# Dynamic Stability of alternative monetary policy rules in a small open economy two-sector New Keynesian model with perfect foresight [1]

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# Dynamic stability of alternative monetary policy rules in a small open economy two-sector New Keynesian model with perfect foresight [1]

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## I. Introduction

This paper studies the dynamic stability of various monetary/exchange regimes in a small open monetary economy model under perfect foresight, perfect capital mobility, and price and wage stickiness. There are two productive sectors: a domestic sector in which monopolistic competition prevails and a competitive export sector. A continuum of households supply differentiated labor services and fix wages, consuming domestic and imported goods and having a transactions technology by which holding money reduces costs. Both firms in the domestic sector and households set prices and wages, respectively, by inter-temporal optimization subject to price or wage inflation adjustment costs. These optimizations generate "Phillips curve" dynamic equations for price and wage inflations and hence a four dimensional dynamic system in the multilateral real exchange rate MRER, the domestic sector product wage (w), and the two inflation rates. The marginal utility of wealth is an additional endogenous variable that is constant except when an unexpected shock hits the economy.

Eight monetary regimes are successively considered within the same general framework, classified according to whether the MRER is a predetermined (FIX regimes) or a jump variable (IT regimes). In the FIX regimes the Central Bank uses exchange market interventions as its sole policy instrument. The UFIX regime is a fixing of the nominal exchange rate with the dollar, the MFIX is a fixing of the nominal exchange rate with a trade-weighted basket of currencies, and the RFIX is a monetary policy that fixes the MRER to a target path that converges to the long run equilibrium MRER. We find that the three FIX systems are unequivocally saddle-path stable, with the two inflation rates being jump variables and the MRER and w predetermined variables. In the five IT regimes the

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Central Bank uses as instrument the nominal interest rate through a feedback rule and gears its intervention policy to gradually achieve a certain backing of the stock of money with international reserves. In the DIT (HIT) regime the feedback rule relates the nominal interest rate to the domestic (headline) inflation rate. In the DIRT regime the feedback rule targets both the domestic inflation rate and the MRER. And in the DIOT (HIOT) the feedback rule targets both the domestic (headline) inflation rate and the output gap. All these IT regimes require some form of "Taylor property" for the coefficient on the targeted inflation rate that are successively more restrictive. In all these IT regimes we find necessary and sufficient conditions for saddle-path stability. However, in the DIOT and HIOT cases they are extremely complicated so we concentrate on more easily interpretable sufficient conditions for saddle-path stability.

In a later section we introduce two extensions to the basic model, each one under one of the policy regimes. In the first we introduce a long interest rate to the model under the DIT. This raises the dimensionality of the dynamical system but presents no problem, maintaining the stability properties intact. In the second extension we introduce inflation stickiness to the model under the UFIX (or MFIX) regime. We there assume that a fraction of firms and households, instead of deciding on their inflation rates through optimization, prefer to simply adjust gradually their inflation rates to the overall inflation rate in a backward looking way, so that the two additional variables of the dynamical system (the backward looking price and wage inflation rates) are predetermined. This increases the dimensionality of the system to 6. Although the necessary and sufficient conditions for saddle-path stability are quite intractable, easily interpretable sufficient conditions are found.

## II. The model

### II.1. Basic price assumptions

We assume there is perfect foresight for endogenous variables, conditional on given unanimous expectations (under certainty) for exogenous variables. Households are assumed to consume imported and domestic goods (i.e. goods that are produced domestically and only consumed domestically). Hence, it is convenient to define the (consumption) multilateral real exchange rate (MRER) as the relative price between imports (M) and domestic goods (N):

$$e \equiv (E_m P_M^*)/P_N.$$

where $E_m$ is the multilateral nominal exchange rate (pesos per a geometrically trade weighted basket of currencies), $P_M^*$ is the geometrically trade weighted basket of import price indexes, and $P_N$ is the peso price of domestic goods. In many less developed countries attention is often concentrated on the nominal exchange rate with the dollar E. Many reasons can account for this: there may be a mismatch between financial dollarization and trade diversification; there may be a fixed exchange regime where the domestic currency (peso) is pegged to the dollar; or simply the large and persistent swings of the MRER of the US may be one of the fundamental external shocks that the economy faces along the economic cycle if the nominal exchange rate does not float freely all the time. For simplicity, let us reduce the country's trade partners to the U.S.A and Europe (which thus represents all trade partners except the U.S.A.) and assume that a significant fraction of trade ($\alpha_{EU}$) is done with the Euro area and the rest ($\alpha_{US} = 1-\alpha_{EU}$) with the U.S.A. Hence, the MRER is:

(1) $$e \equiv (EP_{US}^*/P_N)^{\alpha_{US}}[(E/\rho_{EU/US})P_{EU}^*/P_N]^{\alpha_{EU}} = (E/\rho)/P_N$$

where $\rho_{EU/US}$ is the exogenous euro/dollar nominal exchange rate,

$$\rho \equiv (\rho_{EU/US})^{\alpha_{EU}} \equiv (1)^{\alpha_{US}}(\rho_{EU/US})^{\alpha_{EU}}$$

is the exogenous trade weighted basket of foreign currencies/dollar nominal exchange rate,

$$E_m = E/\rho = (E)^{\alpha_{US}}(E/\rho_{EU/US})^{\alpha_{EU}}$$

is the country's multilateral nominal exchange rate, and we assume in (1) that there is no inflation in import prices and that the multilateral import price index is normalized to one:

$$P_M^* \equiv (P_{US}^*)^{\alpha_{US}}(P_{EU}^*)^{\alpha_{EU}} = 1.$$

Let $P_X^*$ be the analogous geometrically weighted basket of Argentine export price indexes. Then the external terms of trade is defined as $\phi \equiv P_X^*/P_M^*$ and, due to the assumption that $P_M^*=1$, $\phi$ is actually the price index of exports $P_X^*$. Since firms produce export and domestic goods, $\phi e=E_m P_X^*/P_N$ is the relevant relative price for output decisions.

Because the consumption sub-utility function (44) will have a Cobb-Douglas specification for the consumption of imported and domestic goods, the (dual) Consumer Price Index will be a Cobb-Douglas index of these goods:

(2) $P \equiv (E/\rho)^{\theta} P_N^{1-\theta}$.

Hence, the real wage in terms of the consumption basket, $\omega$, is

(3) $\omega \equiv W/P = (W/P_N)/[(E/\rho)/P_N]^{\theta} = w/e^{\theta}$,

where W is the nominal wage, and $w \equiv W/P_N$ is the product wage in the domestic sector. Also, the product wage in the export sector is

(4) $W/[P_X^*(E/\rho)] = (W/P_N)/\{(P_X^*/P_M^*)[(E/\rho)P_M^*/P_N]\} = w/(\phi e)$.

## II.2. Sectoral budget constraints

### Households and the role of money in transactions

We assume that holding money diminishes the cost of transactions in terms of goods.[2] Let M stand for the nominal stock of currency in circulation, which is the only kind of money considered in this paper. Then $m \equiv M/(E/\rho)$ is the foreign currency value of the stock of money and $M/C \equiv m/c$ is the money to consumption ratio, where $c \equiv C/(E/\rho)$ is the foreign currency value of total consumption. We assume that transactions involve the (non utility generating) consumption of real resources (produced goods) and that these transaction costs (per unit of consumption) are a function $\tau$ of the money/consumption ratio:

$$\tau(m/c) \qquad\qquad ( \tau'<0, \tau''>0 ).$$

When the money to consumption ratio increases, transactions costs (per unit of consumption) decrease at a decreasing rate, reflecting a diminishing marginal productivity of money in reducing transaction costs. To obtain private savings we must subtract $(1+\tau)c$ from disposable income (instead of c). Also, for simplicity, we assume that the government can avoid these transaction costs.

[2] This way of modeling money demand has been used by Kimbrough (1992), Agénor (1995) and Montiel (1997), among others.

In this section we will treat households' budget constraints as if there were a representative household and a single domestic good. The model we develop, however, has monopolistic competition in the domestic sector and continuums of households and firms, and corresponding continuums of domestic goods and labor types. We assume that the conditions necessary for all households to face identical budget constraints are satisfied (cfr. Woodford (2003), chapter 3). In particular, if any households face different present values of their wage and profit incomes, their initial financial wealth is assumed to differ by exactly the amount necessary to offset this. In practice, this means that as far as the budget constraints are concerned we can work with a fictitious "representative household".

Households hold financial net wealth that is composed of domestic money (M), peso denominated nominal claims on the government ($B_H$), and net foreign currency denominated foreign debt ($d_H$). For simplicity, we assume that the composition of foreign debt exactly matches the composition of trade.[3] Expressed in foreign currency, household wealth is:

(5) $a = m + b_H - d_H,$

where $b_H \equiv B_H/(E/\rho)$. The household's flow budget constraint is:

(6) $\dot{a} = \dot{m} + \dot{b}_H - \dot{d}_H = y - t - (1+\tau(m/c))c - r\, d_H + i\, b_H - \delta(m+b_H), \quad (\delta \equiv \dot{E}/E),$

where y is pre-tax income, t is the foreign currency value of lump sum taxes net of transfers, r is the interest rate on the foreign debt, i is the domestic interest rate, and $\delta$ is the rate of nominal depreciation of the peso against the dollar. Because we treat $\rho_{EU/US}$ (as well as other relevant parameters such as $\phi$) as an exogenous parameter that only changes through jumps, it is the rate of nominal deprecication of the peso against the dollar that figures in (6) and elsewhere. Net interest payments on the debt $rd_H - ib_H$ must be subtracted from primary savings, as well as the capital losses on the foreign currency value of the stock of money and government bonds due to currency depreciation.

[3] Escudé (2004) takes the extreme opposite assumption that all trade is with Europe and all debts are dollar denominated. Hence, that paper has an additional effect of a strong dollar shock associated with this mismatch.

To simplify, we assume perfect capital mobility. Hence, there is no country risk premium and, by arbitrage, uncovered interest parity must prevail, i.e. the domestic peso-denominated interest rate i must equal the interest rate on foreign debts ($r=r_{US}\alpha_{US}+r_{EU}\alpha_{EU}$), which is assumed to be constant, plus the rate of nominal depreciation:

(7) $$i = r + \delta.$$

Using (5) and (7) gives an alternative expression for the household budget constraint:

(6') $$\dot{a} = y - t - (1+\tau(m/c))c + ra - im.$$

The household's inter-temporal solvency is guaranteed by a "No Ponzi Game" condition:

(8) $$\lim_{T\to\infty} a \exp\left(-\int_0^T r\, ds\right) = \lim_{T\to\infty} a\, e^{-rT} \geq 0,$$

which in the household optimum holds with equality. Integrating (6') forward and using (8) gives the inter-temporal budget constraint: the present value of future consumption (gross of transactions consumption) must be equal to the present value of disposable income plus initial (non-human) wealth ($a_0$).

(9) $$a_0 + \int_0^\infty [\, y - t - (1+\tau(m/c))c - im]\, e^{-rs}\, ds = 0.$$

The public sector

The Central Bank

This paper considers eight alternative monetary regimes, three of which consist in fixing a nominal or real exchange target (FIX) and five different versions of Inflation Targeting (IT). We model the Central Bank's balance sheet and budget constraint in a sufficiently general fashion that it can contain any of these regimes. In the general framework, the Central Bank has currency M ($=mE_m=mE/\rho$) as a liability and domestic currency government bonds $B_{CB}$ and international reserves R as assets. We place no restriction on the sign of $B_{CB}$. If it is negative, it can be interpreted as a stock of bonds issued by the Central Bank which are perfect substitutes to those issued by the Government. The international reserves are assumed to be invested in dollar and euro bonds in the proportions given by $\alpha_{US}$ and $\alpha_{EU}$. Whenever $\delta$ differs from 0, the Central Bank also issues (or withdraws) a certain amount of additional currency $\delta(E_m R)$ which it passes to(or collects from) the government, along with the interest earned on R and $B_{CB}$. Hence, the rate of money growth is:

$$(10) \qquad \dot{M} = E_m \dot{R} + \dot{B}_{CB} + \delta(E_m R).$$

Then the Central Bank's budget constraint expressed in foreign currency is:

$$(10') \qquad \dot{m} - \dot{R} - \dot{b}_{CB} = \delta(R + b_{CB} - m).$$

Assuming the existing monetary policy started at an instant in which there was full backing of monetary liabilities with international reserves and government bonds, i.e. $M=E_m R+ B_{CB}$, the Central Bank is thus assumed to maintain this full backing at all times:

$$(11) \qquad m = R + b_{CB}.$$

Whenever there is an exogenous jump in $\rho$ (due to a jump in $\rho_{EU/US}$) there is a one time capital gain (or loss) on R that is also passed on to (or recovered from) the government in the form of currency.

The Government and fiscal policy

Let G (g) be the primary expenditures in pesos (foreign currency) and assume the government only purchases domestic goods. Then $G = P_N g_N$.

The Government can finance its primary expenditures and its interest payments through lump sum taxes (net of transfers), interests gained on Central Bank reserves, seigniorage revenues, and (foreign and domestic) debt financing. Hence, the government's flow budget constraint is:

$$(12) \qquad \dot{b} + \dot{d}_G = g - t + r(d_G - R) + ib_H - \delta(R+b) = g - t + r(d_G + b_H - R) - \delta(R + b_{CB})$$

where $d_G$ is the government's foreign debt. We assume that government debt instruments are perfect substitutes for securities issued in the rest of the world. The government pays interest on its foreign and domestic debt, and receives from the Central Bank the interest on international reserves and the pesos that are issued with the backing of the capital gains on reserves ($\delta R$).

Define the consolidated government's net liabilities (including money) as

(13) $n = m - R + b_H + d_G = b_{CB} + b_H + d_G = b + d_G$.

This is the same as the financial wealth of the Government because (11) states that the Central Bank's net liabilities are always zero. Then the budget constraint of the consolidated public sector is simply:

(12') $\dot{n} = g - t + rn - im$.

It is assumed that the public sector always plans to be solvent, which implies that it expects to comply with a "no-Ponzi game condition":

(14) $\lim_{T \to \infty} n \exp(-\int_0^T r\, ds) = \lim_{T \to \infty} n\, e^{-rT} = 0.$

This condition implies that the public sector's net debt must eventually grow at a rate that is lower than the interest rate. Integrating (12') forward and using (14) gives the public sector's inter-temporal budget constraint: the present value of its primary expenditures plus its initial debt must be equal to the present value of its revenues (including the interest on the international reserves and seigniorage due to the effect of currency depreciation on monetary liabilities):

(15) $n_0 = \int_0^\infty [\, t - g + im]\, e^{-rs}\, ds.$

Fiscal policy will be extremely simple and common to the different dynamical systems. We assume that the does not consume imported goods, that the government's domestic expenditures are always a fraction *g* of aggregate household consumption of domestic goods and that the foreign exchange value of tax revenues t is adjusted to maintain fiscal balance at all times. Hence, n is always constant at $n_0$.

The balance of payments

Due to the assumption of perfect capital mobility, the public and the private sectors have full access to foreign savings at the international rate r. Let us define the country's net foreign debt as:

(16) $d \equiv d_H + d_G - R = n - a.$

Then, subtracting (12') from (6') gives the country's budget constraint, or balance of payments:

(17) $-\dot{d} = y - g - (1+\tau(m/c))c - rd.$

The country's net foreign position expressed in dollars (-d) evolves according to primary savings (net of transaction costs) minus interest payments on the national debt. Also, (9) and (15) give the country's inter-temporal budget constraint: the present value of future trade surpluses must equal the initial foreign debt.

