the compound parameters used in the preceding equations can be found in Appendix 3.

14.2. Forcing stochastic processes

As long as we have no good reason to believe that some of the exogenous shocks are correlated, we can assume that they follow individual AR(1) processes:

$$\begin{aligned}\widehat{\mu}_t^{z*} &= \rho^{z*} \widehat{\mu}_{t-1}^{z*} + \varepsilon_t^{z*}, \\ \widehat{e}_t &= \rho_\epsilon \widehat{e}_{t-1} + \varepsilon_t^\epsilon,\end{aligned}$$

and similarly for the rest of the exogenous forcing processes. We stack the exogenous variables in a vector $Z_t^R \equiv (Z_t' \ P_{R,t}')'$ which is composed of a subvector which is independent on the monetary policy regime:

$$Z_t \equiv [\widehat{\varepsilon}_t^z \ \widehat{e}_t \ \widehat{z}_t^C \ \widehat{z}_t^V \ \widehat{z}_t^H \ \widehat{v}_t^K \ \widehat{v}_t^W \ \widehat{v}_t^N \ \widehat{\gamma}_t^B \ \widehat{\gamma}_t^R \ \widehat{\ell}_t^G \ \widehat{g}_t \ \widehat{t}_t \ \widehat{\mu}_t^{z*} \ \widehat{x}_t^* \ \widehat{\pi}_t^* \ \widehat{\pi}_t^{*N} \ \widehat{q}_t^* \ \widehat{i}_t^* \ \widehat{\phi}_t^{*B}]'$$

and a subvector $P_{R,t}$ which depends on the policy regime ($R = PEC, IT$). Under a PEC regime:

$$P_{PEC,t} \equiv [\widehat{\delta}_t^T],$$

and under an IT regime:

$$P_{IT,t} \equiv [\widehat{\pi}_t^T \ \widehat{\pi}_t^{wT} \ \widehat{y}_t^T]'$$

However, in our baseline model (which addresses IT-MEF) we will simply assume constant targets, so we only need to use Z_t .

We can express the dynamics for the exogenous forcing variables in the form of a first order VAR process:

$$Z_t^R = M^R Z_{t-1}^R + \varkappa_t, \quad \varkappa_t \sim iid N(0, \Sigma), \quad (131)$$

where M^R is a matrix that is congruent with Z_t^R and has all its eigenvalues inside the unit circle.

14.3 The log-linearized systems in matrix format

We first eliminate \widehat{p}_t^C , \widehat{p}_t^V , and \widehat{C}_t^B , in order to reduce the number of equations without significantly complicating the remaining equations. But we introduce a new variable and equation in order to eliminate $\widehat{\mu}_{t-2}^z$ from the growth equation: $\widehat{\mu}_{1,t}^z \equiv \widehat{\mu}_{t-1}^z$. The 32 remaining equations in each regime determine the paths of the following 32 variables, which we stack in three vectors (W_t, X_t, Y_t) given the paths of the exogenous stochastic or deterministic variables:

$$W_t \equiv [\widehat{i}_t^K \quad \widehat{h}_t \quad \widehat{i}_t^L \quad \widehat{i}_t^D \quad \widehat{b}_t^{CB} \quad \widehat{m}_t^0 \quad \widehat{q}_t \quad \widehat{y}_t \quad \widehat{q}_t^D \quad \widehat{\ell}_t \quad \widehat{d}_t \quad \widehat{m}c_t \quad \widehat{n}_t]'$$

$$X_t \equiv [\widehat{w}_t \quad \widehat{p}_t^N \quad \widehat{p}_t^{*X} \quad \widehat{b}_t^{*B} \quad \widehat{x}_t \quad \widehat{r}_t^{*CB} \quad \widehat{i}_t \quad \widehat{k}_t \quad \widehat{\mu}_t^z \quad \widehat{\mu}_{1,t}^z]'$$

$$Y_t \equiv [\widehat{c}_t \quad \widehat{v}_t \quad \widehat{\pi}_t^w \quad \widehat{\pi}_t \quad \widehat{\pi}_t^N \quad \widehat{\pi}_t^{*X} \quad \widehat{\lambda}_t^\circ \quad \widehat{\zeta}_t^\circ \quad \widehat{e}_t]'$$

W_t includes the variables that can be eliminated by use of the static equations, X_t includes the state (or pre-determined) variables, and Y_t includes the jump (or non-predetermined) variables. We can express the three blocks of equations (static, dynamic with no expectational terms, and dynamic with expectational terms) in the following matrix form:

$$H_w W_t = H^x X_t + H^y Y_t + H^z Z_t \quad (132)$$

$$B_{11} X_t + B_{12} Y_t = C_{11} X_{t-1} + C_{12} Y_{t-1} + D_1 W_t + J_1^1 Z_t + J_1^2 Z_{t-1} + J_1^3 Z_{t-2} \quad (133)$$

$$B_{21} X_t + B_{22} Y_t = A_{21} E_t X_{t+1} + A_{22} E_t Y_{t+1} + C_{22} Y_{t-1} + D_2 W_t + J_2^0 E_t Z_{t+1} \quad (134)$$

Note that the matrix J_1^3 is included to account for the presence of $\widehat{\mu}_{t-2}^{z*}$ in the growth equation, and that J_2^0 is included to account for $E_t \widehat{z}_{t+1}^C$ in the consumption dynamics equation and $E_t \widehat{\pi}_{t+1}^{*N}$ in the bank arbitrage equation. The second and third blocks of equations can be conveniently expressed as:

$$\begin{aligned} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} E_t X_{t+1} \\ E_t Y_{t+1} \end{bmatrix} \\ &+ \begin{bmatrix} C_{11} & C_{12} \\ 0 & C_{22} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} W_t \\ &+ \begin{bmatrix} J_1^1 \\ J_2^1 \end{bmatrix} Z_t + \begin{bmatrix} J_1^2 \\ 0 \end{bmatrix} Z_{t-1} + \begin{bmatrix} J_1^3 \\ 0 \end{bmatrix} Z_{t-2} + \begin{bmatrix} 0 \\ J_2^0 \end{bmatrix} E_t Z_{t+1} \end{aligned} \quad (135)$$

Assuming H_w is non-singular, (132) yields:

$$W_t = H_w^{-1}H^x X_t + H_w^{-1}H^y Y_t + H_w^{-1}H^z Z_t = \quad (136)$$

$$\begin{bmatrix} H_w^{-1}H^x & H_w^{-1}H^y \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} + H_w^{-1}H^z Z_t. \quad (137)$$

Inserting this equation as well as (131) (without the superindex R since we do not consider variable target paths in the baseline case) in (135) and rearranging yields:

$$\begin{aligned} & \begin{bmatrix} B_{11} - D_1 H_w^{-1} H^x & B_{12} - D_1 H_w^{-1} H^y \\ B_{21} - D_2 H_w^{-1} H^x & B_{22} - D_2 H_w^{-1} H^y \end{bmatrix} \begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \\ & \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} E_t X_{t+1} \\ E_t Y_{t+1} \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ 0 & C_{22} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + \\ & + \begin{bmatrix} D_1 H_w^{-1} H^z + J_1^1 \\ D_2 H_w^{-1} H^z + J_2^1 + J_2^0 M \end{bmatrix} Z_t + \begin{bmatrix} J_1^2 \\ 0 \end{bmatrix} Z_{t-1} + \begin{bmatrix} J_1^3 \\ 0 \end{bmatrix} Z_{t-2}. \end{aligned}$$

We now rearrange the system in a form suitable for numerical solution using the Generalized Schur decomposition, as in Klein (2000) (see alternatively Binder and Pesaran (1995), King and Watson (1998), Uhlig (1999), and Sims (2000)). For this we convert the second order matrix difference equation to first order by defining $\bar{X}_t = X_{t-1}$ and $\bar{Y}_t = Y_{t-1}$. The matrix difference equation can hence be expressed as:

$$\begin{aligned} & \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \bar{X}_{t+1} \\ \bar{Y}_{t+1} \\ E_t X_{t+1} \\ E_t Y_{t+1} \end{bmatrix} = \\ & \begin{bmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ -C_{11} & -C_{12} & B_{11} - D_1 H_w^{-1} H^x & B_{12} - D_1 H_w^{-1} H^y \\ 0 & -C_{22} & B_{21} - D_2 H_w^{-1} H^x & B_{22} - D_2 H_w^{-1} H^y \end{bmatrix} \begin{bmatrix} \bar{X}_t \\ \bar{Y}_t \\ X_t \\ Y_t \end{bmatrix} + \\ & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -(D_1 H_w^{-1} H^z + J_1^1) & -J_1^2 & -J_1^3 \\ -(D_2 H_w^{-1} H^z + J_2^1 + J_2^0 M) & 0 & 0 \end{bmatrix} \begin{bmatrix} Z_t \\ Z_{t-1} \\ Z_{t-2} \end{bmatrix}. \end{aligned}$$

