Towards New Money Measures

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September 20, 2001

We would like to thank our colleagues, especially Jack Selody and Dinah Maclean, for helpful comments and suggestions. We are also grateful to Jeannie Kottaras and Debbie Dandy for their hard work preparing the data.

Abstract

Technological innovations in the financial industry pose major problems for the measurement of monetary aggregates. This paper presents preliminary results from a project that aims at developing a new measure of money which has a more satisfactory means to identify and remove the effects of financial innovations. The fundamental contribution is to establish an explicit distinction between the measured data (currency and deposit balances) and the underlying phenomena of interest (intended usage of money for transactions and savings). While the classification scheme used for monetary aggregates was once designed to provide a proxy for the phenomena of interest, it is breaking down. Thus we think it is better to move to an explicit attempt to measure an index of intended usage.

The paper reviews previous methodologies and then outlines a dynamic factor approach for making the explicit distinction between the measured data and the underlying phenomena. Identification issues associated with dynamic factor analysis are discussed and preliminary results with both simulated and real data are presented.

1. Introduction

1.1 Overview

Monetary aggregates have been used for half a century to predict economic activity and inflation, more successfully in some periods than others. However, since the late 1970s, successive waves of financial innovations have made it increasingly difficult to measure the underlying growth of money. In particular, it is hard to differentiate balances used for transactions from those used for savings. Having a good measure of transactions money is important because theory suggests it will have the most predictive power for output and inflation. This paper presents preliminary work from a project to develop a new measure of money that has a more satisfactory means to identify and remove the effects of financial innovations. The proposed measure also differs significantly from previous measures in that it is not an aggregate. By this we mean that it measures activity instead of account balance items. Activity refers to economic agents' behaviour. More precisely, we attempt to establish indexes of intended usage (e.g. transactions and savings) rather than aggregate deposit balances according to some classification scheme. The classification scheme was once designed to provide a proxy for these underlying phenomena but the classification scheme is breaking down and we now think it is better to move to an explicit attempt to measure an index of intended usage.

The next sub-section of this *Introduction* explains the motivation of the larger project, for which this paper presents some preliminary results. The section on *Monetary Aggregates* surveys current methodologies and aggregates, outlining the literature and known problems. The section *A New Approach to Measuring Money* describes the new proposed approach, including some simulation results from studying the estimation techniques and some preliminary results. The final section outlines future directions.

1.2 Motivation

Past attempts to improve Canadian monetary measures have included the development of the narrow aggregates M1+ and M1++ which include a broader range of accounts than M1, and adjusted M1 which is a model based definition of money.[1] However, none of these is completely satisfactory. M1+ and M1++ aggregates include savings balances, and adjusted M1 mutes some of the predictive power of money.

Official monetary aggregates in Canada are a simple sum of currency and various deposits, classified according to their characteristics. Narrow aggregates attempt to measure transactions money and so are composed of currency, demand deposits, and some other deposits traditionally associated with transactions. Broad aggregates also include deposits associated with savings. Technological progress poses two major problems for the measurement of transactions money. Firstly, transaction money is a measure of purchasing power, but this purchasing power can now be accessed in a variety of ways. Savings and transactions balances are not held in clearly defined separate accounts, but rather are mixed together. Also, investment accounts and stock market-oriented deposits have become more popular in the late 1990's. While aimed more at savings balances, the money in these accounts is still very liquid and available for any kind of transaction. Soon many deposits may be in accounts "tailor-made" for the habits of a person, not for the purpose of which the money in the account is intended to be used. Secondly, many transactions balances are held in accounts that are not included in current narrow monetary aggregates and there are new deposit-taking institutions not included in the aggregates, such as investment dealers, life insurance companies and near banks, which offer new types of deposits. Moreover, the information revolution has considerably changed agents' behaviour regarding their money management and, in particular, money can be moved from one account to another very easily and quickly. A simple phone call or a visit on the Internet is sufficient. When this money is transferred between institutions included in the aggregates and those excluded, it produces spurious fluctuations in the aggregates which can reduce their predictive ability.

For these reasons the old classification system is breaking down. Currently individual problems are dealt with on a case-by-case basis but this is becoming increasingly difficult. Research is needed to develop a new measure of money that can be used by analysts as the current classification system continues to break down and eventually fails. These new money measures should not depend on features of the different accounts, as these are becoming increasingly diverse and very difficult to classify and measure.

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1. M1+ is defined as the sum of currency held by the public and all chequable (demand and notice) deposits at chartered banks, credit unions and caisses populaires (CUCPs), and trust and mortgage loan companies (TMLs). M1++ is the sum of M1+ and all non-chequable notice deposits at chartered banks, CUCPs, and TMLs. For general background about analysing the monetary aggregates at the Bank of Canada see Maclean (2001)

We propose using dynamic factors to overcome the two problems identified above. Dynamic factors allow us to focus on measuring the underlying economic activities rather than the amounts in historical deposit classifications. We think this approach offers the best way to address the innovation problems because it distinguishes the underlying economic activities (economic agents intentions to transact or save) from the measured items (balances in accounts), which are affected by the above mentioned financial innovations. Despite the instability in the characteristics of deposit accounts we believe the technology revolution has not changed the fundamental uses of money for economic activities that we are trying to measure.