(18) $d_0 = \int_0^{\infty} [y - g - (1+\tau(m/c))c]\, e^{-rs}\, ds.$

II.3. The price and wage setting framework

We assume that the export sector is perfectly competitive, and that absolute PPP prevails for exports and imports, with full and instantaneous pass-through to peso prices in the case of imports. There is monopolistic competition in the supply of labor services by households and in the supply of goods by firms in the domestic sector. However, these wages and prices are sticky, which means that they are predetermined variables and that the desired wage and domestic price cannot be attained quickly because there are price and wage adjustment costs which must be taken into account. We use adjustment cost functions similar to those in Rotemberg (1982, 1995) and Sbordone (1998). These adjustment costs reflect costs related to optimal decision making, such as information gathering and analysis, evaluation of customer's possible reactions, etc.. Firms' maximization of discounted profits and households' inter-temporal utility maximization in symmetric equilibriums lead to well defined "Phillips curves" for domestic price inflation and wage inflation, respectively. These equations reflect a gradual adjustment of domestic prices and wages, respectively, towards their long-run desired levels, which are the monopolistic competition mark-up over marginal cost and marginal rate of substitution of wealth for leisure, respectively. The resulting dynamic model has as steady state a benchmark economy of full wage and price flexibility, i.e. one in which there are no price and wage adjustment costs. The benchmark model, with flexible domestic prices and wages, is similar to the static Blanchard and Kiyotaki (1987) model, except for the fact that it represents an open two-sector economy. And the dynamic system is similar to those in Erceg et al (2000), Sbordone (2001), and Woodford (2003), except again for openness and for the fact that instead of a Calvo type staggered pricing framework we have price and wage inertia due to explicit price and wage changing costs. There are also similarities with Galí and Monacelli (2003), but theirs is a one sector open economy which does not have monopolistic competition in the household sector. Also, we abstain from modeling the rest of the world (which seems to be more important when there is a single produced good because in this case the “small country” assumption can only be valid as a limit). Soto (2003) has a two sector open economy framework with Calvo type staggered prices that is somewhat more complicated than ours because the export good is also consumed domestically. Monetary policy, however, is less clearly formulated.

In section IV backward looking “rule of thumb” firms and households are introduced in order to have “inflation stickiness”, as the data seem to indicate (cfr. Fuhrer and Moore (1995) and Roberts (1997)). Instead of going through costly price or wage adjustment decisions, these agents prefer to simply adjust their price or wage inflation to the general price or wage inflation index.

II.4. <u>Firm decisions</u>

There are two production sectors that produce exportable (X) and domestic (N) goods, respectively. Capital is fixed in each sector and does not depreciate and labor is perfectly mobile between sectors but immobile internationally. There is a representative firm in the export sector and a continuum of monopolistically competitive firms in the domestic sector, each of which is characterized by the good type $i \in [0,1]$ it produces. Output in each sector is given by production functions: $y_X = F_X(L_X)$, $y_{Ni} = F_N(L_N)$, that have positive and diminishing marginal labor productivities, where $L_X$ and $L_N$ are aggregates of the complete range of labor types $j \in [0,1]$, as we will see in the next section. We assume that there is a single labor market, where all firms (whether in the domestic or export sector) hire the same CES aggregate of all types of labor and face the same wages. As in Erceg et al (2000), assume that there is a competitive “employment agency” (or “representative labor aggregator”) that combines households’ labor types in the same proportion that firms would choose. Define the aggregate of labor types by

$$(19) \qquad L = \left\{ \int_0^1 L_j^{(\psi -1)/\psi} \, dj \right\}^{\psi/(1-\psi)} \qquad (\psi>1).$$

We will refer to L as ‘labor’. The employment agency’s demand for each labor type j is equal to the sum of all firms’ demands. It minimizes the cost of producing a given level of L. Hence, it minimizes

$$(20) \qquad \int_0^1 W_j L_j \, dj$$

subject to (21) with a given value of L, where $W_j$ is the wage rate set by the monopolistic supplier of labor type j. This gives the agency’s demand (and the aggregate demand of all firms) for labor type j as

$$(21) \qquad L_j = L \, (W_j/W)^{-\psi}$$

where W is the aggregate wage index, defined as:

$$(22) \qquad W = \left\{ \int_0^1 W_j^{1-\psi} \, dj \right\}^{1/(1-\psi)},$$

and $\psi$ is the wage elasticity of demand for all types of differentiated labor services. The higher $\psi$ is, the lower is the monopolistic power of households, because the varieties of labor are closer substitutes. Total labor cost is given by

$$(23) \qquad \int_0^1 W_j L_j \, dj = WL.$$

The export sector is assumed to be competitive and has a profit maximizing representative firm that chooses the labor input each instant so that its marginal productivity is equal to the product wage (4):

(24) $F_X'(L_X) = w/(\phi e)$.

Note that in (4) W is the wage index for the complete range of labor types, as given by (22).

Each firm in the domestic sector is constrained in its price setting activity by te assumption that its price level is a predetermined variable (which implies it cannot jump in response to changes in expectations) and the assumption that changing price is costly. For simplicity, we assume that this price changing activity requires the non utility generating consumption of the good the price of which is to be adjusted. Let us temporarily drop the sub-index N, for ease of notation. In a continuous time analogy to Sbordone (1998), let $x(\pi_i)$ represent the cost per unit sale of changing $P_i$ at the rate $\pi_i \equiv dlnP_i/dt$. We assume that this adjustment cost function is twice continuously differentiable and has the following properties:

(25) $x(0) = x'(0) = 0, \qquad x''(0) = a_F > 0.$

Note that while $P_i$ is a predetermined variable, $\pi_i$ is a jump variable which can respond discretely to changes in expectations. Each firm in the domestic sector is also constrained by its technology and by the demand function it faces for its distinct variety i:

(26) $F(L_i)= y_i, \qquad y_i = y(P_i/P)^{-\nu}$.

The demand function for domestic goods will be derived in the next section.

Firm i chooses $\pi_i$ to maximize the present value of future profits:

$$\int_0^{\infty} \{ y_iP_i[1–x(\pi_i)] - WL_i \} e^{-rs} ds = \int_0^{\infty} \{ yP^{\nu} P_i^{1-\nu}[1–x(\pi_i)] - WF^{-1}(yP^{\nu} P_i^{-\nu}) \} e^{-rs} ds,$$

subject to the fact that

(27) $\dot{P}_i = P_i \pi_i$.

Hence, its undiscounted Hamiltonian is:

$$H^i = yP^{\nu} P_i^{1-\nu}(1 - x(\pi_i)) - WF^{-1}(yP^{\nu} P_i^{-\nu}) + \lambda_i P_i \pi_i,$$

where $\lambda_i \equiv \lambda_i^* e^{rt}$ represents the marginal net present value of price increase (and $\lambda_i^*$ is the corresponding co-state variable). Firm i's first order conditions are:[4]

(28) $$H^i_{\pi i} = 0, \qquad \dot{\lambda}_i - r\lambda_i = -H^i_{Pi}.$$

The first of these conditions gives:

(29) $$\lambda_i = x'(\pi_i)y_i.$$

For the second one, it is convenient to define the marginal cost as the wage rate divided by the marginal productivity of labor ($1/z(y_i)$):

(30) $$Wz(y_i) \equiv Wd(F^{-1}(y_i))/dy_i$$

Hence,

$$H^i_{Pi} = yP^{\nu}(1-\nu)P_i^{-\nu}[1 - x(\pi_i)] - Wz(y_i)(-\nu)yP^{\nu} P_i^{-\nu-1} + \lambda_i \pi_i,$$

and therefore, using (31):

$$\dot{\lambda}_i/\lambda_i = r - H^i_{Pi}/\lambda_i = r + [(\nu-1)/x'(\pi_i)]\{ 1-x(\pi_i)-\pi_i x'(\pi_i)/(\nu-1) - \nu/(\nu-1)(W/P_i)z(y_i) \}.$$

On the other hand, (31) implies

$$\dot{\lambda}_i/\lambda_i = x''(\pi_i)\dot{\pi}_i/x'(\pi_i) + \dot{y}_i/y_i .$$

Therefore, the last two equations imply:

$$x''(\pi_i)\dot{\pi}_i/x'(\pi_i) + \dot{y}_i/y_i = r - \pi_i + [(\nu-1)/x'(\pi_i)]\{ 1-x(\pi_i) - \nu/(\nu-1)(W/P_i)z(y_i) \},$$

which can be rearranged to:

(31) $$\dot{\pi}_i = [x'(\pi_i)/x''(\pi_i)][ r - \dot{y}_i/y_i - \pi_i ] + $$

$$+ [(\nu-1)/x''(\pi_i)]\{ 1-x(\pi_i) - \nu/(\nu-1)(W/P_i)z(y_i) \}.$$

In a neighborhood of a steady state with zero inflation (25) applies, and hence (31) reduces to

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[4] There is also the transversality condition: $\lim_{t\to\infty} P_i\lambda_i e^{-rt} = 0$.

(32) $\dot{\pi}_i = [(\nu\text{-}1)/a_F] \{ 1 - \nu/(\nu\text{-}1)(W/P_i)z(y_i) \}.$

Since all domestic firms face the same problem, they all set the same price and inflation rate, so we may drop the subscript i from (34) (but again insert the subscript N) to obtain a domestic price "Phillips Curve" equation:

(33) $\dot{\pi}_N = -\gamma_F G^P(w, y_N)$ $(\gamma_F \equiv (\nu\text{-}1)/a_F)$,

where we defined the percentage gap between the actual domestic price and the benchmark (flex-price) domestic price as:

(34) $G^P(w, y_N) \equiv [\mu_P Wz(y_N) - P_N]/P_N = \mu_P wz(y_N) - 1$ $(\mu_P \equiv \nu/(\nu\text{-}1))$.

Whenever the price gap is positive, the domestic price level is below the desired one (which is the usual mark-up over marginal cost). This will make firms gradually increase their price ($\pi_N$>0) but at a decreasing rate ($d\pi_N/dt$<0).

II.5 Household decisions

Households are also assumed to be monopolistic competitors. They set the wage rate and face wage adjustment costs that make them adjust the wage rate gradually towards the benchmark (flex-wage) nominal wage. Let $x(\pi_{Wj})$ represent the cost of changing $W_j$ at the rate $\pi_{Wj} \equiv d\ln W_j/dt$. We use the same symbol as for firms' cost of adjustment function only for ease of notation. Assume again that this function has the following properties:

(25') $x(0) = x'(0) = 0, \quad x''(0) = a_H > 0.$

Household $j \in [0,1]$ supplies labor of type j and maximizes an inter-temporal utility function which is additively separable in consumption and leisure (or negative work effort):

(35) $\int_0^\infty \{ u(c_M,c_N)^{1-\sigma}/(1-\sigma) - v(L_j) \} e^{-\beta s} ds,$

where $c_M$ is the consumption of imported goods, $c_N$ is the consumption of the domestic goods bundle, and $L_j$ is labor exertion. The consumption part of the instantaneous utility expression is of the constant relative risk aversion (CRRA) family, where $\sigma$>0 is the inverse of the of the inter-temporal elasticity of substitution (as well as the coefficient of relative risk aversion[5]). In (35), u(.) is a private goods consumption sub-utility index, v(.) is the disutility of labor, which is increasing with $L_j$ at an increasing rate (v'>0, v">0),[6] and $\beta$ is the inter-temporal discount factor.

In analogy to the 'employment agency', assume that there is a competitive 'commercial agency' (or 'representative consumption aggregator') that combines the different goods into a single product, that we will refer to as 'domestic good' in the proportions dictated by households' preferences. The commercial agency's composite $c_N$ is defined by:

$$(36) \qquad c_N = \left\{ \int_0^1 c_{Ni}^{(\nu-1)/\nu} \, di \right\}^{\nu/(1-\nu)} \qquad (\nu>1).$$

For any level of the composite $c_N$ the agency minimizes expenditures, given the prices $P_{Ni}$ set by the domestic sector firms. Hence, it minimizes

$$(37) \qquad \int_0^1 P_{Ni} \, c_{Ni} \, di$$

subject to (36) for a given value of $c_N$. This gives total consumption demand for $c_{Ni}$ as:

$$(38) \qquad c_{Ni} = (P_{Ni}/P_N)^{-\nu} \, c_N,$$

where the Lagrange multiplier $P_N$ is the (dual) Dixit-Stiglitz price index for domestic goods

$$(39) \qquad P_N = \left\{ \int_0^1 P_{Ni}^{1-\nu} \, di \right\}^{1/(1-\nu)},$$

and $\nu$ is the price elasticity of demand for all types of (differentiated) goods. The higher $\nu$ is, the lower is the market power of firms because the varieties are closer substitutes. Furthermore, total expenditure on domestic goods is

$$(40) \qquad \int_0^1 P_{Ni} \, c_{Ni} \, di = P_N \, c_N.$$

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[5] Observe that if $u(c)=c^{1-\sigma}/(1-\sigma)$, the coefficient of relative risk aversion is $-cu''(c)/u'(c)=\sigma$. We generally assume below that $\sigma \geq 1$. In some cases we find it useful to specialize to logarithmic utility (where $\sigma=1$).

[6] We could include and additively separable sub-utility index $\upsilon(g_N)$ representing the utility obtained by the household from the quantities of public goods produced by the government (measured through the quantities purchased by the government). However, since $g_N$ is not a decision variable for the household and will be held constant throughout, $\upsilon(.)$ would not play any significant role except as a reminder that government expenditures do generate household utility.

For concreteness, assume that u(.) is Cobb-Douglas:[7]

(41) $u(c_M,c_N) \equiv c_M^{\theta} c_N^{1-\theta}$.

where θ is the intra-temporal elasticity of substitution in consumption between imported and domestic goods, and is also the share of imported goods in total consumption, as shown below. The consumer price index defined by (2) corresponds to the dual of (41). Total consumption expenditure measured in foreign currency is:

(42) $c = c_M + c_N/e$.

Then minimizing (42) subject to a constant (and arbitrary) level of utility $u_0 = c_M^{\theta} c_N^{1-\theta}$ gives:

(43) $ec_M/c_N = \theta/(1-\theta)$

independently of $u_0$. Note that (42) and (43) imply

(44) $c_N = (1-\theta)ec$, $c_M = \theta c$,

(45) $c_M^{\theta} c_N^{1-\theta} = \kappa_0 \, e^{1-\theta} c$ $(\kappa_0 \equiv \theta^{\theta}(1-\theta)^{1-\theta})$.

As the two expressions in (44) show, consumption demands for N and M are easily obtained from c and e, so we will prefer to work with the latter. Using (45) in (41) gives the following expression for (35):

(46) $\int_0^{\infty} \{ \kappa_1 (e^{1-\theta} c)^{1-\sigma}/(1-\sigma) - v(L_j) \} e^{-\beta s} \, ds$ $(\kappa_1 \equiv \kappa_0^{1-\sigma})$.

Let $Y = E_m \phi y_X + P_N \, y_N$ be aggregate output measured in pesos. Then the foreign currency value of aggregate output is $Y/E_m \equiv y = \phi y_X + y_N/e$. In a symmetric equilibrium, aggregate profits are $\Pi/E_m \equiv \phi y_X + (y_N/e)[1-x(\pi_N)]-(W/E_m)L[1-x(\pi_W)]$. We assume that firm ownership is distributed evenly among households. Hence, j's income in foreign currency is $\Pi/E_m + (W_j/E_m)L_j$ and the flow budget constraint (6') may equivalently be expressed as:

(6") $\dot{a} = (W_j/E_m)L_j \, (1 - x(\pi_{Wj})) + \Pi/E_m - t - (1+\tau(m/c))c + ra - im$.