The first order autorregressive equation for the forcing processes can be written as:

$$\begin{bmatrix} Z_t \\ Z_{t-1} \\ Z_{t-2} \end{bmatrix} = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ Z_{t-2} \\ Z_{t-3} \end{bmatrix} + \begin{bmatrix} \varkappa_t \\ \varkappa_{t-1} \\ \varkappa_{t-2} \end{bmatrix}.$$

In compact (and obvious) notation, we have the Klein (2000) format:

$$\begin{aligned} \tilde{A}E_t\tilde{X}_{t+1} &= \tilde{B}\tilde{X}_t + \tilde{C}\tilde{Z}_t \\ \tilde{Z}_t &= \tilde{M}\tilde{Z}_{t-1} + \tilde{\varkappa}_t. \end{aligned}$$

Note that since \tilde{A} is clearly singular, the traditional Blanchard and Kahn (1980) method cannot be used (at least in this particular state space setup). The generalized Schur decomposition (also called QZ decomposition) is particularly appropriate for this case. As long as there exists at least some complex ω such that $\det(\tilde{A}\omega - \tilde{B}) \neq 0$, there exist unitary matrices Q and Z such that $Q\tilde{A}Z \equiv S$ and $Q\tilde{B}Z \equiv T$ are upper triangular and such that for all i the diagonal elements S_{ii} and T_{ii} are not both zero. Also, the set of generalized eigenvalues is the set of ratios T_{ii}/S_{ii} (where, with abuse of notation, when $S_{ii} = 0$ we call the corresponding generalized eigenvalue "infinite"). Furthermore, the pairs (S_{ii}, T_{ii}) can be arranged in any order. Hence the eigenvalues can be arranged so that their moduli are in ascending order. We partition \tilde{X}_t into two parts such that all the predetermined variables come first:

$$\tilde{X}_t = \begin{bmatrix} k_t \\ Y_t \end{bmatrix}, \quad \text{where } k_t = \begin{bmatrix} \bar{X}_t \\ \bar{Y}_t \\ X_t \end{bmatrix}.$$

If the number of eigenvalues within the unit circle (i.e., such that $|T_{ii}| < |S_{ii}|$) is equal to the dimension of k_t and the resulting (after rearrangement) upper left block Z_{11} of Z is non-singular, then there exists a saddlepath solution (which is almost surely (P) unique) and can be expressed as:

$$\begin{aligned} k_{t+1} &= Gk_t + Hz_t \\ Y_t &= Kk_t + Lz_t, \end{aligned}$$

where

$$\begin{aligned} G &= Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1} \\ H &= (GZ_{12} - Z_{11}S_{11}^{-1}T_{12})R + (Z_{11}S_{11}^{-1}S_{12} - Z_{12})R\tilde{M} + Z_{11}S_{11}^{-1}Q_1\tilde{C} \\ K &= Z_{21}Z_{11}^{-1} \\ L &= (Z_{22} - KZ_{12})R \\ \text{vec}(R) &= \left[I - \tilde{M} \otimes (T_{22}^{-1}S_{22}) \right]^{-1} \text{vec} \left(T_{22}^{-1}Q_2\tilde{C} \right). \end{aligned}$$

16. Conclusion

This paper has developed a rational expectations, dynamic and stochastic general equilibrium model for a small economy whose growth stems from a unit root technology shock that is cointegrated with the analogous technology shock in the rest of the world. The model has households and three types of firms (domestic, importing and exporting), all of which are monopolistically competitive, as well as perfectly competitive banks. Importing and exporting firms engage in local currency pricing. The model has four Phillips inflation equations (for wage inflation, domestic goods inflation, imported goods inflation, and exported goods inflation, respectively), in which price (or wage) setters that don't have the opportunity of optimizing fully index to the previous period's inflation rate. Households make the consumption and investment decisions and also decide on the intensity of utilization of the physical capital they rent to domestic firms in a competitive market and their demands for cash and bank deposits. Banks finance a stochastic fraction of the domestic

firms' wage bill, capital rental bill and imported inputs bill and the Government's exogenous demand for loans. They also issue deposits and obtain funds abroad to finance their loans, hold cash and regulatory reserves, and purchase Central Bank (domestic currency denominated) bonds. Their (static) profit maximization yields the model's risk adjusted uncovered interest parity equation. The Central Bank issues currency and domestic currency bonds, and holds foreign currency reserves and regulatory bank reserves. The main focus is on building a common framework where monetary policy can take different forms. Hence, the Central Bank's balance sheet plays an important role. In particular, we develop the extreme cases of a crawling exchange rate peg with a pure interest rate float, an inflation targeting regime with a pure exchange rate float, and an inflation targeting with managed float regime where the Central Bank simultaneously intervenes in the money and the foreign exchange markets with two corresponding policy feedback rules.

In future research we hope to calibrate the model and use it to make numerical simulations under alternative policy regimes. Towards this end the present paper makes a significant advance by specifying reasonable and easy to use adjustment functions, obtaining the log-linear version of the model with the variables in stationary format, and by going a long way towards the solution of the steady state systems. The fiscal aspects have been purposely kept to a bare minimum. We leave for future research the development of a more fiscally oriented version of the model, which should be quite straightforward but highly rewarding for policy analysis.

Appendix 1: Analysis of the steady state

We first define the non-stochastic steady states around which we make log-linear approximations to the different dynamic systems that correspond to the alternative monetary policy regimes. We replace the stationary variables in the system by their non-stochastic steady state values (which we denote as the same variables without any time index) and recall that $\tau_V(\mu^z) = \tau'_V(\mu^z) = 0$ (and hence $\varphi_V(\mu^z) = 1$), and $\tau_u(u) = \tau_u(1) = 0$ (and hence $\Gamma^K(i^K) = i^K$). For simplicity, we normalize the following shocks to unity in the steady state: $q^* = p^{N*} = z^C = z^V = z^H = x^* = \epsilon = 1$. We also eliminate y and q^D from the domestic goods market balance equation, obtaining:

Dynamic equations:

Consumption:

$$\frac{1}{\lambda^\circ c} \frac{\mu^z - \beta\xi}{\mu^z - \xi} = \tilde{\varphi}_M (1 + i^D) \quad (138)$$

Investment:

$$\frac{\zeta^\circ}{\lambda^\circ} = \tilde{\varphi}_M (1 + i^D) \quad (139)$$

Marginal utility of installed physical capital:

$$\frac{\zeta^\circ}{\lambda^\circ} = \frac{1}{\mu^z/\beta - (1 - \delta^K)} i^K \quad (140)$$

Marginal utility of real income:

$$\mu^z \pi = \beta (1 + i^D) \quad (141a)$$

Physical capital accumulation:

$$\frac{k}{v} = \frac{\mu^z}{\mu^z - (1 - \delta^K)}, \quad (142)$$

Wage Phillips equations:

$$\begin{aligned} \tilde{w}^{1+\psi\chi} &= \frac{\psi}{\psi - 1} \frac{\eta_H h^\chi}{\lambda^\circ \bar{w}} \\ 1 &= \tilde{w} \end{aligned} \quad (143)$$

Domestic price Phillips equations:

$$\begin{aligned} \tilde{p} &= \frac{\theta}{\theta - 1} mc, \\ 1 &= \tilde{p}. \end{aligned} \quad (144)$$

Imported goods price Phillips equations:

$$\begin{aligned} \tilde{p}^N &= \frac{\theta_N}{\theta_N - 1} \frac{e}{p^N} \\ 1 &= \tilde{p}^N \end{aligned} \quad (145)$$

Exported goods price Phillips equations:

$$\begin{aligned} \hat{p}^{*X} &= \frac{\theta^*}{\theta^* - 1} \frac{1}{ep^*} \\ 1 &= \hat{p}^{*X} \end{aligned} \quad (146)$$