[7] This follows Montiel (1999). The model can easily be extended to a CES sub-utility index (cfr. Calvo and Végh (1992), or Obstfeld and Rogoff (1996), for example). However, it does not seem to add much in the present context while it complicates the formulas.

The household is also constrained by the demand function it faces for its distinct labor variety j (21) and the fact that

(47) $\dot{W}_j = W_j \pi_{Wj}$.

Hence, the household maximizes (46) subject to (21), (47), (6") and its "no Ponzi-game" condition (8). Its control variables are c, m, and $\pi_{Wj}$, and it takes as given the future paths of $\Pi$, $E_m$, e, t, and i, as well as the values of the parameters involved. Due to the assumption of perfect foresight, unless there is an unexpected shock to any of the parameters, those expected paths will be the actual ones.

The non-discounted Hamiltonian of household j is:

(48) $$H^j = \kappa_1(e^{1-\theta} c)^{1-\sigma}/(1-\sigma) - v(LW^{\psi}W_j^{-\psi}) + \lambda_j\{ LW^{\psi}W_j^{1-\psi}[1 - x(\pi_{Wj})]/E_m$$
$$+ \Pi/E_m - t - (1+\tau(m/c))c + ra - im\}+ \mu_j W_j \pi_{Wj},$$

where $\lambda_j \equiv \lambda_j^* e^{\beta t}$ represents the marginal utility of wealth (and $\lambda_j^*$ is the corresponding co-state variable) and $\mu_j \equiv \mu_j^* e^{\beta t}$ represents the marginal utility of wage increases (and $\mu_j^*$ the corresponding co-state variable). The necessary conditions for an optimum are:

(49) $$H^j_c = 0, \qquad H^j_m = 0, \qquad H^j_{\pi Wj} = 0,$$
$$\dot{\lambda}_j - \beta\lambda_j = -H^j_a, \qquad \dot{\mu}_j - \beta\mu_j = -H^j_{wj},$$

that is,

(50) $\kappa_1 e^{(1-\theta)(1-\sigma)} c^{-\sigma} = \lambda_j \varphi(m/c)$

(50b) $-\tau'(m/c) = i$,

(50c) $\mu_j = x'(\pi_{Wj})\lambda_j L_j/E_m$,

(50d) $\dot{\lambda}_j/\lambda_j = \beta - r$

(50e) $\dot{\mu}_j/\mu_j = \beta - \{ \psi v'(L_j)(L_j/W_j) + (1-\psi)\lambda_j(L_j/E_m)[1-x(\pi_{Wj})] + \mu_j\pi_{Wj} \}/\mu_j$

along with the transversality condition

(50f) $\lim_{t\to\infty} a\lambda_j e^{-\beta t} = 0.$

To alleviate notation, in (50) we have defined the function φ that gives the effect on savings of a marginal increase in utility generating consumption c:

(51) $\varphi(m/c) \equiv 1+\tau(m/c)-(m/c)\tau'(m/c),\qquad \varphi'(m/c) = -(m/c)\,\tau''(\rho m/c) < 0.$

We will call φ the marginal savings function. Equation (50) shows that in equilibrium the utility of a marginal increment in consumption (left side of the equality) must be equal to the marginal disutility of the reduction in wealth that it generates. The latter is equal to the marginal utility of wealth, $\lambda_j$, times the marginal reduction in savings, φ. Observe in (51) that φ varies inversely with m/c and that the reduction in savings generated by a marginal increase in c is given by the increase in consumption gross of existing transaction costs, 1+τ, plus the increase in transaction costs due to the reduction in the money/consumption ratio.

Equation (50b) shows that in the optimum money holdings must be such that the reduction in transaction costs generated by a marginal increase in money holdings equals the opportunity cost of holding money (i). Inverting -τ' gives the following demand function for money:

(52) $m^D = \ell(i)c \qquad (\ell \equiv (-\tau')^{-1},\ \ell'=-1/\tau''<0\ ).$

Observe that this implies that in terms of domestic goods the demand for money is $M/P_N = \ell(i)ec$.

Equation (50d) shows that over time the rate of growth of the marginal utility of wealth must be equal to the difference between the inter-temporal discount rate, β, and the interest rate. This implies that the more impatient the household is (the greater is β), the faster the marginal utility of wealth must increase, that is, the faster the household must reduce its wealth through increased consumption. However, given that β and r are both exogenous constants, in order to have a steady state we make the usual simplifying assumption that β=r. Hence, $\lambda_j$ is constant as long as there are no unanticipated shocks that make the household re-evaluate its consumption decision, in which case $\lambda_j$ (and c) may face a discrete jump.

Taking the derivative of (50c) with respect to time and using (50e) gives an expression entirely analogous to the one obtained for the firms' problem (31)

$$\dot{\pi}_{Wj} = [x'(\pi_{Wj})/x''(\pi_{Wj})]\,[\delta - \dot{L}_j/L_j + \beta - \pi_{Wj}] +$$
$$+ [(\psi-1)/x''(\pi_{Wj})]\{\,1-x(\pi_{Wj}) - \psi/(\psi-1)v'(L_j)(E_m/W_j\lambda_j)\,\}.$$

In a neighborhood of a steady state with zero inflation, using (37) this expression reduces to

(53) $\dot{\pi}_{Wj} = [(\psi-1)/a_H]\,\{\,1 - \psi/(\psi-1)v'(L_j)(E_m/W_j\lambda_j)\,\}.$

Since all households face the same problem, they all set the same wage and wage inflation rate, so we can drop the subscript j from (53) to obtain a wage "Phillips Curve" equation:

(54) $\dot{\pi}_W = -\gamma_H\, G^W(\lambda w/e, L)$ $(\,\gamma_H \equiv (\psi-1)/a_H\,)$,

where L represents total (domestic and export sectors') demand for the labor aggregate as well as the labor consumed in wage decisions (wage adjustment costs) and we defined the percentage gap between the actual wage and the benchmark (flex-wage) wage as:

(55) $G^W(\lambda w/e, L) \equiv [\mu_W\, v'(L)E_m/\lambda - W]/W = [\mu_W e/(\lambda w)]v'(L) - 1$ $(\,\mu_W \equiv \psi/(\psi-1)\,)$.

Whenever the wage gap is positive, the nominal wage is below the desired one, which is a mark-up over the marginal rate of substitution of wealth for leisure. Whenever this is the case, households gradually increase the nominal wage ($\pi_W>0$), but at a decreasing rate ($d\pi_W/dt<0$).

The first two of the first order conditions ((50) and (50b)), along with the budget constraint (6"), equation (47), the Phillips curve (54), the No Ponzi Game condition (8) and the transversality condition (50f), jointly determine the paths of c, m, W, $\pi_W$, a, given the values of exogenous parameters such as $\phi$, the paths of policy variables such as t and the paths of other endogenous variables such as E, e, $\Pi$, L, $\lambda$ and i.

II.6. <u>Money and domestic goods market clearing and the price and wage gaps</u>

Assuming money market equilibrium ($m^D$=m), (52) gives m/c as a decreasing function of i. Hence, $\tau$ and $\varphi$ are both increasing functions of i:

$\bar{\tau}(i) \equiv \tau(\ell(i))$, $\bar{\varphi}(i) \equiv \varphi(\ell(i))$ $(\;\bar{\tau}'>0,\;\; \bar{\varphi}'>0\;)$.

Also, (44) and (50) give household demand for domestic goods as a function of e, i and $\lambda$:

(56) $c_N = (1-\theta)(\bar{\varphi}(i)\lambda/\kappa_1)^{-1/\sigma}e^{\theta+(1-\theta)/\sigma} \equiv c_N(e,i,\lambda)$ $(c_{Ne}>0,\ c_{Ni}<0,\ c_{N\lambda}<0)$.

To simplify, assume that government demand for each type of domestic good is a fraction $g$ of private consumption demand for that good $g_{Ni} = gc_{Ni}$. Hence, for domestic goods market clearing total output of domestic good i must be:

$$y_{Ni} = [1 + g + \bar{\tau}(i)]c_{Ni}/[1-x(\pi_{Ni})] = [1 + g + \bar{\tau}(i)]c_{Ni}/[1-x(\pi_N)]$$

where total domestic goods consumption demand by households (including transaction costs) and the government must be grossed up to include the real resources used up in the price adjustment decision process. The second equality is derived from the fact that all firms in a symmetric equilibrium will face the same price adjustment costs. Since every firm has the same decision process, the use of (38) yields the domestic goods demand functions (26) used in section II.4. Aggregating over domestic goods as in (36) gives domestic goods output:

(57) $y_N(e,i,\pi_N,\lambda;g) = [1 + g + \bar{\tau}(i)]c_N(e,i,\lambda)/[1-x(\pi_N)] =$

$= (1-\theta)f(i,g)\kappa_2\, e^{\theta+(1-\theta)/\sigma}\lambda^{-1/\sigma}/[1-x(\pi_N)]$

$(\kappa_2 \equiv \kappa_1^{1/\sigma})$ $(y_{Ne}>0,\ y_{Ni}<0,\ y_{N\lambda}<0,\ y_{Ng}>0)$

where we have defined the function f(i, $g$) that combines the influence of the interest rate and government demand on the output of domestic goods:

$$f(i,g) \equiv [1 + g + \bar{\tau}(i)]/[\bar{\varphi}(i)]^{1/\sigma}.$$

The influence of government expenditures on domestic output is positive but the influence of the interest rate is in general ambiguous. First, an increase in i induces lower cash balances in relation to consumption (m/c) and hence a higher marginal saving rate ($\bar{\varphi}$), reducing consumption. However, the reduction in relative money balances also increases transaction costs per unit of consumption ($\tau$), which have to be produced. The net effect on domestic goods output is given by the sign of the corresponding partial derivative:

$$\partial f(i,g)/\partial i = -\ell' f\ \{-\tau'/(1 + g + \tau) - (1/\sigma)\ \ell\tau''/(1+\tau + i\ell)\}.$$

It is readily seen that the sign of this expression is negative if and only if

$\sigma\varepsilon_{m/c} < (1+\tau+g)/(1+\tau+i\ell)$

where $\varepsilon_{m/c} \equiv -\tau'/(\tau''\ell)$ is the elasticity of $\ell(i)$ (=m/c) and we have used (50b) and (51).

We assume that this inequality is satisfied. Hence, an increase in the nominal interest rate not only reduces consumption of domestic goods but also the production of domestic goods. Formally, we have the following assumption:

<u>Assumption 1</u>: The relative money demand m/c is sufficiently inelastic and/or government expenditures $g$ is sufficiently large with respect to the opportunity cost of holding money to consumption ratio im/c that the following inequality holds: $\sigma\varepsilon_{m/c} < (1+\tau+g)/(1+\tau+i\ell)$. Hence, $\partial y_N/\partial i<0$.

The first expression in (26) gives firm i's demand for labor as $L_{Ni} = F_N^{-1}(y_{Ni})$. Since all domestic sector firms produce the same amount (of their specific type of goods), they all produce $y_N$ using the same combination of labor types $L_N$. Hence, aggregating over firms as in (36) gives labor demand in the domestic sector:[8]

(58) $L_N(e,i,\pi_N,\lambda; g) = F_N^{-1}(y_N(e,i,\pi_N,\lambda; g))$ $(L_{Ne}>0, L_{Ni}<0, L_{N\lambda}<0, L_{Ng}>0)$.

From (24), labor demand by the export sector is:

(59) $L_X(w/\phi e) \equiv (F_X')^{-1}(w/\phi e)$ $(L_X'<0)$.

Therefore, total labor requirements (including labor used up in wage adjustment decisions) can be defined as:

(60) $L(w,e,i,\pi_N,\pi_W,\lambda; \phi,g) \equiv [L_N(e,i,\pi_N,\lambda; g) + L_X(w/\phi e)]/[1-x(\pi_W)]$

$(L_w<0, L_e>0, L_i<0, L_\lambda<0, L_\phi>0, L_g>0)$.

Hence, the final expressions for the price gap (34) and the wage gap (55) are:

(34') $G^P(w,e,i,\pi_N,\lambda; g) = \mu_P\, w\, z(y_N(e,i,\pi_N,\lambda; g)) - 1$

$(G^P_w>0, G^P_e>0, G^P_i<0, G^P_\lambda<0, G^P_g>0)$

(55') $G^W(w,e,i,\pi_N,\pi_W,\lambda; \phi,g) = [\mu_W e/\lambda w]\, v'(L(w,e,i,\pi_N,\pi_W,\lambda; \phi,g)) - 1$

---

[8] In the following definitions, when we show the signs of the partial derivatives in parentheses we will omit the partial derivatives with respect to $\pi_N$ and $\pi_W$, since in the steady state equilibrium they will all be zero, according to (25) and (25').

$(G^W_w<0,\ G^W_e>0,\ G^W_i<0,\ G^W_\lambda<0,\ G^W_\phi>0,\ G^W_g>0)$.

For later use, let us define "natural" (or "potential") output as the output that prevails in the steady state, when there are no longer any price or wage adjusting costs. Output is always at its natural level in the export sector because we assumed perfect competition and no price stickiness there. In the domestic sector "natural" output is:

$$y_N^n(\mu_P w) \equiv F_N(L_N^n(\mu_P w)) \equiv F_N(F_N'^{-1}(\mu_P w))$$

Hence, using (57) the output gap is:

$$(61) \qquad G^N(w, e, i, \pi_N, \lambda; \wp) \equiv y_N(e, i, \pi_N, \lambda; g) - y_N^n(\mu_P w).$$

$(G^N_w>0,\ G^N_e>0,\ G^N_i<0,\ G^N_\lambda<0,\ G^N_g>0)$.

III. The alternative monetary regimes and the saddle-path stability of the corresponding dynamical systems

In the eight monetary/exchange regimes we shall consider the Central Bank follows simple feedback rules for the rate of accumulation of international reserves and/or the nominal interest rate. These regimes can be classified according to whether the MRER is a predetermined (FIX regimes) or a jump variable (IT regimes). In the three alternative FIX regimes, the Central Bank will not have a policy rule for the nominal interest rate but will gear its intervention in the foreign exchange market towards achieving an exchange rate goal. In the five alternative IT regimes, the Central Bank will have a Taylor rule for the interest rate and will gear its intervention policy towards gradually achieving a specified target for the fraction of the money stock that is backed by international reserves. In all cases, there will always be a full backing the money stock with either international reserves or domestic bonds. Under the IT regimes, bond market intervention (or issuance) will be used residually for mopping up excess liquidity or generating the necessary liquidity to ensure the attainment of money market equilibrium according to the demand generated by (among other factors) the level of the policy interest rate.