Identities:

$$1 = \frac{\pi^w}{\pi \mu^z} \quad (147)$$

$$1 = \frac{\pi^N}{\pi} \quad (148)$$

$$1 = \frac{\pi^{*X}}{\pi^*} \quad (149)$$

$$1 = \frac{\delta \pi^{*N}}{\pi} \quad (150)$$

Balance of Payments cum Fiscal:

$$\begin{aligned} r^{*CB} \left(1 - \frac{1+i^*}{\mu^z \pi^{*N}} \right) - b^{*B} \left(1 - \frac{(1+i^*) [1 + \phi_t^{*B} + p_* (b^{*B} e)]}{\mu^z \pi^{*N}} \right) \\ = (p^{*X} x - n) + \frac{g - t + i^L \ell^G}{e}. \end{aligned}$$

Bank arbitrage:

$$\frac{1+i}{\pi} = \delta \left(\frac{1+i^*}{\pi^{*N}} \right) \varphi_B (eb^{*B}) \quad (151)$$

Static equations:**Market balance equations:** Physical capital rental market:

$$ki^K [1 + v^K i^L] / \mu^z = abmc [q + F^D]$$

Labor market:

$$h\bar{w} [1 + v^W i^L] = (1 - a)bmc [q + F^D]$$

Loan market:

$$\ell = f_L (1 + i^L) mc [q + F^D] + \ell^G \quad (152)$$

Deposit market:

$$d = \frac{1}{a^B} \{ a_L^B [(1 - \gamma^B - \gamma^R)(1 + i) - (1 + i^D)] + a_0^B [(1 + i^L) - (1 + i)] \} \quad (153)$$

Interbank cum Central Bank bond market:

$$b^{CB} = (1 - \gamma^B - \gamma^R)d + eb^{*B} - \ell.$$

Cash market:

$$m^0 = \mathcal{L} (1 + i^D) [p^C c + p^V v] + \gamma^B d.$$

Domestic goods market:

$$q = a_D p^C c + b_D p^V v + x + g + \tilde{\tau}_M (1 + i^D) (p^C c + p^V v) + C^B$$

Other non-policy static equations: Loan supply

$$\ell = \frac{1}{a^B} \{ a_D^B [(1 + i^L) - (1 + i)] + a_0^B [(1 - \gamma^B - \gamma^R)(1 + i) - (1 + i^D)] \} \quad (154)$$

Bank real cost:

$$C^B = \frac{1}{2} [a_L^B \ell^2 + a_D^B d^2 - 2a_0^B \ell d] \quad (155)$$

Central Bank balance sheet:

$$b^{CB} = er^{*CB} - m^0 - \gamma^R d$$

Real marginal cost:

$$mc = \frac{1}{\kappa} f_{MC} (1 + i^L) (i^K)^{ab} \bar{w}^{(1-a)b} (p^N)^{1-b}.$$

Export demand:

$$x = (p^*)^{-\theta^*}$$

Import demand:

$$p^N n = (1 - a^D) p^C c + (1 - b^D) p^V v + \frac{(1 - b)mc}{1 + v^N i^L} [q + F^D]$$

Consumption relative price:

$$p^C = \left[a^D + (1 - a^D) (p^N)^{1-\theta_C} \right]^{\frac{1}{1-\theta_C}}.$$

Investment relative price:

$$p^V = \left[b^D + (1 - b^D) (p^N)^{1-\theta_V} \right]^{\frac{1}{1-\theta_V}}.$$

Policy equations: Pure Exchange rate Crawl regimes (PEC)

$$b^{CB} = b_0^C.$$

$$\frac{\pi}{\pi^{*N}} = \delta^T.$$

Inflation Targeting under a Pure Exchange rate Float regime (IT-PEF)

$$(1+i)^{1-h_0} = \left(\frac{\pi^T}{\pi^{*N}} (1+i^*) \varphi_B (eb^{*B}) \right)^{1-h_0} \left(\frac{\pi}{\pi^T} \right)^{h_1} \left(\frac{\pi^w}{\pi^{wT} \mu^z} \right)^{h_2} \quad (156)$$

$$r^{*CB} = r_0.$$

Inflation Targeting under a Managed Exchange rate Float regime (IT-MEF)

$$(1+i)^{1-h_0} = \left(\frac{\pi^T}{\pi^{*N}} (1+i^*) \varphi_B (eb^{*B}) \right)^{1-h_0} \left(\frac{\pi}{\pi^T} \right)^{h_1} \left(\frac{\pi^w}{\pi^{wT} \mu^z} \right)^{h_2}$$

$$r^{*CB} = \gamma^T \frac{m^{0,H} + d + eb^{*B}}{e}$$

A first glance at these equations shows that several of the steady state variables are easily determined. This is the case of \tilde{w} , \tilde{p} , \tilde{p}^N , \tilde{p}^{*X} , which are all equal to one. This implies that in the steady state there is no distinction between agents that optimize and those that index and, in particular (from (144)):

$$mc = \frac{\theta - 1}{\theta} \equiv s_\theta^{-1}.$$

Also, (147)-(150) imply

$$\pi = \pi^N = \delta \pi^{*N}, \quad \pi^w = \pi \mu^z \quad \text{and} \quad \pi^{*X} = \pi^*. \quad (157)$$

The second of these equalities implies that consistency in price and wage inflation targets requires $\pi^{wT} = \pi^T \mu^z$, which we assume is the case. In the case of a Pure Crawl regime, $\delta = \delta^T$. Hence, in terms of exogenous variables, the endogenous rates of inflation are:

$$\pi = \pi^N = \delta^T \pi^{*N}, \quad \pi^w = \delta^T \pi^{*N} \mu^z \quad \text{and} \quad \pi^{*X} = \pi^*. \quad (158)$$

Therefore, (141a) implies that the steady state deposit rate is:

$$1+i^D = \frac{\delta^T \pi^{*N} \mu^z}{\beta} = \frac{\pi^w}{\beta}.$$

In the case of Inflation Targeting, first note that introducing the second equality of (157) in (156) simplifies the feedback rule to:

$$(1+i)^{1-h_0} = \left(\frac{\pi^T}{\pi^{*N}} (1+i^*) \varphi_B (eb^{*B}) \right)^{1-h_0} \left(\frac{\pi}{\pi^T} \right)^{h_1+h_2} \quad (160)$$

Second, introducing (151) in (160) and recalling that $h_1 > 1$ (and hence $h_0 + h_1 + h_2 > 1$) gives $\pi = \pi^T$. Therefore, we again have all the steady state inflation rates:

$$\pi = \pi^N = \delta \pi^{*N} = \pi^T, \quad \pi^w = \pi^T \mu^z \quad \text{and} \quad \pi^{*X} = \pi^*.$$

We now let δ^T stand for π^T/π^{*N} in the IT regimes. Hence, from now on we can use the same notation in all regimes: δ^T for the steady state rate of capital depreciation, and π^T for the steady state rate of inflation.

Using (141a) we verify that the steady state gross deposit rate under IT regimes is also equal to the wage inflation rate divided by the time discount factor:

$$1 + i^D = \frac{\pi^T \mu^z}{\beta} = \frac{\pi^w}{\beta}.$$

Therefore, from now on we can use $\tilde{\tau}_M$, $\tilde{\varphi}_M$, and ϖ , to denote $\tilde{\tau}_M (\pi^T \mu^z / \beta)$, $\tilde{\varphi}_M (\pi^T \mu^z / \beta)$, and $\mathcal{L} (\pi^T \mu^z / \beta)$, respectively (which are detailed in (124), (125) and (123)). Hence, from (139) and (140) we obtain the steady state value of i^K as:

$$i^K = s_0, \quad s_0 \equiv (\mu^z / \beta - 1 + \delta^K) \tilde{\varphi}_M.$$

We now concentrate on the remaining variables and equations. Combining (152) with (154), we obtain the following relation between the loan rate and the interbank rate:

$$1 + i = \frac{a_D^B (1 + i^L) - a^B f_L (1 + i^L) s_\theta^{-1} [q + F^D] - a^B \ell^G - a_0^B \pi^w / \beta}{a_D^B - a_0^B (1 - \gamma^B - \gamma^R)}$$

Note that, according to (59), the numerator is strictly increasing in $1 + i^L$ and strictly decreasing in q . Also, since we assume that $\gamma^B + \gamma^R < 1$ and that $a_D^B > a_0^B$ (see (80)), the denominator is positive. Hence, we can invoke the implicit function theorem to obtain $1 + i^L$ as a function $I(\cdot)$ that is strictly increasing in $1 + i$ and strictly decreasing in q :

$$1 + i^L \equiv I_L(1 + i, q). \quad (162)$$

Using (155) to eliminate C^B , and inserting (162) in (153) and (154) and adopting the following shorthand notation:

$$d(1 + i, q) \equiv \frac{1}{a^B} \{ a_L^B [(1 - \gamma^B - \gamma^R) (1 + i) - \pi^w / \beta] + a_0^B [I_L(1 + i, q) - (1 + i)] \} \quad (163)$$

$$\ell(1 + i, q) \equiv \frac{1}{a^B} \{ a_D^B [I_L(1 + i, q) - (1 + i)] + a_0^B [(1 - \gamma^B - \gamma^R) (1 + i) - \pi^w / \beta] \} \quad (164)$$

$$p^C(e) \equiv \left[a^D + (1 - a^D) (s_{\theta_N} e)^{1 - \theta_C} \right]^{\frac{1}{1 - \theta_C}}$$

$$p^V(e) \equiv \left[b^D + (1 - b^D) (s_{\theta_N} e)^{1 - \theta_V} \right]^{\frac{1}{1 - \theta_V}},$$

we can write the remaining equations as:

$$\lambda^\circ = \frac{s_1}{\tilde{\varphi}_M^C}, \quad \left(s_1 \equiv \frac{\mu^z - \beta \xi}{\mu^z - \xi} \right) \quad (165)$$

$$\zeta^\circ = \tilde{\varphi}_M \lambda^\circ = \frac{s_1}{c} \quad (166a)$$

$$k = v s_2, \quad \left(s_2 \equiv \frac{\mu^z}{\mu^z - (1 - \delta^K)} \right) \quad (167)$$

$$\bar{w} = \frac{s_\psi \eta_H h^\chi}{\lambda^\circ} = s_3 h^\chi c, \quad \left(s_3 \equiv \frac{\tilde{\varphi}_M s_\psi \eta_H}{s_1} \quad s_\psi \equiv \frac{\psi}{\psi - 1} \right) \quad (168)$$

$$p^N = s_{\theta_N} e, \quad \left(s_{\theta_N} \equiv \frac{\theta_N}{\theta_N - 1} \right) \quad (169)$$

$$p^* = \frac{s_{\theta^*}}{e}, \quad \left(s_{\theta^*} \equiv \frac{\theta^*}{\theta^* - 1} \right) \quad (170)$$

$$\begin{aligned} r^{*CB} \left(1 - \frac{1 + i^*}{\mu^z \pi^{*N}} \right) - b^{*B} \left(1 - \frac{1 + i^*}{\mu^z \pi^{*N}} \left[1 + \phi^{*B} + \alpha_1^{RP} (eb^{*B})^{\alpha_2^{RP}} \right] \right) \\ = \left(\frac{s_{\theta^*}}{e} x - n \right) + \frac{g - t + i^L \ell^G}{e} \\ 1 + i = \delta^T (1 + i^*) \left[1 + \phi^{*B} + (\alpha_2^{RP} + 1) \alpha_1^{RP} (eb^{*B})^{\alpha_2^{RP}} \right] \end{aligned} \quad (171)$$

$$k = \frac{a b s_\theta^{-1}}{s_0 [1 - v^K + v^K I_L(1 + i, q)]} [q + F^D] \quad (172)$$

$$h = \frac{(1 - a) b s_\theta^{-1}}{\bar{w} [1 - v^W + v^W I_L(1 + i, q)]} [q + F^D]$$

and hence, using (168) to eliminate \bar{w} from this equation:

$$h^{1+\chi} = \frac{(1 - a) b s_\theta^{-1}}{s_3 c [1 - v^W + v^W I_L(1 + i, q)]} [q + F^D] \quad (173)$$

$$b^{CB} = (1 - \gamma^B - \gamma^R) d(1 + i, q) + eb^{*B} - \ell(1 + i, q) \quad (174)$$

$$m^0 = \varpi [p^C(e)c + p^V(e)v] + \gamma^B d(1 + i, q). \quad (175)$$

$$\begin{aligned} q = (a^D + \tilde{\tau}_M) p^C(e)c + (b^D + \tilde{\tau}_M) p^V(e)v + x + g \\ + \frac{1}{2} \{ a_L^B \ell(1 + i, q)^2 + a_D^B [d(1 + i, q)]^2 - 2a_0^B \ell(1 + i, q) d(1 + i, q) \} \end{aligned} \quad (176)$$

$$b^{CB} = e r^{*CB} - m^0 - \gamma^R d(1 + i, q) \quad (177)$$

$$\bar{w}^{(1-a)b} = \frac{\kappa s_\theta^{-1}}{f_{MC}(I_L(1 + i, q)) s_0^{ab} (s_{\theta_N} e)^{1-b}}, \quad (178)$$

$$x = \left(\frac{s_{\theta^*}}{e} \right)^{-\theta^*} \quad (179)$$

$$s_{\theta_N} e n = (1 - a^D) p^C(e)c + (1 - b^D) p^V(e)v + \frac{1 - b}{1 - v^N + v^N I_L(1 + i, q)} s_\theta^{-1} [q + F^D] \quad (180)$$

Pure Exchange rate Crawl regimes (PEC)

$$b^{CB} = b_0^{CB}.$$

Inflation Targeting under a Pure Exchange rate Float regime (IT-PEF)

$$r^{*CB} = r_0.$$

Inflation Targeting under a Managed Exchange rate Float regime (IT-MEF)

$$er^{*CB} = \gamma^T [m^0 + (1 - \gamma^B)d(1 + i, q) + eb^{*B}].$$

A steady state system of 8 equations

Now, we eliminate h and \bar{w} from (168), (173), and (178), to obtain consumption in terms of i and q and e ((185) below), use (167) in (172) to obtain investment in terms of those same variables, use (179) and (180) to eliminate x and n , respectively from the balance of payments cum fiscal equation, and use (171) to eliminate b^{*B} ((182) below). Also, to abbreviate we use the following notation for the trade balance and the domestic currency value of bank foreign debt:

$$tb(c, v, 1 + i, q, e) \equiv \frac{1}{e} \left\{ s_{\theta^*} \left(\frac{e}{s_{\theta^*}} \right)^{\theta^*} - \frac{1}{s_{\theta_N}} [(1 - a^D) p^C(e)c + (1 - b^D) p^V(e)v + \frac{1 - b}{1 - v^N + v^N I_L(1 + i, q)} s_{\theta}^{-1} (q + F^D)] \right\}. \quad (181)$$

$$eb^{*B} = I_B(1 + i) \equiv \left[\frac{\frac{1+i}{\delta^T(1+i^*)} - (1 + \phi^{*B})}{(\alpha_2^{RP} + 1) \alpha_1^{RP}} \right]^{\frac{1}{\alpha_2^{RP}}}. \quad (182)$$