A general version for the Central Bank's accumulation of international reserves is the following:

$$(62) \qquad \dot{R} = \ell_U(1 - E) + \ell_M(\bar{E}_m - E_m) + \ell_R(\bar{e} - e) + \ell_B(\beta m - R).$$

$$(\ell_U, \ell_M, \ell_R, \ell_B \geq 0)$$

In the FIX regimes the Central Bank uses its intervention policy towards achieving its exchange rate target. In the regime in which the Unilateral Nominal Exchange Rate is Fixed (UFIX), the peso/dollar nominal exchange rate target is unity. In the regime in which the multilateral nominal exchange rate is fixed (MFIX), the Central Bank has a target path for the multilateral nominal exchange rate $\bar{E}_m$, and when the MRER is "fixed" the Central Bank has a target path for the MRER $\bar{e}$. Finally, it is assumed that under all the alternative IT regimes, the Central Bank follows a policy of gradually achieving a backing of a fraction $\beta$ (<1) of the money stock with international reserves. Hence, the international reserves policy under the alternative monetary regimes can be defined by the following restrictions on the coefficients $\ell_i$ :

| | | | |
|---|---|---|---|
| UFIX: | $\ell_U \to \infty$, | $\ell_i = 0$, | i = M, R, B. |
| MFIX: | $\ell_M \to \infty$, | $\ell_i = 0$, | i = U, R, B. |
| RFIX: | $\ell_R \to \infty$, | $\ell_i = 0$, | i = U, M, B. |
| IT: | $\ell_B > 0$, | $\ell_i = 0$, | i = U, M, R. |

One of the equations in all of the dynamical systems is the balance of payments constraint in its intertemporal formulation (18). Inserting in this equation the domestic goods market equilibrium (57), the definitions of the foreign currency value of aggregate output ($y = \phi y_X + y_N/e$) and of the function $f(i,g)$, as well as the first order condition (50), yields the equivalent expression:

$$(63) \qquad d_0 = \int_0^{\infty} [\ \phi y_X(\phi e/w) - \theta\kappa_2 f(i,0) e^{(1-\theta)(1/\sigma-1)} \lambda^{-1/\sigma}\ ]\ e^{-rs}\ ds,$$

where $y_X(\phi e/w) \equiv F_X(L_X(\phi e/w))$. This equation is necessary for obtaining the (constant) value of the marginal utility of wealth $\lambda$, which may jump to a new level whenever there is an exogenous shock that impacts on the country's net wealth. Hence, (62) plays a crucial role in determining the system's steady state and, therefore, the paths of the endogenous variables.

III.1. The FIX monetary regimes

A. The fixed unilateral nominal exchange rate (UFIX) regime

We assume that the Central Bank fixes E at unity. We may think of this as a feedback equation for the instantaneous rate of change in international reserves, whereby the Central Bank purchases (sells) an amount of foreign exchange that is a constant fraction $k_U$ of the positive (negative) gap between the target nominal exchange rate (unity) and the actual nominal exchange rate Since we assume that $k_U$ tends to infinity, the Central Bank in effect fixes E at 1. The Central Bank must also ensure that the money market clears at all times. Hence, (52) gives the endogenous stock of money that guarantees money market equilibrium:

$$(64) \qquad M = \ell(i)E_m c = \ell(i)c/\rho.$$

By uncovered interest parity (7) and the assumption that r is an exogenous constant, the domestic nominal interest rate is constant (at r) under the FIX regime. This implies that $\ell$, and hence $\tau$, $\varphi$, and m/c are also constant. Since $P_N$ is predetermined, if the Central Bank devalues there must be a one time change in the nominal stock of money so as to accommodate the required change in e as well as whatever discrete jump in c may take place. By our assumption on the full backing of m (11), this implies a one time discrete exchange market intervention (apart from the usual flow interventions). Since i is constant we may leave it out of the gap functions (34') and (55') in the FIX regime.

The dynamical system is made up of the balance of payments equation (66) and the following additional equations:

$$(65) \qquad \dot{w}/w = \pi_W - \pi_N,$$

$$(65b) \qquad \dot{e}/e = -\pi_N.$$

$$(65c) \qquad \dot{\pi}_N = -\gamma_F G^P(w, e, \pi_N, \lambda; p)$$

(65d) $\dot{\pi}_W = -\gamma_H G^W(w, e, \pi_N, \pi_W, \lambda; p)$

where $p$=($g$, r, $\phi$) is a vector of parameters (which in $G^P$ is merely ($g$, r)). The first two equations are directly derived from the definitions of w and e. Note that in the steady state, due to (25) and (25'), the partial derivatives of $G^P$ with respect to $\pi_N$ and of $G^W$ with respect to $\pi_N$ and $\pi_W$ are zero. Equations (65) give a dynamical system that determines the paths of w, e, $\pi_N$, and $\pi_W$, given the value of $\lambda$ (which is endogenous) and of the exogenous parameters. Since $\lambda$ is constant except when it jumps to a new level when there is an unexpected shock that makes households adjust their level of consumption, its level affects the steady state of the subsystem given by (65) and therefore the paths of w, e, $\pi_N$, and $\pi_W$, for any initial value. In this section we analyze the stability properties of this subsystem for any given value of $\lambda$.

For the linear approximation to this subsystem it is convenient to define the vector of relative prices and inflation rates: $x^T \equiv (w, e, \pi_N, \pi_W)^T$, (where the symbol $^T$ means transposition). Then the linearized system can be written as:

(66) $\dot{x} = C(x-x^{\bullet})$,

where the matrix C is defined as:

$$C \equiv \begin{bmatrix} 0 & 0 & -w & w \\ 0 & 0 & -e & 0 \\ -\gamma_F G^P_w & -\gamma_F G^P_e & 0 & 0 \\ -\gamma_H G^W_w & -\gamma_H G^W_w & 0 & 0 \end{bmatrix}.$$

The elements of C are at their steady state values. The characteristic equation of the linearized system is:

(67) $\Delta(\lambda) \equiv \det(\lambda I - C) = \lambda^4 - p_2 \lambda^2 + \det = 0$,

where

$p_2 = \gamma_F wG^P{}_w - \gamma_H wG^W{}_w + \gamma_F eG^P{}_e > 0.$

$\det \equiv \det(C) = \gamma_F\gamma_H weH > 0.$ $\quad ( H \equiv G^P{}_w G^W{}_e - G^P{}_e G^W{}_w )$

Define $\mu \equiv \lambda^2$. Then (67) can be written as a quadratic equation:

(68) $\mu^2 - p_2 \mu + \det = 0$,

which has the solutions

(69) $\mu_1 = (1/2)\{ p_2 + [ (p_2)^2 - 4\det ]^{1/2} \} > 0$

$\mu_2 = (1/2)\{ p_2 - [(p_2)^2 - 4\det ]^{1/2} \} > 0.$

The four characteristic roots of C are the following:

(70) $\lambda_1 = -(\mu_1)^{1/2}$ , $\lambda_2 = -(\mu_2)^{1/2}$ , $\lambda_3 = +(\mu_2)^{1/2}$ , $\lambda_4 = +(\mu_1)^{1/2}$ ,

Hence, the number of roots with positive real part, which we shall denominate k(C), is 2. Furthermore, as long as the discriminant is non-zero, $\mu_1 > \mu_1$andall four roots are different: and they are either all real or two pairs of complex conjugates. Variables w and e are predetermined, because of wage and price setting by households and domestic sector firms, respectively, and nominal exchange rate fixing by the Central Bank. On the other hand, the inflation rates $\pi_N$ and $\pi_W$ are jump variables.[9] Hence, there is the same number of roots with negative real parts as there are predetermined variables and the same number of roots with positive real parts as there are jump variables. Therefore, the equilibrium is saddle-path stable.

Since $G^W_e$ is in det but not in $p_2$ it is readily seen that if $G^W_e$ is sufficiently small, which amounts to having a sufficiently inelastic labor supply (v' and v" small), the discriminant is positive, and hence the four roots are real. In that case,

(71) $0 < \mu_2 < \mu_1 < p_2$ and $\lambda_1 < \lambda_2 < 0 < \lambda_3 = -\lambda_2 < \lambda_4 = -\lambda_1$.

The following Proposition gathers the main conclusions:

<u>Proposition A</u>: Under the UFIX regime, the equilibrium is always saddle-path stable. If labor supply is sufficiently inelastic, the roots are all real.

The fact that the system is saddle-path stable should not lead to unwarranted conclusions. We have made the strong assumption of perfect capital mobility because it makes the system considerably simpler. But in the real world, with uncertainty, risk, and the possibility of having surprise shocks that lead to sudden reversals in capital flows, this

[9] In section IV.2 we will consider sticky rates of inflation.

regime is extremely vulnerable to strong and persistent shocks that lead, via consumption smoothing, to foreign indebtedness. A persistent strong dollar shock, for example, leads to a long deflationary adjustment process with recession and unemployment.[10]

B. The fixed multilateral nominal exchange rate (MFIX) regime

Knowing the vulnerabilities generated by a unilateral fixed exchange rate system, the Central Bank has a similar but more prudent and balanced regime at hand based on fixing the multilateral nominal exchange rate $E_m$ at a target level $\bar{E}_m$. Operationally, the Central Bank "fixes" the nominal exchange rate with the dollar at $E = \rho\ \bar{E}_m$. Whenever the dollar appreciates (depreciates) with respect to other currencies ($\rho$ increases), the Central Bank raises (decreases) the nominal exchange rate with the dollar E (pesos per dollar) in the same proportion. This maintains the external competitiveness of the economy, completely neutralizing the direct external shock[11] that is generated by the international dollar cycle. Except for this (important) modification, the dynamical system is formally the same as in the UFIX regime. Since the MRER is still a predetermined variable, we need k(C)=2 for saddle-path stability. Hence, we have the following Proposition:

Proposition B: Under the MFIX regime, the equilibrium is always saddle-path stable. If labor supply is sufficiently inelastic, the roots are all real.

C. The pegged multilateral real exchange rate (RFIX) regime

In the two previous regimes, the exchange rate was fixed to a constant target value. It could, however, have been "pegged" to a moving target value. When the target is the MRER, however, the target value must necessarily be moving along a path that eventually leads to the long run equilibrium value of the MRER. If that path does not lead to that equilibrium value, the model would not have a clearly defined steady state nor a well defined saddle path that leads there.

[10] Escudé (2004) uses the same basic framework as this paper to model the functioning and ultimate collapse of Argentina's Convertibility regime. In that paper, the vulnerability is compounded by the assumption that all trade is with Europe while all foreign debt is dollar denominated (which was closer to Argentina's situation, with only 15% of its trade with the dollar area).
[11] Note that there may be other shocks indirectly related to the dollar cycle, as the terms of trade or the international interest rate.

In this regime the Central Bank purchases (sells) foreign exchange whenever e is below (above) the target value $\bar{e}$:

(72) $\dot{R} = k_R(\bar{e} - e).$ $(k_2 \to \infty)$

Since we assume that $k_R$ tends to infinity, the Central Bank actually pegs e to its desired path. As mentioned above, in order to have a steady state it is necessary that the path itself converge to the steady state value of the MRER ($e^{\bullet}$). For simplicity, we will assume that the Central Bank follows a simple disequilibrium correcting rule for the target path.:

(73) $\dot{\bar{e}} / \bar{e} = s\,(e^{\bullet} - \bar{e}).$ $(s > 0)$

where $s$ is a positive constant. In practice, this makes the MRERT regime quite different from the preceding regimes. In the actual (non perfect foresight) world, the requirement that the path for the target converge to the system's steady state is very stringent in terms of information due to the fact that, as we will see below in section IV, the steady state is dependent on various shocks, even temporary ones. However, in our idealized perfect foresight world the assumption places no additional informational requirements.

Since the exchange market intervention keeps the actual MRER pegged to the target path, below we can just as well use e instead of $\bar{e}$ in the dynamical equation (73). The nominal rate of depreciation that is compatible with the target path (73) is then:

(74) $\delta = \pi_N + s(e^{\bullet} - e).$

The dynamical sub-system under MRERT is hence the following:

(75) $\dot{e} = s\,(e^{\bullet} - e).$

(75b) $\dot{w}/w = \pi_W - \pi_N,$

(75c) $\dot{\pi}_N = -\gamma_F G^P(w, e, r + \pi_N + s(e^{\bullet}-e), \pi_N, \lambda; p)$

(75d) $\dot{\pi}_W = -\gamma_H G^W(w, e, r + \pi_N + s(e^{\bullet}-e), \pi_N, \pi_W, \lambda; p).$

We have inverted the order of the first two equations to stress the decomposability of the matrix of the linear approximation around the steady state:

$$C_R \equiv \begin{bmatrix} -se & 0 & 0 & 0 \\ 0 & 0 & -w & w \\ -\gamma_F (G_e^P - sG_i^P) & -\gamma_F G_w^P & -\gamma_F G_i^P & 0 \\ -\gamma_H (G_e^W - sG_i^W) & -\gamma_H G_w^W) & -\gamma_H G_i^W & 0 \end{bmatrix},$$

Hence, the characteristic equation is:

$$\Delta(\lambda) = (\lambda + se)\,(\lambda^3 - \text{tr}\,\lambda^2 - p_2\lambda - \det),$$

where tr, $p_2$, and det refer to the lower right hand side (3 by 3) sub-matrix of $C_R$ and hence:

(76) $\text{tr} = -\gamma_F G^P{}_i > 0,$

$p_2 = w(\gamma_F G^P{}_w - \gamma_H G^W{}_w) > 0$

$\det = -w\, \gamma_F \gamma_H L < 0.$ $( L \equiv G^W{}_w G^P{}_i - G^P{}_w G^W{}_i > 0 ).$

One of the roots of $C_R$ is clearly $\lambda_1 = -se^\bullet$, and is hence negative. The other three roots correspond to the submatrix with the coefficients of (76). The negative determinant implies that for this 3 by 3 matrix k is either 2 or 0. On the other hand, the positive trace implies that k cannot be 0 leaving k=2 as the only possibility. Since the remaining root ($\lambda_1$) is negative, we conclude that $C_R$ also has two roots with positive real part. Hence, we have the following Proposition:

<u>Proposition C</u>: Under the MFIX regime, the equilibrium is always saddle-path stable.

There are some complementary matters that are worth considering. First, the dynamical equation for R (72) is a part of the system. When e (and hence $\bar{e}$) converges to its steady state level from above, $e^\bullet - \bar{e} = e^\bullet - e$ is negative. Without Central Bank intervention, e would tend to fall and, hence $\bar{e} - e$ would tend to be negative. To prevent the real appreciation beyond the targeted amount, the Central Bank purchases whatever amount of foreign exchange is needed. Hence, there is a long process of foreign exchange purchases by the Central Bank, with R converging towards a steady state level that depends on the targeted path, the deviation from the path and the speed of convergence of the targeted path. In the real world, of course, this can be quite costly. If we made a slight change in our framework by positing a constant (risk) premium p in (7), the government would have an opportunity cost of holding reserves equal to pR, since it could earn the premium by reducing more expensive foreign debt. Since R would be growing, this cost would also be growing. According to our simple fiscal policy, the cost of holding reserves would be financed through a concomitant increase in t. With perfect foresight, the private sector would know of this gradual increase in t as soon as the MRERT policy was established and would adjust $\lambda$, and hence consumption, to this reality. Aside from this, however, there would be no impediments to such a policy. Its financial cost that would presumably have to be weighed against its benefits when the government decided to establish it.