We are left with the following equations:

$$er^{*CB} \left(1 - \frac{1 + i^*}{\mu^z \pi^{*N}} \right) + I_B(1 + i) = etb(c, v, 1 + i, q, e) + g - t + [I_L(1 + i, q) - 1] \ell^G. \quad (183)$$

$$q = (a^D + \tilde{\tau}_M) p^C(e)c + (b^D + \tilde{\tau}_M) p^V(e)v + \left(\frac{e}{s_{\theta^*}} \right)^{\theta^*} + g + \frac{1}{2} [a_L^B \ell(1 + i, q)^2 + a_D^B [d(1 + i, q)]^2 - 2a_0^B \ell(1 + i, q)d(1 + i, q)] \quad (184)$$

$$c = \frac{\left[\frac{\kappa s_{\theta}^{-1}}{f_{MC}(I_L(1+i,q)) s_{\theta}^{ab} (s_{\theta_N} e)^{1-b}} \right]^{\frac{1+\chi}{(1-a)b}}}{s_3 \left[\frac{(1-a) b s_{\theta}^{-1} [q + F^D]}{1 - v^W + v^W I_L(1+i,q)} \right]^{\chi}} \equiv c(1 + i, q, e) \quad (185)$$

$$v = \frac{ab s_{\theta}^{-1} [q + F^D]}{s_2 s_0 [1 - v^K + v^K I(q, 1 + i)]} \equiv v(1 + i, q) \quad (186)$$

$$m^0 = \varpi [p^C(e)c + p^V(e)v] + \gamma^B d(1 + i, q). \quad (187)$$

$$b^{CB} = (1 - \gamma^B - \gamma^R)d(1 + i, q) + I_B(1 + i) - \ell(1 + i, q). \quad (188)$$

$$b^{CB} = er^{*CB} - m^0 - \gamma^R d(1 + i, q) \quad (189)$$

Pure exchange rate Crawl regimes

$$b^{CB} = b_0^{CB}. \quad (190)$$

Inflation Targeting under a Pure Exchange rate Float regime (IT-PEF)

$$r^{*CB} = r_0. \quad (191)$$

Inflation Targeting under a Managed Exchange rate Float regime (IT-MEF)

$$er^{*CB} = \gamma^T \{m^0 + (1 - \gamma^B)d(1 + i, q) + I_B(1 + i)\} \quad (192)$$

For any of the regimes we have 8 equations to determine the 8 variables: r^{*CB} , e , q , c , v , $1 + i$, m^0 , b^{CB} . The steady state values of the remaining variables can then be obtained from the equations above.

A steady state system of 3 equations

Finally, we now obtain a core system of only 3 equations in $1 + i$, q and e , two of which are specific to the monetary policy regime. First, (185), and (186) can be used to eliminate c and v , and hence express the stock of currency in circulation (187), the trade balance (181), and the domestic output balance as:

$$\begin{aligned} m^0(1 + i, q, e) &\equiv \varpi [p^C(e)c(1 + i, q, e) + p^V(e)v(1 + i, q)] + \gamma^B d(1 + i, q) \\ tb(1 + i, q, e) &\equiv tb(c(1 + i, q, e), v(1 + i, q), 1 + i, q, e) \end{aligned}$$

Domestic output balance:

$$\begin{aligned} q &= (a^D + \tilde{\tau}_M) p^C(e)c(1 + i, q, e) + (b^D + \tilde{\tau}_M) p^V(e)v(1 + i, q) \quad (193) \\ &+ \left(\frac{e}{s_{\theta^*}}\right)^{\theta^*} + g + \frac{1}{2} [a_L^B \ell(1 + i, q)^2 + a_D^B [d(1 + i, q)]^2 - 2a_0^B \ell(1 + i, q)d(1 + i, q)] \end{aligned}$$

This last equation is common to the three regimes. The other two equations are specific to the alternative monetary regimes.

Pure Exchange rate Crawl regimes (PEC)

Under a Pure Crawl, (190) eliminates b^{CB} . Hence (189) becomes

$$er^{*CB} = b_0^{CB} + m^0(1 + i, q, e) + \gamma^R d(1 + i, q),$$

which can be used to eliminate r^{*CB} . Hence, we have:

Balance of payments cum fiscal balance:

$$\begin{aligned} &\{b_0^{CB} + m^0(1 + i, q, e) + \gamma^R d(1 + i, q)\} \left(1 - \frac{1 + i^*}{\mu^z \pi^{*N}}\right) + I_B(1 + i) \\ &= etb(1 + i, q, e) + g - t + [I_L(1 + i, q) - 1] \ell^G. \end{aligned}$$

Banking system balance sheet:

$$b_0^{CB} = (1 - \gamma^B - \gamma^R)d(1 + i, q) + I^B(1 + i) - \ell(1 + i, q). \quad (194)$$

Inflation Targeting under a Pure Exchange rate Float regime

In the case of an IT-PEF regime, (191) eliminates r_0 . Eliminating b^{CB} from (188) and (189) we obtain the financial sector (Central Bank and Banking system) consolidated balance sheet (196). Hence, the two equations are:

Balance of payments cum fiscal balance:

$$er_0 \left(1 - \frac{1+i^*}{\mu^z \pi^{*N}} \right) + I_B(1+i) = etb(1+i, q) + g - t + [I_L(1+i, q) - 1] \ell^G. \quad (195)$$

Financial system balance sheet:

$$er_0 = m^0(1+i, q, e) + (1 - \gamma^B)d(1+i, q) + I_B(1+i) - \ell(1+i, q). \quad (196)$$

Inflation Targeting under a Managed Exchange rate Float regime (IT-MEF)

Finally, under a Managed Float, eliminating b^{CB} from (188) and (189) we obtain:

$$er^{*CB} = m^0(1+i, q, e) + (1 - \gamma^B)d(1+i, q) + I_B(1+i) - \ell(1+i, q).$$

Hence, using (192) to eliminate er^{*CB} , we obtain (198), which combines the Financial System balance sheet with the Central Bank's feedback rule for its international reserves. Hence, the two equations are:

Balance of payments cum fiscal balance:

$$\frac{\gamma^T}{1 - \gamma^T} \ell(1+i, q) \left(1 - \frac{1+i^*}{\mu^z \pi^{*N}} \right) + I_B(1+i) = etb(1+i, q, e) + g - t + [I_L(1+i, q) - 1] \ell^G. \quad (197)$$

Financial system balance sheet:

$$(1 - \gamma^T) \{ m^0(1+i, q, e) + (1 - \gamma^B)d(1+i, q) + I_B(1+i) \} = \ell(1+i, q). \quad (198)$$

Appendix 2: Log-linearization of the Phillips equations

Phillips equation for domestic goods

We first log-linearize the Phillips equations for domestic goods, since the procedure is simpler than with the one for the wage rate. We rewrite the equations to be log-linearized for the reader's convenience:

$$\pi_t^{1-\theta} = \alpha \pi_{t-1}^{1-\theta} + (1 - \alpha) (\tilde{p}_t \pi_t)^{1-\theta}. \quad (199)$$

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha)^j \bar{\Lambda}_{t+j}^{\circ} q_{t+j} (\pi_{t+j})^{\theta} \left\{ \frac{\tilde{p}_t \pi_t}{\pi_{t+j}} - \frac{\theta}{\theta - 1} m c_{t+j} \right\}, \quad (200)$$

First we express these equations in terms of the log deviations from steady state values. For a variable π_t , for example, we define the log deviation from steady state as:

$$\hat{\pi}_t \equiv \log \left(\frac{\pi_t}{\pi} \right).$$

Take (199) first. Dividing through by $\pi_t^{1-\theta}$ and taking logs yields:

$$0 = \log \left\{ \alpha \left(\frac{\pi_{t-1}}{\pi_t} \right)^{1-\theta} + (1-\alpha) \tilde{p}_t^{1-\theta} \right\}.$$

The steady state values for π_t and \tilde{p}_t are π and 1, respectively, so we can write this expression in terms of the ratios of the variables and their steady state values:

$$\begin{aligned} 0 &= \log \left\{ \alpha \left(\frac{\pi_{t-1}/\pi}{\pi_t/\pi} \right)^{1-\theta} + (1-\alpha) \tilde{p}_t^{1-\theta} \right\} \\ &= \log \left\{ \alpha \exp \left[(1-\theta) \left(\log \frac{\pi_{t-1}}{\pi} - \log \frac{\pi_t}{\pi} \right) \right] + (1-\alpha) \exp \left[(1-\theta) \log \tilde{p}_t \right] \right\} \\ &= \log \left\{ \alpha \exp \left[(1-\theta) (\hat{\pi}_{t-1} - \hat{\pi}_t) \right] + (1-\alpha) \exp \left[(1-\theta) \hat{p}_t \right] \right\} \\ &\equiv G(\hat{\pi}_t, \hat{\pi}_{t-1}, \hat{p}_t). \end{aligned}$$

Second, a linear approximation of this expression is:

$$G(\hat{\pi}_t, \hat{\pi}_{t-1}, \hat{p}_t) \simeq G + G_1 \hat{\pi}_t + G_2 \hat{\pi}_{t-1} + G_3 \hat{p}_t,$$

where G is the value of the function at the steady state values of the variables, and G_j is the partial derivative of G with respect to its j th variable, valued at the steady state values of the variables. Taking the corresponding partial derivatives gives:

$$0 = -\alpha(1-\theta)(\hat{\pi}_t - \hat{\pi}_{t-1}) + (1-\alpha)(1-\theta)\hat{p}_t$$

and hence:

$$\hat{p}_t = \frac{\alpha}{1-\alpha} (\hat{\pi}_t - \hat{\pi}_{t-1}). \quad (201)$$

Now take (200) and simplify the notation to:

$$0 = E_t \sum_{j=0}^{\infty} \gamma^j \Gamma_{t+j} \{ \tilde{p}_t \Omega_{t+j} - s_{\theta} m c_{t+j} \}, \quad (202)$$

by defining:

$$\begin{aligned} \Gamma_{t+j} &\equiv \bar{\Lambda}_{t+j}^{\circ} q_{t+j} (\pi_{t+j})^{\theta}, & \Omega_{t+j} &\equiv \frac{\pi_t}{\pi_{t+j}}, \\ s_{\theta} &\equiv \frac{\theta}{\theta-1}, & \gamma &\equiv \beta\alpha. \end{aligned}$$

Now rewrite (202) as:

$$\tilde{p}_t E_t \sum_{j=0}^{\infty} \gamma^j \Gamma_{t+j} \Omega_{t+j} = s_{\theta}^{-1} E_t \sum_{j=0}^{\infty} \gamma^j \Gamma_{t+j} m c_{t+j}. \quad (203)$$

Recall that the steady state value of \tilde{p}_t is equal to 1, as is that of Ω_{t+j} by construction. Then, since $\gamma < 1$, the steady state for (203) is:

$$\Gamma \sum_{j=0}^{\infty} \gamma^j = \frac{\Gamma}{1-\gamma} = s_{\theta}^{-1} \frac{\Gamma}{1-\gamma} m c = s_{\theta}^{-1} \Gamma m c \sum_{j=0}^{\infty} \gamma^j. \quad (204)$$

Dividing term by term (203) by (204), and taking logs, yields:

$$\begin{aligned} & \widehat{p}_t + \log \left((1 - \gamma) E_t \sum_{j=0}^{\infty} \gamma^j \exp \left(\log \frac{\Gamma_{t+j}}{\Gamma} \right) \exp (\log \Omega_{t+j}) \right) \\ &= \log \left((1 - \gamma) E_t \sum_{j=0}^{\infty} \gamma^j \exp \left(\log \frac{\Gamma_{t+j}}{\Gamma} \right) \exp \left(\log \frac{mc_{t+j}}{mc} \right) \right). \end{aligned}$$

We rewrite this as:

$$\widehat{p}_t + H(\widehat{\Gamma}_t, \widehat{\Omega}_t, \widehat{\Gamma}_{t+1}, \widehat{\Omega}_{t+1}, \dots) = J(\widehat{\Gamma}_t, \widehat{mc}_t, \widehat{\Gamma}_{t+1}, \widehat{mc}_{t+1}, \dots). \quad (205)$$

where

$$\begin{aligned} H(\widehat{\Gamma}_t, \widehat{\Omega}_t, \widehat{\Gamma}_{t+1}, \widehat{\Omega}_{t+1}, \dots) &\equiv \log \left((1 - \gamma) E_t \sum_{j=0}^{\infty} \gamma^j \exp \left(\widehat{\Gamma}_{t+j} \right) \exp \left(\widehat{\Omega}_{t+j} \right) \right) \\ J(\widehat{\Gamma}_t, \widehat{mc}_t, \widehat{\Gamma}_{t+1}, \widehat{mc}_{t+1}, \dots) &\equiv \log \left((1 - \gamma) E_t \sum_{j=0}^{\infty} \gamma^j \exp \left(\widehat{\Gamma}_{t+j} \right) \exp \left(\widehat{mc}_{t+j} \right) \right). \end{aligned}$$

Now we log-linearize H and J , as we did above for G , noting that 1) $\Omega_t \equiv 1$, so the corresponding term disappears, and 2) the partial derivatives of H and J with respect to $\widehat{\Gamma}_{t+j}$ are the same, so that the corresponding terms cancel out in the linear approximation of (205). Hence, we are left with:

$$\begin{aligned} & \widehat{p}_t + \gamma (1 - \gamma) E_t \widehat{\Omega}_{t+1} + \gamma^2 (1 - \gamma) E_t \widehat{\Omega}_{t+2} + \dots \\ &= (1 - \gamma) \widehat{mc}_t + \gamma (1 - \gamma) \widehat{mc}_{t+1} + \gamma^2 (1 - \gamma) \widehat{mc}_{t+2} \dots \end{aligned} \quad (206)$$

Using the definition of Ω_t , its log-linear deviation from steady state is:

$$\widehat{\Omega}_{t+j} \equiv \widehat{\pi}_t - \widehat{\pi}_{t+j},$$

so (206) becomes:

$$\begin{aligned} & \widehat{p}_t + \gamma (1 - \gamma) E_t (\widehat{\pi}_t - \widehat{\pi}_{t+1}) + \gamma^2 (1 - \gamma) E_t (\widehat{\pi}_t - \widehat{\pi}_{t+2}) + \dots \\ &= (1 - \gamma) \widehat{mc}_t + \gamma (1 - \gamma) \widehat{mc}_{t+1} + \gamma^2 (1 - \gamma) \widehat{mc}_{t+2} \dots \end{aligned}$$

which can be rearranged to:

$$\widehat{p}_t + \widehat{\pi}_t = (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j E_t (\widehat{mc}_{t+j} + \widehat{\pi}_{t+j}).$$

Now, note that this implies:

$$\widehat{p}_t + \widehat{\pi}_t = (1 - \gamma) (\widehat{mc}_t + \widehat{\pi}_t) + \gamma E_t (\widehat{p}_{t+1} + \widehat{\pi}_{t+1}),$$

and hence:

$$\widehat{p}_t = (1 - \beta\alpha) \widehat{mc}_t + \beta\alpha E_t (\widehat{p}_{t+1} + \widehat{\pi}_{t+1} - \widehat{\pi}_t),$$

(where we have replaced γ by its original expression). Now we use (201) to eliminate \widehat{p}_t and \widehat{p}_{t+1} , and finally obtain the log-linearized Phillips equation:

$$\widehat{\pi}_t - \widehat{\pi}_{t-1} = \frac{(1 - \alpha)(1 - \beta\alpha)}{\alpha} \widehat{mc}_t + \beta E_t (\widehat{\pi}_{t+1} - \widehat{\pi}_t).$$

Phillips equation for wages

In this case the equations are:

$$(\pi_t^w)^{1-\theta} = \alpha_W (\pi_{t-1}^w)^{1-\theta} + (1 - \alpha_W) (\tilde{w}_t \pi_t^w)^{1-\theta}, \quad (207)$$

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_W)^j \lambda_{t+j}^\circ h_{t+j} \bar{w}_{t+j} (\pi_{t+j}^w)^\psi \left\{ \left(\frac{\tilde{w}_t \pi_t^w}{\pi_{t+j}^w} \right) - \frac{\psi}{\psi - 1} \frac{\eta_H z_{t+j}^H h_{t+j}^\chi}{\lambda_{t+j}^\circ \bar{w}_{t+j}} \left(\frac{\tilde{w}_t \pi_t^w}{\pi_{t+j}^w} \right)^{-\psi\chi} \right\}. \quad (208)$$

Repeating the procedure used for (199), the log-linear version of (207) is:

$$\widehat{\tilde{w}}_t = \frac{\alpha_W}{1 - \alpha_W} (\widehat{\pi}_t^w - \widehat{\pi}_{t-1}^w). \quad (209)$$