When the currency is overvalued and the Central Bank tries to gradually adjust it to its steady state equilibrium level, the gradual loss of reserves would eventually deplete them. But with our assumption of perfect capital mobility this would pose no problem, since the Central Bank could finance the necessary reserves by borrowing abroad. If we assumed the possibility of failure of perfect capital mobility because, say, foreign investors unexpectedly decide to withdraw their credit and recover their loans, then the anticipation of a limit to the gradual reduction of Central Bank international reserves (with no possibility of indebtedness) would force a change of policy through the possibility of a speculative attack against the currency. Hence, under such an assumption of possible unanticipated sudden capital reversals, a rational Central Bank would calibrate the target path so that there are sufficient reserves to withstand the whole process of reserve losses.[12]

A second related issue is the distributional consequence of a policy of gradually approaching the long run equilibrium MRER. We have made the simplifying assumption above that each household is equally benefited by firm profits because the property of firms is evenly distributed. A more realistic assumption would consist in recognizing that only a fraction of households earn interest and profits and the rest only have wage incomes. In such a context, a monetary policy of very gradually approaching the long run equilibrium MRER from above would have very important distributional effects if the economy is Domestically Biased (in Consumption in relation to Production), as we will specify in

[12] Bofinger and Wollmershaeuser (2001) emphasize the asymmetry with the case of reserve accumulation.

section IV below. In such economies there is a clear inverse relation between the real wage $\omega$ (measured against the consumption basket) and the MRER e. If, say, an external shock generates an overshooting of e and the Central Bank initiates a policy such as RFIX before the overshooting has been corrected, with a very gradual reduction of $\bar{e}$ towards $e^{\bullet}$, the real wage $\omega = w/e^{\theta}$ will be held systematically below its long run equilibrium level while profit income will be systematically above. In foreign currency, profits are:

$$\Pi/E_m \equiv \phi y_X(\phi e/w) + [y_N(e, r+\pi_N+\mathcal{s}(e^{\bullet}-e), \pi_N, \lambda; \mathcal{p})/e][1-x(\pi_N)] -$$

$$- (w/e)L(w, e, r+\pi_N+\mathcal{s}(e^{\bullet}-e), \pi_N, \pi_W, \lambda; \mathcal{p})[1-x(\pi_W)].$$

With e systematically above the long run equilibrium level (and hence w below), the export sector produces with a larger than long run share of real resources because its product wage is low and, concomitantly, the domestic sector functions with less real resources, even though the real interest rate in terms of domestic goods is systematically below the international real rate due to the low rate of nominal currency depreciation. Furthermore, labor costs are systematically below long run levels because w/e is low. Obviously, such a policy can well be contested by wage earners (or their representatives) at the union or electoral levels. Symmetrically, attempting to gradually depreciate the currency in real terms when it is overvalued implies having the use of resources biased towards the domestic sector and income distribution biased towards wage earners.

### III.2. The IT monetary regimes

When there is Inflation Targeting, the Central Bank essentially lets the level of the exchange rate be determined by the market. Therefore, the nominal and real exchange rates are jump variables. This does not imply, however, that Central Bank actions do not influence the exchange rate. First, the level of the nominal and real exchange rates can be influenced by the Central Bank's policy with respect to international reserves. We assume that this policy consists in gradually achieving a certain fractional backing $\mathcal{b}$ of the money stock with international reserves.

(77) $\dot{R} = \mathcal{k}_B(\mathcal{b}m - R).$

Second, the Central Bank influences the exchange rate through its use of the interest rate as instrument of monetary policy. Here the interrelation between the uncovered interest parity equation (7) and the Central Bank's feedback rule is crucial, since it determines, as we develop below, a linear relation between the rate of nominal depreciation δ and the rate of inflation.

D. The Domestic Inflation Targeting (DIT) regime

has a feedback rule ("Reaction Function" or "Taylor rule") by which it

increases the nominal interest rate whenever the rate of domestic inflation is above a target. In the case of DIT, when the target is $\bar{\pi}_N$ we can write the feedback rule as:

(78) $i = r + \pi_N + \hbar_1 (\pi_N - \bar{\pi}_N)$ $(\hbar_1 > 0)$.

Note that we assume that the feedback rule has the "Taylor property" ($\hbar_1 > 0$), by which the Central Bank increases the real rate in terms of domestic goods ($i - \pi_N$) whenever inflation is above target. On the other hand, the uncovered interest parity condition (7) implies that the nominal interest rate is linearly related to the expected (and actual) rate of nominal depreciation, δ. These two equations together imply that the Central Bank actions are such that the rate of real depreciation is proportional to the gap between the rate of domestic inflation and the target rate:

(79) $\delta - \pi_N = \hbar_1(\pi_N - \bar{\pi}_N)$.

If, to simplify, we further assume that the target rate of inflation is zero, the Central Bank must gear monetary policy to make the nominal rate of depreciation proportional to the current rate of domestic inflation :

(80) $\delta = (\hbar_1 + 1)\pi_N$. .

Hence, under DIT the dynamical subsystem is:

(81) $\dot{w}/w = \pi_W - \pi_N$,

(80b) $\dot{e}/e = \hbar_1 \pi_N$.

(80c) $\dot{\pi}_N = -\gamma_F G^P(w, e, r + (\hbar_1 + 1)\pi_N, \pi_N, \lambda; p)$

(80d) $\dot{\pi}_W = -\gamma_H G^W(w, e, r+(\hbar_1+1)\pi_N, \pi_N, \pi_W, \lambda; p)$.

where we used (7) and (78) to express i as a (linear) function of $\pi_N$ in the gap functions. As under the fixed exchange rate regime, we can express the system as in (66). The matrix C, however, must now be modified to include the partial derivatives of the gap functions with respect to the interest rate as well as the effect of monetary policy on the rate of real depreciation:

$$C_I \equiv \begin{bmatrix} 0 & 0 & -w & w \\ 0 & 0 & eh_1 & 0 \\ -\gamma_F G_w^P & -\gamma_F G_e^P & -\gamma_F G_i^P(h_1+1) & 0 \\ -\gamma_H G_w^W & -\gamma_H G_e^W & -\gamma_H G_i^W(h_1+1) & 0 \end{bmatrix}$$

Upon expansion, the characteristic polynomial of $C_I$ is:

$$\Delta(\lambda) = \det(\lambda I - C_I) = \lambda^4 - tr\lambda^3 - p_2\lambda^2 - p_3\lambda + \det,$$

where:

(82) $tr = -(\hbar_1+1)\gamma_F G^P{}_i > 0$

$p_2 = w[\gamma_F G^P{}_w - \gamma_H G^W{}_w] - \hbar_1 \gamma_F e G^P{}_e$

$p_3 = -(\hbar_1+1)w\gamma_F\gamma_H L < 0$

$\det = -\hbar_1 we\, \gamma_F\gamma_H H < 0,$

where H and L were defined in (67) and (76), respectively. We prove the following Proposition:

<u>Proposition D</u>: Under the DIT regime, the equilibrium is saddle-path stable if and only if the Taylor property holds, i.e. if and only if $\hbar_1 > 0$. Also, $\hbar_1 < 0$ implies indeterminacy of the equilibrium path.

<u>Proof:</u> We now have one predetermined variable (w) and three jump variables (e, $\pi_N$, and $\pi_W$). Hence, saddle-path stability requires that three of the eigenvalues of $C_I$ have positive real parts. The negative determinant implies that $k(C_I)$ is either 3 or 1. In the Appendix we show that application of the theorem of Routh and Hurwitz to a 4 by 4 system implies that

$$k(C_I) = V(1, -tr, -q_2, -q_3, det)$$

where $q_2 \equiv p_2+p_3/tr$ and $q_3 \equiv p_3+tr(det/q_2)$, and the function V(.) is defined as the number of times there is a change of sign as we proceed from left to right along its arguments. According to (82), tr>0, $p_3<0$, and det<0. Hence, k = V( +, - , $-q_2$, $-q_3$, - ). The signs of $q_2$ and $q_3$ are ambiguous due to the presence of $p_2$ in $q_2$. Discarding the freak cases in which one or both of them are zero (which could be eliminated by a small change in $\hbar$), the only case in which the number of sign changes along the arguments of V(.) is 1 (in which case k=1) is when $q_2$ and $q_3$ are both positive. In all the other cases k=3. But if $q_2>0$, it is immediate from the signs of tr, $p_3$ and det that $q_3$ must be negative. Hence, for any (non-zero) sign of $q_2$ we have $k(C_I)=3$ and, hence, saddle-path stability. As for the second part of the Proposition, notice that if $\hbar_1$ were negative the determinant would be positive. In that case, $k(C_I)$ would have to be either 4 or 2 or 0. Assuming $\hbar_1>-1$, the positive trace would rule out k=0 and the negative $p_3$ would rule out k=4. Hence, a negative $\hbar_1$ implies k=2, i.e. indeterminacy. QED

E. The Headline Inflation Targeting (HIT) regime

Assume now that instead of targeting domestic inflation the Central Bank targets CPI inflation. Its feedback rule is hence:

(83) $$i = r + \pi + \hbar_1 (\pi - \bar{\pi}) \qquad (\hbar_1 >0).$$

By (7) and the assumption of a zero target rate of inflation, the Central Bank must ensure that the rate of nominal depreciation is proportional to the rate of CPI inflation:

(84) $$\delta = (\hbar_1 +1)\pi = (\hbar_1 +1) [\theta\delta + (1-\theta)\pi_N]$$

where the second equality is obtained by differentiating the CPI index (2). Hence,

(85) $$\delta = (\hbar^\circ_1 +1) \pi_N, \qquad (\hbar^\circ_1 \equiv \hbar_1/[1-(\hbar_1+1)\theta]).$$

The dynamical subsystem is now:

(86) $$\dot{w}/w = \pi_W - \pi_N,$$

(86b) $$\dot{e}/e = \hbar^\circ_1 \pi_N.$$

(86c) $\dot{\pi}_N = -\gamma_F G^P(w, e, r+(\hbar^\circ_1+1)\pi_N, \pi_N, \lambda; p)$

(86d) $\dot{\pi}_W = -\gamma_H G^W(w, e, r+(\hbar^\circ_1+1)\pi_N, \pi_N, \pi_W, \lambda; p)$.

The system is almost the same as under domestic inflation targeting, except that $\hbar^\circ_1$ must be substituted for $\hbar_1$ and $\hbar^\circ$ must be substituted for $\hbar$ in matrix $C_I$. The trace, $p_3$, and determinant of $C_I$ are now:

$$tr = -(\hbar^\circ_1+1)\gamma_F G^P_i, \qquad p_3 \equiv (\hbar^\circ_1+1)w\gamma_F\gamma_H[G^P_w G^W_i - G^W_w G^P_i], \qquad det = -\hbar^\circ_1 we\gamma_F\gamma_H H.$$

As before, we assume that the Central Bank's feedback rule has the Taylor property ($\hbar_1>0$). If additionally $\hbar_1<1/\theta -1$, then $\hbar^\circ_1>0$, which implies a positive trace and negative $p_3$ and determinant. This is the same configuration of signs of the coefficients of the characteristic equation as we had under DIT, yielding again k=3 and saddle-path stability by use of the Routh-Hurwitz theorem. The condition is also necessary for saddle-path stability because both a negative $\hbar_1$ (assuming it is greater than –1) and a $\hbar_1$ that is greater than $1/\theta -1$ imply a positive determinant. But they also imply that tr>0 and $p_3>0$, and hence k=2 and indeterminacy by the arguments used for DIT. In conclusion, we have saddle-path stability under HIT if and only if the Central Bank is wise enough to calibrate its reaction function so that $0<\hbar_1<1/\theta -1$. Hence, we have the following Proposition:

Proposition E: Under the HIT regime, the equilibrium is saddle-path stable if and only if the following generalized Taylor property holds: $0<\hbar_1<1/\theta -1$. Also, $-1<\hbar_1<0$, or $\hbar_1>1/\theta -1$ imply indeterminacy of the equilibrium path.

The problem with $\hbar_1>1/\theta -1$ is that by reacting too strongly to CPI inflation the Central Bank actually reduces the real rate in terms of domestic goods whenever domestic inflation rises. From (84), we have:

$$i - \pi_N = r + \hbar^\circ_1 \pi_N.$$

If $\hbar_1>1/\theta -1$, the Central Bank reaction to increases in the domestic inflation rate are insufficient to increase the real rate of interest. This highlights the fact that the required Taylor property concerns the real interest rate in terms of domestic goods. Through the sign of the determinant, saddle-path stability requires that a positive relation exist between the rate of domestic inflation and the rate of real depreciation. And this happens if and only if $\delta$ is greater than $\pi_N$. An exaggerated response to headline inflation in the Taylor rule leads to an insufficient increase in the rate of nominal depreciation when there is an acceleration domestic inflation. The bottom line is that both the coefficient for domestic inflation ($\hbar^{\circ}_1$) and the coefficient for headline inflation ($\hbar_1$) must have the Taylor property.

It may be of interest to note that a monetary regime that responds to domestic inflation, as in the previous sub-section, but also responds (positively) to the rate of nominal depreciation, is equivalent to the HIT regime. If the Taylor rule were, instead of (78):

$$i = r + (\hbar_1+1)\pi_N + \hbar_2\,\delta \qquad (\hbar_1>0,\ \hbar_2>0),$$

where $\hbar_2$ represents a systematic tendency to raise the interest rate whenever the rate of nominal currency depreciation increases (leaning against the wind), then instead of (85) we have:

(85') $$\delta = (\hbar^{\circ}_1 + 1)\,\pi_N, \qquad (\hbar^{\circ}_1 \equiv (\hbar_1+\hbar_2)/(1-\hbar_2)).$$

If $\hbar^{\circ}_1$ is positive, this extended DIT regime is exactly analogous to the HIT regime. The new definition for $\hbar^{\circ}_1$ implies that it is positive if and only if, in addition to $\hbar_1$ and $\hbar_2$ being positive, $\hbar_2<1$. Hence, in this case the generalized Taylor property would be: $\hbar_1>0$, $0<\hbar_2<1$. If this holds, for any calibration of a HIT Taylor rule there is a calibration of the extended DIT Taylor rule that yields the same system, and vice versa.

F. <u>The domestic inflation and real exchange rate targeting (DIRT) regime</u>

Under this extended domestic inflation targeting regime the Central Bank includes the multilateral real exchange rate gap (with respect to its steady state value) in its feedback rule for interest rate policy (which we here assume is of the DIT variety). Assuming as usual a zero inflation target, we define the interest rate feedback rule as:

(87) $$i = r + \pi_N + \hbar_1\pi_N + \hbar_2\,(e - e^{\bullet}) \qquad (\hbar_1,\ \hbar_2 > 0).$$

Using the uncovered interest parity condition (7), we obtain:

(88) $\delta = (\hbar_1+1)\pi_N + \hbar_2(e - e^\bullet)$.

Hence, under DIRT the dynamical subsystem is:

(89) $\dot{w}/w = \pi_W - \pi_N$,

(89b) $\dot{e}/e = \hbar_1\pi_N + \hbar_2(e - e^\bullet)$

(89c) $\dot{\pi}_N = -\gamma_F G^P(w, e, r+(\hbar_1+1)\pi_N+\hbar_2(e - e^\bullet), \pi_N, \lambda; p)$

(89d) $\dot{\pi}_W = -\gamma_H G^W(w, e, r+(\hbar_1+1)\pi_N+\hbar_2(e - e^\bullet), \pi_N, \pi_W, \lambda; p)$.

We must now use the following matrix in place of C in (66):

$$C_{IR} \equiv \begin{bmatrix} 0 & 0 & -w & w \\ 0 & eh_2 & eh_1 & 0 \\ -\gamma_F G_w^P & -\gamma_F(G_e^P + G_i^P h_2) & -\gamma_F G_i^P(h_1+1) & 0 \\ -\gamma_H G_w^W & -\gamma_H(G_w^W + G_i^W h_2) & -\gamma_H G_i^W(h_1+1) & 0 \end{bmatrix}.$$

The coefficients of the corresponding characteristic equation are now:

(90) $tr = e\hbar_2 - (\hbar_1+1)\gamma_F G^P_i > 0$

$p_2 = -e\gamma_F[\hbar_1 G^P_e - \hbar_2 G^P_i] + w(\gamma_F G^P_w - \gamma_H G^W_w)$

$p_3 = -w\{(\hbar_1+1)\gamma_F L + \hbar_2 e(\gamma_F G^P_e - \gamma_H G^W_e)\} < 0$

$det = -we\gamma_F\gamma_H[\hbar_1 H + \hbar_2 L] < 0$

where H (>0) was defined in section III.1.A and L (>0) in section III.1.C.