Now divide through (208) by $(\tilde{w}_t \pi_t^w)^{-\psi\chi}$ and simplify the notation to:

$$0 = E_t \sum_{j=0}^{\infty} \gamma_W^j \Gamma_{t+j}^w \left\{ (\tilde{w}_t \pi_t^w)^{1+\psi\chi} - s_w \Psi_{t+j}^w (\pi_{t+j}^w)^{1+\psi\chi} \right\},$$

by defining:

$$\begin{aligned} \Gamma_{t+j}^w &\equiv \lambda_{t+j}^\circ h_{t+j} \bar{w}_{t+j} (\pi_{t+j}^w)^{\psi-1}, & \Psi_{t+j}^w &\equiv \frac{\eta_H z_{t+j}^H h_{t+j}^\chi}{\lambda_{t+j}^\circ \bar{w}_{t+j}}, \\ s_w &\equiv \frac{\psi}{\psi - 1}, & \gamma_W &\equiv \beta \alpha_W. \end{aligned}$$

and rewrite it as:

$$(\tilde{w}_t \pi_t^w)^{1+\psi\chi} E_t \sum_{j=0}^{\infty} \gamma_W^j \Gamma_{t+j}^w = s_w E_t \sum_{j=0}^{\infty} \gamma_W^j \Gamma_{t+j}^w \Psi_{t+j}^w (\pi_{t+j}^w)^{1+\psi\chi}, \quad (210)$$

The steady state value of \tilde{w}_t is 1, so the steady state for (210) is:

$$\pi^w \frac{\Gamma^w}{1 - \gamma_W} = s_w \frac{\Gamma^w}{1 - \gamma_W} \Psi^w (\pi^w)^{1+\psi\chi}. \quad (211)$$

Dividing term by term, the last two equations yields:

$$(1 + \psi\chi) (\widehat{\tilde{w}}_t + \widehat{\pi}_t^w) + H^w(\widehat{\Gamma}_t^w, \widehat{\Gamma}_{t+1}^w, \dots) = J^w(\widehat{\Gamma}_t^w, \widehat{\Psi}_t^w, \widehat{\Gamma}_{t+1}^w, \widehat{\Psi}_{t+1}^w, \dots), \quad (212)$$

where

$$\begin{aligned} H^w(\widehat{\Gamma}_t^w, \widehat{\Gamma}_{t+1}^w, \dots) &\equiv \log \left((1 - \gamma_W) E_t \sum_{j=0}^{\infty} \gamma_W^j \exp \left(\widehat{\Gamma}_{t+j}^w \right) \right) \\ &J^w(\widehat{\Gamma}_t^w, \widehat{\Psi}_t^w, \widehat{\pi}_t^w, \widehat{\Gamma}_{t+1}^w, \widehat{\Psi}_{t+1}^w, \widehat{\pi}_{t+1}^w, \dots) \\ &\equiv \log \left((1 - \gamma_W) E_t \sum_{j=0}^{\infty} \gamma_W^j \exp \left(\widehat{\Gamma}_{t+j}^w \right) \exp \left(\widehat{\Psi}_{t+j}^w \right) \exp \left((1 + \psi\chi) \widehat{\pi}_{t+j}^w \right) \right). \end{aligned}$$

As above, we log-linearize H^w and J^w , noting that the partial derivatives of H^w and J^w with respect to $\widehat{\Gamma}_{t+j}$ are the same and cancel out in the linear approximation to (212). We obtain:

$$(1 + \psi\chi) \left(\widehat{w}_t + \widehat{\pi}_t^w \right) = (1 - \gamma^w)_t \sum_{j=0}^{\infty} \gamma_W^j E_t \left(\widehat{\Psi}_{t+j}^w + (1 + \psi\chi) \widehat{\pi}_{t+j}^w \right)$$

which implies:

$$(1 + \psi\chi) \left(\widehat{w}_t + \widehat{\pi}_t^w \right) = (1 - \gamma^w) \left(\widehat{\Psi}_t^w + (1 + \psi\chi) \widehat{\pi}_t^w \right) \\ + (1 + \psi\chi) \gamma^w E_t \left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1}^w \right),$$

and hence:

$$\widehat{w}_t = \frac{1 - \beta\alpha_W}{1 + \psi\chi} \widehat{\Psi}_t^w + \beta\alpha_W E_t \left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1}^w - \widehat{\pi}_t^w \right).$$

Now use (209) to eliminate \widehat{w}_t and \widehat{w}_{t+1} :

$$\widehat{\pi}_t^w - \widehat{\pi}_{t-1}^w = \frac{(1 - \alpha_W)(1 - \beta\alpha_W)}{\alpha_W(1 + \psi\chi)} \widehat{\Psi}_t^w + \beta E_t \left(\widehat{\pi}_{t+1}^w - \widehat{\pi}_t^w \right).$$

Finally, the definition of $\widehat{\Psi}_t^w$ implies:

$$\widehat{\Psi}_t^w = \chi \widehat{h}_t + \widehat{z}_t^H - \widehat{\lambda}_t^\circ - \widehat{w}_t,$$

so substituting in the last expression yields the log-linearized Phillips equation for wages:

$$\widehat{\pi}_t^w - \widehat{\pi}_{t-1}^w = \frac{(1 - \alpha_W)(1 - \beta\alpha_W)}{\alpha_W(1 + \psi\chi)} \left(\chi \widehat{h}_t + \widehat{z}_t^H - \widehat{\lambda}_t^\circ - \widehat{w}_t \right) + \beta E_t \left(\widehat{\pi}_{t+1}^w - \widehat{\pi}_t^w \right).$$

Appendix 3: Definitions of compound parameters

The non-policy primitive parameters involved in the log-linearized structural equations are the following: $\beta, \xi, \mu^z, \delta^K, \cdot \phi^{*B}, \alpha_1^{RP}, \alpha_2^{RP}, g, t, \ell^G, i^*, F^D, a, b, v^K, v^W, v^N, \gamma^B, \gamma^R, \beta^B, a_0^B, a_L^B, a_D^B, a_u, a_v, a_M, b_M$, and the policy parameters are π^T (or δ^T) and h_0, h_1, h_2 , and in the case of IT_MEF, k_0, k_1 , and γ^T .

The following compound parameters have been used in the log-linearization of the systems:

Structure of firm factor/loan demands

$$\alpha_K^{MC} \equiv \frac{i^L}{1/v^K + i^L}, \quad \alpha_W^{MC} \equiv \frac{i^L}{1/v^W + i^L}, \quad \alpha_N^{MC} \equiv \frac{i^L}{1/v^N + i^L},$$

Structure of the real marginal cost base:

$$\alpha_q \equiv \frac{q}{q + F^D}$$

Structure of firm loan demand:

$$\begin{aligned}\gamma_1^{LM} &\equiv \frac{\frac{ab}{1/v^K+i^L}}{\frac{ab}{1/v^K+i^L} + \frac{(1-a)b}{1/v^W+i^L} + \frac{1-b}{1/v^N+i^L}}, \\ \gamma_2^{LM} &\equiv \frac{\frac{(1-a)b}{1/v^W+i^L}}{\frac{ab}{1/v^K+i^L} + \frac{(1-a)b}{1/v^W+i^L} + \frac{1-b}{1/v^N+i^L}}, \\ \delta_K^{LM} &\equiv \gamma_1^{LM} \frac{\alpha_K^{MC}}{v^K i^L}, \quad \delta_W^{LM} \equiv \gamma_2^{LM} \frac{\alpha_W^{MC}}{v^W i^L} \\ \delta_N^{LM} &\equiv (1 - \gamma_1^{LM} - \gamma_2^{LM}) \frac{\alpha_N^{MC}}{v^N i^L}\end{aligned}$$

Elasticity of private loan demand w.r. to the gross loan interest rate:

$$\gamma^{LM} \equiv (\delta_K^{LM} v^K + \delta_W^{LM} v^W + \delta_N^{LM} v^N) (1 + i^L)$$

Structure of total loan demand:

$$a^{LM} \equiv \frac{f_L(1+i^L)mc(q+F^D)}{f_L(1+i^L)mc(q+F^D) + \ell^G} = \frac{\ell - \ell^G}{\ell}$$

Structure of bank deposit supply:

$$\alpha_{DS}^B \equiv \frac{a^B d}{a^B d + a_L^B [(1+i^D) - (1-\gamma^B - \gamma^R)(1+i)]}$$

Structure of bank deposit margin:

$$\alpha^{MD} \equiv \frac{(1-\gamma^B - \gamma^R)(1+i)}{(1+i^D) - (1-\gamma^B - \gamma^R)(1+i)}$$

Structure of bank lending margin:

$$\alpha^{ML} \equiv \frac{1+i}{i^L - i}$$

Structure of deposit drains:

$$\alpha_B^{MD} \equiv \frac{\gamma^B}{1-\gamma^B-\gamma^R}, \quad \alpha_R^{MD} \equiv \frac{\gamma^R}{1-\gamma^B-\gamma^R} = \frac{\gamma^R}{1-\gamma^R} \alpha_B^{MD}$$