Notice that if $\hbar_1$, and $\hbar_2$ are both positive we have exactly the same configuration of signs in tr, $p_3$, and det as in the previous IT regimes. Hence, $k(C_{IR})=3$ and we have saddle-path stability:

Proposition F: Under the DIRT regime, the equilibrium is saddle-path stable if the Taylor property holds ($\hbar_1 > 0$) and $\hbar_2 > 0$.

G. <u>The domestic inflation and output targeting (DIOT) regime</u>

Under this extended domestic inflation targeting regime the Central Bank includes the output gap in its feedback rule for interest rate policy (which we here assume is of the DIT variety). We now define the interest rate feedback rule as:

$$(91) \qquad i = r + \pi_N + h_1(\pi_N - \bar{\pi}_N) + h_2\, G^N(w, e, i, \pi_N, \lambda; p) \qquad (h_1, h_2 \geq 0)$$

where the output gap has been defined in (61). Using the uncovered interest parity condition (7) and a zero target rate of inflation, we obtain:

$$(92) \qquad \delta = (h_1+1)\,\pi_N + h_2\, G^N(w, e, r+\delta, \pi_N, \lambda; p)$$

from which, after making a linear approximation of the output gap around the steady state, we can obtain $\delta$ as a linear function of the deviations of $\pi_N$, w and e from their steady state values:[13]

$$(93) \qquad \delta = h'\pi_N + h_2'(e - e^{\bullet}) + h_3'(w - w^{\bullet}), \qquad h' \equiv (h_1+1)/[1-h_2 G^N_{\,i}\,] > 0$$

$$h_2' \equiv h_2 G^N_{\,e}/[1-h_2 G^N_{\,i}\,] > 0 \qquad h_3' \equiv h_2 G^N_{\,w}\,/[1-h_2 G^N_{\,i}\,] > 0.$$

Hence, under DIOT the dynamical subsystem is:

$$(94) \qquad \dot{w}/w = \pi_W - \pi_N,$$

$$(94b) \qquad \dot{e}/e = (h'-1)\pi_N + h_2'(e - e^{\bullet}) + h_3'(w - w^{\bullet})$$

$$(94c) \qquad \dot{\pi}_N = -\gamma_F G^P(w, e, r+ h'\pi_N + h_2'(e - e^{\bullet}) + h_3'(w - w^{\bullet}), \pi_N, \lambda; p)$$

$$(94d) \qquad \dot{\pi}_W = -\gamma_H G^W(w, e, r+ h'\pi_N + h_2'(e - e^{\bullet}) + h_3'(w - w^{\bullet}), \pi_N, \pi_W, \lambda; p).$$

We must now use the following matrix in place of C in (65):

$$C_{IO} \equiv \begin{bmatrix} 0 & 0 & -w & w \\ eh'_3 & eh'_2 & e(h'-1) & 0 \\ -\gamma_F(G^P_w + G^P_i h'_3) & -\gamma_F(G^P_e + G^P_i h'_2) & -\gamma_F G^P_i h' & 0 \\ -\gamma_H(G^W_w + G^W_i h'_3) & -\gamma_H(G^W_w + G^W_i h_2) & -\gamma_H G^W_i h' & 0 \end{bmatrix}.$$

[13] Note that we use the fact that $\partial y_N/\partial \pi_N$, valued at $\pi_N$=0, is zero by use of (25) and (57).

The coefficients of the corresponding characteristic equation are now:

(95) $tr = e\hbar_2' - \hbar'\gamma_F G^P_i > 0$

$p_2 = -e\gamma_F [(\hbar'-1)G^P_e + \hbar_2' G^P_i ] + w\{\hbar_3'(\gamma_F G^P_i - \gamma_H G^W_i ) + (\gamma_F G^P_w - \gamma_H G^W_w )\}$

$p_3 = ew\{\hbar_2'(\gamma_H G^W_w - \gamma_F G^P_w ) + \hbar_3'(\gamma_F G^P_e - \gamma_H G^W_e )\} - w\gamma_F\gamma_H \hbar' L$

$det = -we\gamma_F \gamma_H \{ (\hbar'-1) H + \hbar_2' L + \hbar_3'(G^W_e G^P_i - G^P_e G^W_i ) \}.$

where H and L were defined previously. Using the definitions in (93), we have:

(96) $det [1-\hbar_2 G^N_i ] = we\gamma_F \gamma_H\{ -(\hbar_1+\hbar_2 G^N_i) H + \hbar_2[G^P_i J + G^W_i K] \}$

$p_3 [1-\hbar_2 G^N_i ] = -w\{ (\hbar_1+1)\gamma_F \gamma_H L + \hbar_2 e[\gamma_H J + \gamma_F K] \},$

where we define:

$J \equiv G^W_e G^N_w - G^W_w G^N_e >0,$ $K \equiv G^P_w G^N_e - G^P_e G^N_w .$ .

From (95), we conclude that the determinant is negative if and only if

(97) $\hbar_1 > \hbar_2 [(-G^N_i) H + G^P_i J + G^W_i K] \equiv \hbar_2 \Omega$

and that $p_3$ is negative if and only if

(98) $(\hbar_1+1) > \hbar_2 e[\gamma_H J + \gamma_F K]/(\gamma_H\gamma_F L) \equiv \hbar_2 \Psi$

The sign configuration is now much more complex then in previous cases, but we can draw some interesting conclusions nonetheless. First, since a negative determinant is necessary for saddle-path stability, $\hbar_1$ must be sufficiently great in relation to $\hbar_2$, according to (97). In that case k is either equal to 3 or to 1. Second, if $\hbar_1$ is also greater than $\hbar_2$ according to (98), then $p_3$ is negative. In that case, we have exactly the same configuration of signs as in the previous cases of IT and, therefore, saddle-path stability. Hence, we have the following Proposition:

<u>Proposition G</u>: Under the DIOT regime, the equilibrium is saddle-path stable if the following generalized Taylor property holds: $\hbar_1 > \max(\hbar_2\Omega, \hbar_2\Psi-1)$. Also, $\hbar_1 > \hbar_2\Omega$ is necessary for saddle-path stability under the DIOT regime.

H. The headline inflation and output targeting (HIOT) regime

Assume now that the Central Bank includes the output gap along with CPI inflation in the interest rate feedback rule:

(99) $i = r + \pi + \hbar_1\pi + \hbar_2\, G^N(w, e, i, \pi_N, \lambda; p)$ $\quad (\hbar_1, \hbar_2 \geq 0)$

Using (7) and linearizing the output gap now yields the following relation between the rate of nominal depreciation and the deviations of $\pi_N$, e and w from their steady state values:

(100) $\delta = \hbar''\pi_N + \hbar_2''(e - e^{\bullet}) + \hbar_3''(w - w^{\bullet})$, $\quad \hbar'' \equiv (1-\theta)(\hbar_1+1)/[1-\theta(\hbar_1+1)-\hbar_2 G^N_i]$,

$\hbar_2'' \equiv \hbar_2 G^N_e /[1-\theta(\hbar_1+1)-\hbar_2 G^N_i]$, $\quad \hbar_3'' \equiv \hbar_2 G^N_w /[1-\theta(\hbar_1+1)-\hbar_2 G^N_i]$.

The dynamical system is then the same as the one for DIOT, except that we must replace $\hbar''$, $\hbar_2''$ and $\hbar_3''$ for $\hbar'$, $\hbar_2'$ and $\hbar_3'$. The only additional complication arises from the fact that, in order for (97) and (98) to continue being necessary and sufficient for a negative determinant and $p_3$, respectively, it is necessary that the denominator in the expressions for the $\hbar''$, $\hbar_2''$ and $\hbar_3''$ be positive. This places a cap on $\hbar_1$, in analogy to the one we had under HIT except for the fact that now the cap is related to the magnitude of $\hbar_2$: $\hbar_1<[1-\theta+\hbar_2(-G^N_i]/\theta$. Hence, we have the following Proposition:

Proposition H: Under the HIOT regime, the equilibrium is saddle-path stable if the following generalized Taylor property holds: $\max(\hbar_2\Omega,\ \hbar_2\Psi-1)< \hbar_1 < [1-\theta+\hbar_2(-G^N_i]/\theta$. Also, $\hbar_1 > \hbar_2\Omega$ is necessary for saddle-path stability under the HIOT regime.

IV. Extensions: the presence of a long rate of interest and inflation stickiness

In this section we make two extensions to the framework we have developed to analyze the dynamical stability properties of the various monetary regimes we consider. First, we introduce a long rate of interest in a quite conventional way and illustrate by means of the DIT regime that it does not alter previous conclusions in a significant way. Second, we introduce segments of "rule of thumb" firms and households that instead of making costly price or wage adjustment decisions prefer to simply adjust their price of wage inflation rates to the overall price or wage inflation rate (that includes both types of firms or households) in a backward looking way. In this case we use the UFIX regime to illustrate that a sufficiently high speed of adjustment of these backward looking rules yields saddle-path stability.

III.3. Domestic Inflation Targeting in the presence of a long rate of interest

In this section we assume that in addition to the (short) rate of interest that the Central Bank operationally targets (and is subject to the uncovered interest parity condition), there is a long interest rate which is the relevant one for households and the government. We use the DIT regime for illustration but of course the same extension is possible for the other regimes. For simplicity, we assume that the long rate corresponds to a perpetuity that pays out one unit of currency each instant. Let S be the market price of this perpetuity. Then the long interest rate is $i_L = 1/S$. The rate of return on the long bond is the current yield plus the rate of expected capital gains: 1/S + dlog(S)/dt. By arbitrage (and under risk neutrality) the rate of return between the short bond and the long one must be equalized. Hence, the following equality must hold:

$$i = 1/S + \dot{S}/S,$$

or, equivalently, the rate of increase in the long rate must be equal to the spread between the long and short rates (or slpope of the "yield curve"):

$$\dot{i}_L/i_L = i_L - i \ .$$

Solving this differential equation yields the long rate as a function of the expected future path of short rates:

$$i_L = \frac{1}{\left\{\int_t^{\infty} e^{-\int_t^{\tau} i(s)ds} d\tau\right\}} .$$

On the other hand, the dependence of demand on financial conditions is now given by the function $f(i_L, g)$, where the long rate is substituted for the short rate.

The Taylor rule (77) and uncovered interest parity (7) both pertain to the short rate and again yield the necessary nominal depreciation in terms of the rate of domestic inflation (79) and the real rate of depreciation (80b). Consequently, the dynamical system for the DIT regime with long rate is now of dimension 5:

(101) $\dot{w}/w = \pi_W - \pi_N,$

(101b) $\dot{e}/e = \hbar_1 \pi_N.$

(101c) $\dot{\pi}_N = -\gamma_F G^P(w, e, i_L, \pi_N, \lambda; p)$

(101d) $\dot{\pi}_W = -\gamma_H G^W(w, e, i_L, \pi_N, \pi_W, \lambda; p).$

(101e) $\dot{i}_L/i_L = i_L - r - (\hbar_1+1)\pi_N.$

Linearizing around the steady state yields system (66) with $x^T \equiv (w, e, \pi_N, \pi_W, i_L)^T$ and matrix $C_{IL}$:

$$C_{IL} \equiv \begin{bmatrix} 0 & 0 & -w & w & 0 \\ 0 & 0 & h_1 e & 0 & 0 \\ -\gamma_F G_w^P & -\gamma_F G_e^P & 0 & 0 & -\gamma_F G_i^P \\ -\gamma_H G_w^W & -\gamma_H G_e^W & 0 & 0 & -\gamma_H G_i^W \\ 0 & 0 & -r(h_1+1) & 0 & r \end{bmatrix}$$

The characteristic polynomial of $C_{IL}$ is:

$$\Delta(\lambda) = \lambda^5 - tr\lambda^4 - p_2\lambda^3 - p_3\lambda^2 - p_4\lambda - det,$$

where:

(102) $tr = r > 0$

$p_2 = -\gamma_F[\hbar_1(eG^P{}_e - rG^P{}_i) - (wG^P{}_w + rG^P{}_i)] - \gamma_H wG^W{}_w$

$p_3 = r\{\hbar_1\gamma_F eG^P{}_e - w[\gamma_F G^P{}_w - \gamma_H G^W{}_w]\}$

$p_4 = \gamma_F\gamma_H w\,[\hbar_1 eH - (\hbar_1+1)r(G^P{}_w G^W{}_i - G^W{}_w G^P{}_i)\,] > 0$

$det = -\hbar_1 r\gamma_F\gamma_H\, we\, H < 0.$

We have the following Proposition:

Proposition J: Under the DIT regime with a long rate of interest in addition to the short rate, the equilibrium is saddle-path stable if and only if the Taylor property holds, i.e. if and only if $\hbar_1 > 0$.

Proof: Since the long rate must be a jump variable, along with e and the two inflation rates, we need k=4 for saddle-path stability. The negative determinant implies that $k(C_{IL})$ must be either 4, 2 or 0. The positive trace implies that k cannot be equal to 0. Hence k must be either 4 or 2. We now prove $k(C_{IL})$ is indeed equal to 4. According to the theorem of Routh-Hurwitz in the 5 by 5 case (see the Appendix),

$$k(C_{IL}) = V( 1, -tr, -q_2, -q_3, -q_4, -det ) \qquad q_2 \equiv p_2+p_3/tr,$$

$$q_3 \equiv p_3 - [tr\ p_4 +det]/q_2 \qquad q_4 \equiv p_4 + det/tr - [q_2\ det]/q_3$$

From (102) it is readily seen that $q_2 = (\hbar_1+1)\,\gamma_F G^P_i < 0$. Hence, $k(C_{IL}) = V( +, - , + , -q_3, -q_4, + )$. There is a minimum of two changes in sign as we proceed from left to right along the arguments of V(.). Hence, as long as $q_3$ and $q_4$ are not both negative (nor zero), k must be equal to 4. We now prove that this is indeed the case by showing that a $q_3<0$ implies $q_4>0$. From (102) we obtain $p_4 + det/tr = -\gamma_F\gamma_H w\, (\hbar_1+1)(G^P_w G^W_i - G^P_i G^W_w)>0$. Since $q_2$ det is positive, the formula for $q_4$ above implies that if $q_3<0$, then $q_4>0$. Hence, as long as $q_3$ is different from zero, $q_4>0$, which yields k=4. The Taylor property ($\hbar_1>0$) is again necessary for saddle path stability because a negative $\hbar_1$ turns the sign of the determinant positive, which means that k can only be equal to 5, 3, or 1. QED.

IV.2. Introduction of backward looking “rule of thumb” price and wage setting

The framework developed above is too forward looking to be realistic. It is well known that the data confirm that there is not only price level inertia but also inflation rate inertia (cfr. Fuhrer and Moore (1995), Roberts (1997) and Galí and Gertler (1999)). In this section we show one way in which the above framework can be modified to handle inflation inertia. Assume that along with the continuum of domestic sector firms that are completely forward looking in adjusting prices there is another continuum that is composed of completely backward looking monopolistic competitors that, instead of making costly decisions for price changes, smoothly adjust their inflation rates towards the average inflation rate.

Symmetrically, assume that there is a continuum of households that is backward looking in the same sense. We take the UFIX (or MFIX) regime to illustrate.

In this section we make a slight change of notation for forward looking firms' and households' price and wage levels and inflation rates to $P_{Nf}$, $W_f$, $\pi_{Nf}$, and $\pi_{Wf}$, and introduce the price and inflation rates of backward looking firms as $P_{Nb}$ and $\pi_{Nb}$ and the wage and wage inflation rates for backward looking households as $W_b$ and $\pi_{Wb}$. Assume that a fraction $\alpha_N$ of firms and $\alpha_W$ of households are forward looking. The price and wage indexes that include both forward and backward looking agents are hence:

(103) $P_N \equiv (P_{Nf})^{\alpha_N} (P_{Nb})^{1-\alpha_N}$.