Structure of Bank assets:

$$\alpha_A^B \widehat{\ell}_t + (1 - \alpha_A^B) \widehat{b}_t^{CB} = \alpha_L^B \left(\widehat{d}_t - \alpha_B^{MD} \widehat{\gamma}_t^B - \alpha_R^{MD} \widehat{\gamma}_t^R \right) + (1 - \alpha_L^B) \left(\widehat{e}_t + \widehat{b}_t^{*B} \right)$$

$$\alpha_A^B \equiv \frac{\ell}{\ell + b^{CB}}$$

Structure of Bank liabilities:

$$\alpha_L^B \equiv \frac{d(1-\gamma^B-\gamma^R)}{d(1-\gamma^B-\gamma^R) + eb^{*B}}$$

Structure of cash demand:

$$a^{CM} \equiv \frac{\varpi [p^C c + p^V v]}{\varpi [p^C c + p^V v] + \gamma^B d}$$

Structure of private absorption:

$$\alpha_A^{CM} \equiv \frac{p^C c}{p^C c + p^V v}$$

Elasticity of household cash-absorption ratio w.r. to the gross deposit interest rate:

$$\alpha_D^{CM} \equiv \frac{1/2}{(a_M + 1)(1 + i^D) - 1}$$

Structure of domestic output:

$$a_y^Q \equiv \frac{y}{y + q^D + q^N}, \quad a_D^Q \equiv \frac{q^D}{y + q^D + q^N}$$

Structure of aggregate supply:

$$\begin{aligned} a^Y &\equiv \frac{y}{y + p^N n}, & \alpha_c^Y &\equiv \frac{p^C c}{p^C c + p^V v + x + g} \\ \alpha_v^Y &\equiv \frac{p^V v}{p^C c + p^V v + x + g}, & \alpha_x^Y &\equiv \frac{x}{p^C c + p^V v + x + g} \end{aligned}$$

Structure of loan supply:

$$\alpha_{LS}^B \equiv \frac{a^B \ell}{a^B \ell + a_0^B [(1 + i^D) - (1 - \gamma^B - \gamma^R)(1 + i)]}$$

Elasticity of intermediate domestic demand w.r. to the rental rate:

$$a_0^{qD} \equiv \frac{ki^K}{a_u q^D} = \frac{ki^K / a_u}{\tilde{\tau}_M [p^C c + p^V v] + C^B}$$

Structure of intermediate domestic demand:

$$a_1^{qD} \equiv \frac{\tilde{\tau}_M [p^C c + p^V v]}{\tilde{\tau}_M [p^C c + p^V v] + C^B}$$

Elasticity of auxiliary transactions cost function w.r. to the gross deposit interest rate:

$$\begin{aligned} b_\tau^Q &\equiv \frac{b_M \varpi^{-1} - a_M \varpi}{a_M \varpi + b_M \varpi^{-1}} \alpha_D^{CM} = \left(1 - \frac{2}{1 + \frac{b_M}{a_M \varpi^2}} \right) \alpha_D^{CM} \\ \tilde{\tau}_M &= a_M \varpi + \frac{b_M}{\varpi}. \end{aligned}$$

Structure of Central Bank liabilities:

$$\begin{aligned} a_1^{CB} &\equiv \frac{m^0}{m^0 + \gamma^R d + b^{CB}} \in (0, 1) \\ a_2^{CB} &\equiv \frac{\gamma^R d}{m^0 + \gamma^R d + b^{CB}} \in (0, 1) \end{aligned}$$

Elasticity of real marginal cost w.r. to the gross loan interest rate:

$$\alpha_L^{MC} \equiv [a b \alpha_K^{MC} + (1-a) b \alpha_W^{MC} + (1-b) \alpha_N^{MC}] \frac{1+i^L}{i^L} < 1$$

Structure of imports:

$$a_1^N \equiv \frac{(1-a^D) p^C c}{(1-a^D) p^C c + (1-b^D) p^V v + (1-b) m c [q + F^D] / (1+v^N i^L)} \in (0, 1)$$

$$a_2^N \equiv \frac{(1-b^D) p^V v}{(1-a^D) p^C c + (1-b^D) p^V v + (1-b) m c [q + F^D] / (1+v^N i^L)} \in (0, 1)$$

Structure of bank cost:

$$a^{BC} \equiv \frac{\ell (a_L^B \ell - a_0^B d)}{a_L^B \ell^2 + a_D^B d^2 - 2a_0^B \ell d}$$

Structure of consumption price index:

$$a_{PC} \equiv \frac{(1-a_D) (p^N)^{1-\theta_C}}{a_D + (1-a_D) (p^N)^{1-\theta_C}} \in (0, 1)$$

Structure of investment price index:

$$a_{PV} \equiv \frac{(1-b_D) (p^N)^{1-\theta_V}}{b_D + (1-b_D) (p^N)^{1-\theta_V}} \in (0, 1)$$

Structure of physical capital formation:

$$a_K \equiv \frac{(1-\delta^K) k}{(1-\delta^K) k + v \mu^z} = \frac{1-\delta^K}{\mu^z} \in (0, 1)$$

Structure of uses of foreign exchange resources:

$$a_1^{BP} \equiv \frac{r^{*CB}}{r^{*CB} + n + (1+i^*) \left[1 + \phi^{*B} + \alpha_1^{BP} (b^{*B} e)^{\alpha_2^{BP}} \right] \frac{b^{*B}}{\mu^z \pi^{*N}}}$$

$$a_2^{BP} \equiv \frac{n}{r^{*CB} + n + (1+i^*) \left[1 + \phi^{*B} + \alpha_1^{BP} (b^{*B} e)^{\alpha_2^{BP}} \right] \frac{b^{*B}}{\mu^z \pi^{*N}}}$$

Structure of sources of foreign exchange resources:

$$a_3^{BP} \equiv \frac{b^{*CB}}{b^{*CB} + p^{*X} x + (1+i^*) \frac{r^{*B}}{\mu^z \pi^{*N}} + \frac{g-t+i^L \ell^G}{e}}$$

$$a_4^{BP} \equiv \frac{p^{*X} x}{b^{*CB} + p^{*X} x + (1+i^*) \frac{r^{*B}}{\mu^z \pi^{*N}} + \frac{g-t+i^L \ell^G}{e}}$$

$$a_5^{BP} \equiv \frac{(1+i^*) \frac{r^{*B}}{\mu^z \pi^{*N}}}{b^{*CB} + p^{*X} x + (1+i^*) \frac{r^{*B}}{\mu^z \pi^{*N}} + \frac{g-t+i^L \ell^G}{e}}$$

Structure of net primary deficit:

$$\alpha_1^{BP} \equiv \frac{t}{g + i^L \ell^G - t}, \quad \alpha_2^{BP} \equiv \frac{i^L \ell^G}{g + i^L \ell^G - t}$$

Structure of marginal utility of consumption:

$$a_C \equiv \frac{\beta \xi}{\mu^z - \beta \xi} > 0, \quad \alpha_C \equiv \frac{\xi}{\mu^z - \xi} > 0$$

Elasticity of auxiliary function $\tilde{\varphi}_M$ w.r. to the gross domestic interest rate:

$$\varepsilon_M = \frac{2b_M}{2b_M + \varpi} \frac{1/2}{(a_M + 1)(1 + i^D) - 1}$$

Structure of bank risk premium:

$$a^{RP} \equiv \frac{1 + \phi^{*B}}{1 + \phi^{*B} + \alpha_1^{RP} (b^{*B} e)^{\alpha_2^{RP}}}$$

Structure of bank arbitrage premium:

$$\bar{a}^{RP} \equiv \frac{1 + \phi^{*B}}{1 + \phi^{*B} + (1 + \alpha_2^{RP}) \alpha_1^{RP} (b^{*B} e)^{\alpha_2^{RP}}} = \frac{1}{(1 + \alpha_2^{RP}) \frac{1}{a^{RP}} - \alpha_2^{RP}}$$

Structure of consumer price index:

$$a_{\Pi C} \equiv \frac{(\pi^T)^{1-\theta_C}}{(\pi^T)^{1-\theta_C} + (a_N/a_D) (p^N)^{1-\theta_C}} \in (0, 1).$$

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