(104) $W \equiv (W_f)^{\alpha_W} (W_b)^{1-\alpha_W}$.

We assume that the backward looking firms and households adjust their inflation rates according to:

(105) $\dot{\pi}_{Nb} = \zeta_N(\pi_N - \pi_{Nb}) = \zeta_N\alpha_N (\pi_{Nf} - \pi_{Nb})$ $\quad ( 0<\alpha_N<1,\ \zeta_N >0 )$

(106) $\dot{\pi}_{Wb} = \zeta_W(\pi_W - \pi_{Wb}) = \zeta_W\alpha_W (\pi_{Wf} - \pi_{Wb})$ $\quad ( 0<\alpha_W<1,\ \zeta_W>0 )$.

The second equalities are derived from the time derivatives of (102) and (103). When the first two equations of (65) are modified accordingly, the system becomes:

(107) $\dot{w}/w = \alpha_W \pi_{Wf} + (1-\alpha_W) \pi_{Wb} - \alpha_N \pi_{Nf} - (1-\alpha_N) \pi_{Nb}$,

(107b) $\dot{e}/e = -\alpha_N \pi_{Nf} - (1-\alpha_N) \pi_{Nb}$.

(107c) $\dot{\pi}_{Nf} = -\gamma_F G^P(w,e,\pi_{Nf},\lambda)$

(107d) $\dot{\pi}_{Wf} = -\gamma_H G^W(w,e,\pi_{Nf},\pi_{Wf},\lambda)$

(107e) $\dot{\pi}_{Nb} = \zeta_N\alpha_N (\pi_{Nf} - \pi_{Nb})$

(107f) $\dot{\pi}_{Wb} = \zeta_W\alpha_W (\pi_{Wf} - \pi_{Wb})$.

This system can again be represented as in (66) by defining:

$$x^T \equiv (w, e, \pi_{Nf}, \pi_{Wf}, \pi_{Nb}, \pi_{Wb})^T,$$

and replacing matrix C by the expanded matrix F:

$$F \equiv \begin{bmatrix} 0 & 0 & -w\alpha_N & w\alpha_W & -w(1-\alpha_N) & w(1-\alpha_N) \\ 0 & 0 & -e\alpha_N & 0 & -e(1-\alpha_N) & 0 \\ -\gamma_F G_w^P & -\gamma_F G_e^P & 0 & 0 & 0 & 0 \\ -\gamma_H G_w^W & -\gamma_H G_w^W & 0 & 0 & 0 & 0 \\ 0 & 0 & \zeta_N \alpha_N & 0 & -\zeta_N \alpha_N & 0 \\ 0 & 0 & 0 & \zeta_W \alpha_W & 0 & -\zeta_W \alpha_W \end{bmatrix}$$

The characteristic polynomial of F is:

$$\Delta(\lambda) = \lambda^6 - \mathrm{tr}\lambda^5 - p_2\lambda^4 - p_3\lambda^3 - p_4\lambda^2 - p_5\lambda + \det,$$

where:

(108) $\mathrm{tr} = -(\zeta_N\alpha_N+\zeta_W\alpha_W) < 0,$

$p_2 = \alpha_N A+\alpha_W(B+\zeta_N\alpha_N\alpha_W),$ $\qquad A\equiv\gamma_F(wG^P{}_w+eG^P{}_e),$ $\qquad B\equiv\gamma_H w(-G^W{}_w)$

$p_3 = \alpha_N\gamma_F(\zeta_N+\zeta_W\alpha_W)A + \alpha_W\gamma_H(\zeta_N\alpha_N+\zeta_W)B > 0,$

$p_4 = \alpha_N\alpha_W \{\zeta_F\zeta_H[A+B] - \gamma_F\gamma_H \, we \, H\},$

$p_5 = -(\zeta_N+\zeta_W) \alpha_N\alpha_W\gamma_F\gamma_H \, we \, H < 0,$

$\det = \gamma_F\gamma_H \, \alpha_N\alpha_W\zeta_N\zeta_W \, we \, H > 0.$

The two new variables, $\pi_{Nb}$ abd $\pi_{Wb}$ are predetermined variables. Hence, the real parts of the two new characteristic roots should be negative and we must have k(F)=2 for saddle-path stability, as we did under the UFIX and MFIX regimes.

We now prove the following Proposition:

Proposition K: Under the UFIX regime, given the proportions of forward looking firms and households $\alpha_N$, $\alpha_W$, if the speeds of adjustments of backward looking firms and households $\zeta_N$, $\zeta_W$ are sufficiently high, the equilibrium is saddle-path stable.

Proof: The positive determinant implies that k(F) must be either 6, 4, 2 or 0. The negative trace excludes k=6, and the positive $p_3$ (which equals the sum of all triple product of the roots) excludes k=0. Hence, we are left with only k=4 and k=2 as possibilities. The 6 by 6 version of the Routh-Hurwitz theorem (Appendix) shows that k(F) is:

$$k(F) = V(1, -tr, -q_2, -q_3, -q_4, -q_5, det ).$$

$$q_2 \equiv p_2+p_3/tr, \qquad q_3 \equiv p_3 - [tr\, p_4 + p_5]/q_2$$

$$q_4 \equiv p_4 + p_5/tr - [q_2\, p_5 + tr\, det]/q_3 \qquad q_5 \equiv p_5 + [tr\, det]/q_2 + [q_3 det]/q_4$$

Using (108) it is readily verified that:

$$q_2 = -\{ [\zeta_N\alpha_N/(\zeta_N\alpha_N+\zeta_W\alpha_W)](1-\alpha_N)A+[\zeta_W\alpha_W/(\zeta_N\alpha_N+\zeta_W\alpha_W)](1-\alpha_W)B+\zeta_N\alpha_N\zeta_W\alpha_W\}<0.$$

Hence, we have the following residual sign pattern: $k(F)=V(1,+,+,-q_3,-q_4,-q_5,+)$. The signs of $q_3$, $q_4$, and $q_5$ are ambiguous. However, it is clear that only under the particular combination of signs $q_3>0$, $q_4<0$, and $q_5>0$ will k be equal to 4. In all other cases k(F)=2. We now prove that if $\zeta_N$ and $\zeta_W$ are sufficiently large, given $\alpha_N$ and $\alpha_W$, then $q_3<0$, which implies that k=2. Without loss of generality, we assume in the following the simpler symmetric case in which $\alpha_N=\alpha_W=\alpha$ and $\zeta_N=\zeta_W=\zeta$. Then

$$q_3 = 2\alpha\zeta\{ (1+\alpha)\Phi - \alpha[(1-\alpha)\gamma^2 ewH+2\alpha\zeta^2\Phi]/[(1-\alpha)\Phi+\alpha^2\zeta^2] \},$$

where $\Phi = (A+B)/2>0$. Hence, $q_3$ is negative if and only if the following inequality holds:

(109) $$(\zeta^2\Phi)\alpha^2 + (\gamma^2 ewH -\Phi^2)\alpha - \Phi^2 > 0.$$

This is a quadratic in-equation in $\alpha$. Given $\zeta$, if $\alpha$ is sufficiently low, then the inequality does not hold, $q_3$ is positive, and we cannot be conclusive with respect to k(F) without consideration of the remaining $q_i$ . On the other hand, given any value of $\alpha$, if

$$\zeta^2 > [\Phi^2 + (\Phi^2 - \gamma^2 ewH)\alpha]/\Phi\alpha^2$$

then the in-equation holds, $q_3$ is negative and k(F)=2. QED

V. The steady states of the dynamical systems

The steady state values of $\pi_N$ and $\pi_W$ are zero by assumptions on monetary policy. The steady state values of w, e and $\lambda$ must be obtained simultaneously by the zero pricing gaps

$G^P$=0, $G^W$=0, and the intertemporal balance of payments equation (63). From the first two (and the zero steady state values of the inflation rates) we can obtain w and e as functions of $\lambda$ and the exogenous parameters. These functions are common to all the dynamical systems. These steady state values are determined simultaneously with the equilibrium paths of these variables. First we obtain the paths of w and e as functions of time, $\lambda$ the exogenous parameters, and the pertinent initial values. Inserting these paths in (63), along with the specification of the monetary regime, gives an equation in $\lambda$ that determines the constant equilibrium value of that variable, and hence the steady state values of w and e as well as the equilibrium paths for these variables. Under the UFIX and MFIX regimes, for example, subsystem (65) gives the paths of w, e, $\pi_N$ and $\pi_W$ as functions of time, $\lambda$, and the initial values of w and e. Inserting these paths in (63) gives the expression that determines the equilibrium value of $\lambda$. Under the DIT regime, subsystem (80) gives the corresponding paths as functions of time, $\lambda$, and the initial value of w. In this case, e is also a jump variable (as $\pi_N$ and $\pi_W$) and hence, its initial value is irrelevant. The procedure is analogous for the other monetary regimes.

In all the dynamical subsystems the steady state conditions that w, e, $\pi_N$ and $\pi_W$ be constant boil down to the following equations:

(110) $\mu_P w\, z(y_N(e, \lambda; r, g))) = 1$ $\qquad$ ( $G^P = 0$ )

(111) $[\mu_W e/\lambda w]\, v'(L(w, e, \lambda; r, \phi, g)) = 1,$ $\qquad$ ( $G^W = 0$ )

where for ease of notation we have omitted from the functions $y_N$ and L the argument i (because it is constant at r in the steady state) and the arguments $\pi_N$ and $\pi_W$ (because they are zero in the steady state). Instead of working with these conditions for obtaining the steady state as described above we will use the equivalent market clearing conditions in the domestic goods and labor markets we now derive. From (108) and the definition of z(.) (in (30)) we obtain the labor cost minimization condition in the domestic sector:

(112) $F_N'(L_N(e,\lambda; r, g))) = \mu_P w$

which implies:

(113) $L_N(e,\lambda; r, g) = (F_N')^{-1}(\mu_P w) \equiv L_N^n(\mu_P w).$

The last term is the definition of the domestic sector's labor demand function in the benchmark flex-price economy where price and wage adjustment costs disappear. Furthermore, combining (57), (58) and (112) in the steady state, yields:

(114) $(1\text{-}\theta)\kappa_2 f(r,g)\lambda^{-1/\sigma} e^{\theta+(1-\theta)/\sigma} = F_N(L_N^{\ n}(\mu_P w)) \equiv y_N^{\ n}(\mu_P w)$, $(\kappa_2 \equiv \kappa_1^{\ 1/\sigma})$

where the last expression is the definition of the domestic supply function in the benchmark economy. The left hand side of (114) is actual domestic output, while the right hand side is "potential" (or "natural") domestic output, the one that prevails when there are no price or wage adjustments (nor associated costs). Hence, (114) is the zero domestic output gap condition:

(114') $G^N(w, e,\lambda; r, g) = 0$.

On the other hand, using the zero wage gap condition (111) and the definition of total labor requirements (60), the labor market clearing condition is:

(115) $L_N(e,\lambda; r,g) + L_X(w/\phi e) = (v')^{-1} (\lambda w/e\mu_W) \equiv \bar{L}(\lambda w/e\mu_W)$.

where the last term defines the labor supply function. This expression reflects the steady state labor market clearing. Using (113), which was derived from the zero domestic price gap condition, yields a simpler steady state labor market equilibrium condition as:

(116) $L_N^{\ n}(\mu_P w) + L_X(w/\phi e) = \bar{L}(\lambda w/\mu_W e)$.

We may call this equation the zero labor gap condition:

(116') $G^L(w,e,\lambda; \phi) = 0$,

if we define the labor gap as:

$$G^L(w,e,\lambda;\phi) \equiv L_N^{\ n}(\mu_P w) + L_X(w/\phi e) - \bar{L}(\lambda w/e\mu_W).$$

Conditions (114) and (116) (or (114') and (116')) constitute a pair of steady state conditions that give the long run equilibrium values of w and e, as functions of $\lambda$ and the values of exogenous parameters such as $\phi$, r and $g$. They represent long run market equilibrium conditions (for domestic goods and labor, respectively) and are derived from the zero price and wage gap conditions that define the system's steady state. We now derive the partial derivatives for w and e with respect to $\lambda$ and the exogenous parameters, and then consider the determination of the equilibrium value of $\lambda$ and its response to exogenous shocks. Only then will we be able to obtain the long run multipliers of w, e, $\lambda$, and d with respect to the exogenous parameters.[14]

Define the following elasticities and the auxiliary expression N:

(117) $\varepsilon \equiv -(e/w)/(\partial w/\partial e)=[\bar{L}'\lambda/(\mu_W e)-L_X'/\phi e]/[\bar{L}'\lambda/(\mu_W e)-L_X'/\phi e-(L_N^n)'\mu_P]\in (0,1).$

$\varepsilon_{FN} \equiv L_N^n F_N'/F_N$ $\qquad$ $\varepsilon_{LN} \equiv -\mu_P w L_N^{n\prime}/L_N^n$

$\varepsilon_{LS} \equiv \bar{L}'(\lambda/\rho)(w/e)/[\bar{L}\mu_W]$ $\qquad$ $\xi \equiv [\varepsilon_{LS}\bar{L}]/[\varepsilon_{LN}L_N^n]$

$\varepsilon_{fr} \equiv -(r/f(.))/(\partial f/\partial r)$ $\qquad$ $\varepsilon_{fg} \equiv -(g/f(.))/(\partial f/\partial g)$

$N \equiv 1-\theta + \sigma\theta + \sigma\,\varepsilon_{FN}\,\varepsilon_{LN}(1-\varepsilon) > 0,$

$\varepsilon$ is the elasticity of the domestic sector product wage w with respect to e along the labor market clearing condition, $\varepsilon_{FN}$ is the elasticity of output with respect to labor in the domestic sector, $\varepsilon_{LN}$ is the elasticity of labor demand in the domestic sector of the benchmark economy, $\varepsilon_{LS}$ is the elasticity of labor supply, $\xi$ is derived from the last two and $\varepsilon_{fr}$ and $\varepsilon_{fg}$ are the elasticities of $f(r,g)$ with respect to its arguments.

Tedious calculations show that the partial derivatives (in log form) are the following:

$N(\lambda/e)(\partial e/\partial\lambda) = 1+\sigma\,\varepsilon_{FN}\,\varepsilon_{LN}\,\varepsilon\xi > 0,$

$N(\phi/e)(\partial e/\partial\phi) = -\sigma\,\varepsilon_{FN}\,\varepsilon_{LN}(1-\varepsilon-\varepsilon\xi) < 0,$

$N(r/e)(\partial e/\partial r) = \sigma\varepsilon_{fr} > 0,$

$N(g/e)(\partial e/\partial g) = -\sigma\varepsilon_{fg} < 0,$

$N(\lambda/w)(\partial w/\partial\lambda) = 1-\varepsilon-\varepsilon\xi(1-\theta+\sigma\theta) > 0$ if $\xi$ is small,

[14] A note of caution is in order with respect to the interpretation of the long run multipliers with respect to r. The existence of a steady state requires that r and $\beta$ be equal. Hence, any change in r must be accompanied by an equivalent change in $\beta$. This should not be interpreted literally and one must remember that the assumption of perfect capital mobility is hardly realistic. Turnovsky (2000) shows alternative and, in some cases, more realistic assumptions. In particular, Bhandari, Ul Haque and Turnovsky (1990) assume that there is an endogenous risk premium that is dependent on the foreign debt. This leads to a dynamic equation in $\lambda$ as in (50d), except that r becomes an increasing function of d. The marginal utility of wealth is then no longer constant along the equilibrium path and the dynamical system has an additional dimension. In our case this would complicate matters considerably so we prefer to stick to the perfect capital mobility assumption.

$N(\phi/w)(\partial w/\partial \phi) = (1-\varepsilon-\varepsilon\xi)(1-\theta+\sigma\theta) > 0.$

$N(r/w)(\partial w/\partial r) = (1-\varepsilon)\sigma\varepsilon_{fr} > 0,$

$N(g/w)(\partial w/\partial g) = -(1-\varepsilon)\sigma\varepsilon_{fg} < 0.$

If labor supply is sufficiently inelastic (which we assume) all eight multipliers have unambiguous signs. An increase in the marginal utility of wealth or in the international interest rate have the effect of increasing both e and w. The same effect is produced by a reduction in government spending. And an increase in the terms of trade decreases e and increases w.

It is also of some interest to derive the long run effects on the real wage from $d\omega/\omega = dw/w - \theta de/e$ (which is implied by $\omega = w/e^{\theta}$) and the above expressions yielding:

$N(\lambda/\omega)(\partial\omega/\partial\lambda) = -[\varepsilon+\theta-1+\sigma\theta\varepsilon\xi(1+\sigma\ \varepsilon_F\ \varepsilon_N)] < 0$ if $\varepsilon+\theta>1$,

$N(\phi/\omega)(\partial\omega/\partial\phi) = (1-\varepsilon-\varepsilon\xi)[1-\theta+\sigma\theta(1+\sigma\ \varepsilon_F\ \varepsilon_N)] > 0$

$N(r/\omega)(\partial\omega/\partial r) = (1-\varepsilon-\theta)\sigma\varepsilon_{fr} < 0$ iff $\varepsilon+\theta>1$,

$N(g/\omega)(\partial\omega/\partial g) = -(1-\varepsilon-\theta)\sigma\varepsilon_{fg} > 0$ iff $\varepsilon+\theta>1$.

The effect of an increase in $\lambda$ on the real wage is ambiguous in general. It is negative if and only if $1-\theta < [1+\sigma\xi(1+\varepsilon_{FN}\ \varepsilon_{LN})]/[(1/\varepsilon)+\sigma\xi(1+\varepsilon_{FN}\ \varepsilon_{LN})]$. The latter condition essentially states that the elasticity of the domestic sector product wage w with respect to e along the labor market equilibrium condition ($\varepsilon$) is sufficiently large in relation to the share of domestic goods in consumption ($1-\theta$). An increase in e generates an increase in labor demand in the export sector and a fall in labor supply, requiring a fall in the product wage in the domestic sector that sufficiently diminishes labor demand to maintain equilibrium. The increase in e also generates an increase in the relative demand for domestic goods in relation to imported goods, which tends to increase labor demand by the domestic sector. Hence, the condition for a negative effect of $\lambda$ on $\omega$ requires that the first effect is sufficiently large in relation to the second. We call Domestically Biased Economies (DBEs) (in production relative to consumption) those economies where $\varepsilon>1-\theta$.[15] Hence, under the

[15] This condition is studied extensively in Escudé (2004).

assumptions of a relatively small $\xi$ and a DBE, an increase in the marginal utility of wealth $\lambda$ increases the long run values of both w and e, while it reduces the long run value of $\omega$. A necessary and sufficient condition for either an increase in $g$ or a fall in r to increase the real wage is that that we have a DBE. This condition is also sufficient for an increase in $\lambda$ to reduce $\omega$. Also, an increase in the terms of trade $\phi$ unambiguously increases the real wage.

To obtain the long run multipliers we need the balance of payments equilibrium condition in its intertemporal formulation (63), which depends on w, e, i and $\lambda$. There are several possible specifications for the nominal interest rate i, corresponding to the four possible monetary/exchange rate regimes. Each of the regimes yields a distinct specification for the intertemporal balance of payments equilibrium condition (63). To illustrate, let us take the logarithmic utility case ($\sigma$=1). In the UFIX (or MFIX) regime, (63) reduces to

$$(118) \qquad \theta\kappa_1 f(r,0)/\lambda = r \left\{ \int_0^\infty \phi y_X(\phi e/w) e^{-rs}\, ds - d_0 \right\}.$$

This expression shows total consumption of imports (including those used up in transactions) as the international interest rate (which equals the discount rate $\beta$) times the country's net wealth, which itself is the present value of future exports net of the initial foreign debt . Whenever a shock reduces the country's net wealth, the consumption of imports, and hence all consumption, must fall in the same proportion to comply with intertemporal solvency. Hence, the marginal utility of wealth $\lambda$ must increase sufficiently to reduce the constant foreign currency value of imports. A permanent fall in the terms of trade, for example, reduces the value of future exports and hence $\lambda$ must increase. Then the total long run effect on e is given by the sum of the direct increase through the direct partial derivative above and the indirect increase due to the increase in $\lambda$.

Under the IT regimes the corresponding expression is somewhat more complicated due to the dependence of the policy instrument i on the domestic inflation rate. In the case of DIT, for example, we have instead of (118):

$$(119) \qquad \theta\kappa/\lambda = \left\{ \int_0^\infty [f(r+\hbar\pi_N,0)\, e^{-rs}\, ds \right\}^{-1} \left\{ \int_0^\infty [\phi y_X(\phi e/w) e^{-rs}\, ds - d_0 \right\}.$$

Summing up, in the general case (for $\sigma$) the conditions for the steady state values of the endogenous variables e, w, and $\lambda$, are (114), (116), (63) and (in the case of IT) the corresponding specification of the interest rate feedback rule. Notice that the complete paths of the endogenous variables are determined simultaneously.

It is important to emphasize that the monetary regime in place has an incidence on the long run effects of shocks. The exogenous variables whose long run effects we are mainly interested in are the international value of the dollar $\rho$, the terms of trade $\phi$, the international interest rate r, and government spending $g$. Except for $\rho$, the remaining variables figure explicitly in at least one of these equations. Although dollar strength $\rho$ does not figure explicitly, it is within the definition of the MRER (1). And the nature of its impact, medium and long run effect is heavily dependent on the exchange regime. Under a UFIX regime, for example, a strong dollar shock (whether it be permanent or temporary) has the impact effect of immediately producing a real appreciation of the peso, and a concomitant loss in output and employment, which under perfect capital mobility is financed with debt. These effects, which are a consequence of price and wage stickiness, reduce private sector wealth and require an immediate increase in $\lambda$, thus increasing the long run values of e and w and reducing the long run real wage (assuming we have a DBE). The UFIX regime generates a perverse impact effect because it reduces e and increases $\omega$ while the long run effects are exactly the opposite.[16] In contrast, under the MFIX and any of the IT regimes, the rise in $\rho$ produces an immediate nominal depreciation of the peso that exactly compensates for the effect of $\rho$ on e. The vagaries of the international value of the dollar are completely compensated, either by the choice of the basket of currencies in the MFIX regime, or by the floating exchange rate in the IT regimes. Hence, there is no loss of output, employment or wealth. In fact, under these regimes an increase in $\rho$ has no effect at all on the economic system! Under the RFIX regime, the impact effect of an increase in $\rho$ on e is the same as under the UFIX or MFIX regimes. However, since this is a regime in which the government is specifically interested in the dynamic path of the MRER, the Central Bank has the ability to adopt compensating actions to completely cancel these

[16] This is studied extensively in Escudé (2004) as an explanation for the devastating effects the increasingly strong dollar experienced since 1995 had on Argentina's Convertibility regime.

effects. In the case of an increase in ρ, for example, the Central Bank could instrument a one time devaluation that cancels the unexpected appreciation and then continue with its previous policy.

## III. Conclusions

This paper has studied the dynamic stability of various monetary/exchange regimes in a small open monetary economy under perfect capital mobility and perfect foresight in which there is price and wage stickiness. There are two productive sectors: a monopolistically competitive domestic sector and a competitive export sector. A continuum of households supply differentiated labor services and set wages, consuming domestic and imported goods and having a transactions technology by which holding money reduces costs. Both firms in the domestic sector and households set prices and wages, respectively, by inter-temporal optimization subject to price or wage inflation adjustment costs.

Eight monetary regimes are considered within the same general framework, classified according to whether the MRER is a predetermined (FIX regimes) or a jump variable (IT regimes). In the FIX regimes the Central Bank uses exchange market interventions as its sole policy instrument. The UFIX regime is a fixing of the nominal exchange rate with the dollar, the MFIX is a fixing of the nominal exchange rate with a trade-weighted basket of currencies, and the RFIX is a monetary policy that fixes the MRER to a target path that converges to the long run equilibrium MRER. We find that the three FIX systems are unequivocally saddle-path stable, with the two inflation rates being jump variables and the MRER and w predetermined variables. The UFIX regime is particularly vulnerable to real dollar shocks because of its intrinsic asymmetry. Although the RFIX regime requires that the Central Bank have precise information on the steady state value of the MRER in order to be able to design its Taylor rule, this is no problem in our framework due to the perfect foresight assumption. A long period of gradual adjustment in the MRER towards its long run equilibrium has important income distribution consequences, especially in a Domestically Biased Economy (DBE) in which the MRER and the real wage are inversely related. Also, there is a long process of international reserve accumulation or reduction.

In the five IT regimes the Central Bank uses as instrument the nominal interest rate through a feedback rule and gears its intervention policy to gradually achieve a certain backing of the stock of money with international reserves. The IT regimes differ according to what endogenous variables the Central Bank react to: the domestic inflation rate, the headline inflation rate, the domestic inflation rate and the MRER, the domestic inflation rate and the output gap, and the headline inflation rate and the output gap. All these IT regimes require that the coefficient on the targeted inflation rate satisfy some form of "Taylor property". These become successively more restrictive as headline inflation, the MRER or the output gap are included. In all these IT regimes we found necessary and sufficient conditions for saddle-path stability. However, in the DIOT and HIOT cases they were extremely complicated so we concentrated on more easily interpretable sufficient conditions for saddle-path stability.

Two extensions are made to the basic framework: the inclusion of a long interest rate, and the inclusion of backward looking "rule of thumb" firms and households, each considered for only one of the monetary regimes. We find that the DIT system with a long rate is saddle-path stable if and only if the basic Taylor property holds, without placing any additional requirement than in the basic framework. We find that the UFIX (or MFIX) regime with "rule of thumb" firms and households is saddle-path stable if, given the proportion of these backward looking agents, the adjustment of their respective inflation rates towards the overall inflation rate is sufficiently fast.

Appendix 1: The Routh-Hurwitz theorem

The Routh-Hurwitz criterion gives necessary and sufficient conditions for all the roots of a real polynomial to have negative real parts. However, Routh and Hurwitz obtained a much more powerful theorem that gives the exact number of roots with positive real parts. Gantmacher (1977) provides an excellent reference for the statement and proof of this theorem.[17] We use this theorem in the text to determine the number of eigenvalues with positive real part k(A) of square matrices of dimensions 4, 5, and 6. We here apply the Routh-Hurwitz theorem to these cases but show the 4 by 4 case with more detail.

The characteristic polynomial of a 4 by 4 real matrix A is:

$$\Delta(\lambda) \equiv \det(\lambda I - A) = \lambda^4 - p_1\lambda^3 - p_2\lambda^2 - p_3\lambda + \det.$$

[17] Cfr. Gantmacher (1977), Volume II, Chapter XV.6.

where the coefficients $p_s$ are functions of the elements of A. To obtain the number of roots of $\Delta(\lambda)$ in the right-half of the complex plane (i.e. with positive real parts) one first forms the following Hurwitz matrix H with the coefficients of the characteristic polynomial:

$$H = \begin{bmatrix} -tr & -p_3 & 0 & 0 \\ 1 & -p_2 & \det & 0 \\ 0 & -tr & -p_3 & 0 \\ 0 & 1 & -p_2 & \det \end{bmatrix}.$$

The first row contains the coefficients of the odd powers of $\lambda$ followed by zeros; the second row contains the coefficients of the even powers of $\lambda$ followed by zeros; the third and fourth rows repeat the first and second, respectively, after an initial zero. Now obtain the Routh matrix R by successively: 1) subtracting from the second (fourth) row 1/-tr times the first (third) row, 2) subtracting from the third row of the resulting matrix, -tr(A)/($-q_2$) times the second row, where $q_2 \equiv p_2 + p_3/tr$, and 3) subtracting from the fourth row of the matrix resulting from 2), the third row multiplied by $-q_2/(-q_3)$, where $q_3 \equiv p_3 + tr(\det/q_2)$. In each step, the operation on the rows of the matrix are designed to convert H to the upper diagonal Routh matrix R:

$$R = \begin{bmatrix} -tr & -p_3 & 0 & 0 \\ 0 & -q_2 & \det & 0 \\ 0 & 0 & -q_3 & 0 \\ 0 & 0 & 0 & \det \end{bmatrix}$$

According to the theorem, assuming the non-singular case in which none of the elements in the main diagonal of R is zero (in which case the determination of k is somewhat more complicated and can be found in Gantmacher (1977)), k is given by:

(A1) $k(A) = V(1, -tr, -q_2, -q_3, \det)$

$q_2 \equiv p_2 + p_3/tr, \qquad q_3 \equiv p_3 + tr(\det/q_2).$

where the function V(.) is defined as the number of times there is a change of sign as we proceed from left to right along its arguments which, after the initial 1, are the elements of the main diagonal of R.

When the system matrix A is 5 by 5, the characteristic equation is:

$$\Delta(\lambda) = \lambda^5 - tr\lambda^4 - p_2\lambda^3 - p_3\lambda^2 - p_4\lambda - det.$$

A procedure analogous to the one developed above for the 4 dimensional case yields the following Hurwitz and Routh matrices:

$$H = \begin{bmatrix} -tr & -p_3 & -\det & 0 & 0 \\ 1 & -p_2 & -p_4 & 0 & 0 \\ 0 & -tr & -p_3 & -\det & 0 \\ 0 & 1 & -p_2 & -p_4 & 0 \\ 0 & 0 & -tr & -p_3 & -\det \end{bmatrix}; \quad R = \begin{bmatrix} -tr & -p_3 & -\det & 0 & 0 \\ 0 & -q_2 & -(p_4 + \det/tr) & 0 & 0 \\ 0 & 0 & -q_3 & -\det & 0 \\ 0 & 0 & 0 & -q_4 & 0 \\ 0 & 0 & 0 & 0 & -\det \end{bmatrix}$$

$q_2 \equiv p_2 + p_3/tr$, $q_3 \equiv p_3 - [tr\ p_4 + det]/q_2$ $q_4 \equiv p_4 + det/tr - [q_2\ det]/q_3$

Assuming again that none of the elements in the main diagonal of R are zero, the theorem of Routh-Hurwitz says that the number of roots with positive real part is given by

(A2) $k(A) = V( 1, -tr, -q_2, -q_3, -q_4, -det )$.

When the system matrix A is 6 by 6, the characteristic equation is:

$$\Delta(\lambda) = \lambda^6 - tr\lambda^5 - p_2\lambda^4 - p_3\lambda^3 - p_4\lambda^2 - p_5\lambda + det.$$

Carrying out the same procedure as in the previous cases, k(A) and the main diagonal elements of the resulting Routh matrix are:

(A3) $k(A) = V( 1, -tr, -q_2, -q_3, -q_4, -q_5, det )$.

$q_2 \equiv p_2 + p_3/tr$, $q_3 \equiv p_3 - [tr\ p_4 + p_5]/q_2$

$q_4 \equiv p_4 + p_5/tr - [q_2\ p_5 + tr\ det]/q_3$ $q_5 \equiv p_5 + [tr\ det]/q_2 + [q_3\ det]/q_4$

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