One important difference between the proposed dynamic factor approach and the traditional aggregation approach is that it is no longer necessary to include all deposit-taking institutions to compute a valid measure. Only a good sample of deposits is required to get a measure representative of the fundamental activities, while aggregation requires correct classification and data from all institutions to build good aggregates.

2. Monetary Aggregates

2.1 Existing Methodologies

Official monetary aggregates in Canada are a simple sum of currency and various deposits with weights for all components set to one. This implies that all monetary assets should be dollar-for-dollar perfect substitutes. This is not true since some are clearly less liquid and give a higher yield than currency and demand deposits. Hence, the monetary aggregates constructed by a simple summation provide a good measure of the stock of nominal monetary wealth but are not a structural economic variable.

To account for substitutability, and for the fact that certain kinds of accounts have both a transaction and a saving nature, attempts have been made to consider weights for components. Barnett (1980)[2] suggests the Divisia index. This monetary aggregate is constructed by combining monetary theory with statistical index number theory and micro economic aggregation theory. It measures the flow of services produced by the component assets.

The Divisia index is a time-varying weighted monetary aggregate where the weights are expressed in terms of the contribution of each component to the total value of services provided by all monetary assets. This index is derived from the optimization behaviour of economic agents. It is reputed to have better theoretical foundations than the simple-sum monetary aggregates. Also, some consider that the Divisia index is better adapted to the context of continuous financial innovations because it internalizes substitution effects. However, monetary authorities are reluctant to publish these monetary aggregates because their construction requires various subjective choices that make them almost impossible to reproduce[3,4].

Others have worked to measure transaction balances. Spindt (1985) suggests a weighted monetary aggregate (MQ) derived from the quantity theory of money equation, MV=PQ. Weights are based on each monetary asset's velocity (turnover rate). Another attempt to measure the liquidity services is the currency-equivalent (CE) monetary aggregate proposed by Rotemberg, Driscoll and Poterba (1995). This aggregate has some improvements but is similar to

2. See also Barnett and Serletis (2000).
3. The Bank of England and the Federal Reserve Bank of St-Louis are the only institutions that publish Divisia indices in their official statistics.
4. For a detailed discussion on the disadvantages of Divisia indices, see Cockerline and Murray (1981), Fisher, Hudson and Pradhan (1993) and Longworth and Atta-Mensah (1995).

Divisia in the sense that it is derived from an optimization problem. Nevertheless, it has not been used because practical issues in addition to those related to the Divisia index have emerged. For example, weights tend to be highly volatile, which complicates interpretation and empirical use.

2.2 Empirical Evidence in Canada

Many studies have assessed the performance of monetary aggregates in terms of various criteria such as their information content, money-income causality, and stability in money demand equations. In general, the results are mixed. For Canada, Cockerline and Murray (1981) find that Divisia aggregates contain less information on contemporaneous and future levels of income than summation aggregates. Summation aggregates also appear to be superior in causality tests. On the other hand, the study finds Divisia indices to be more stable in money demand equations, which is consistent with the fact that these aggregates tend to follow more consistent time paths than their summation counterparts.

Hostland, Poloz and Storer (1987) also look at the information content of alternative monetary aggregates. They compare summation aggregates with Fisher ideal indices of monetary services[5]. They conclude that the information loss through simple-sum aggregation is not significant. In other words, the Fisher ideal aggregates add very little information to improve income and price forecasts. Serletis and King (1993) examine the empirical relationships between money, income and prices, comparing summation aggregates to Divisia. They find that the growth rates of Divisia aggregates are more useful than summation aggregates for forecasting nominal income fluctuations, while the growth rate of the summation aggregate M2+ is the best leading indicator of inflation.

The results in these Canadian studies are consistent with those of other researchers using data for different countries.[6] Despite the theoretical advantages of Divisia aggregates, they have not been shown clearly superior to summation aggregates.

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5. Like Divisia, Fisher ideal monetary aggregates are known as *superlative* indexes.
6. See, for example, Bailey et al.(1982a, 1982b), Driscoll et al. (1985), Horne and Martin (1989) and Subrahmanyam and Swami (1991).

2.3 Adjusted M1

In recent years, movements in M1, the traditional measure of transactions money used by the Bank of Canada, have been affected by financial innovations[7]. This has changed the relationships between money, output and inflation and, as a result, the M1-based models have been unstable. Since alternative aggregates described above were not very successful, economists at the Bank of Canada create a new model-based measure of transaction balances called adjusted M1[8].

The objective of adjusted M1 was to correct instability in the main money-based model used at the Bank of Canada (the M1-VECM model)[9]. It is obtained in two steps. First, using the money-forecasting equation from a M1 VECM estimated with a sample ending in 1993 (the beginning of the second wave of innovations according to Aubry and Nott (2000)), a forecast of M1 is obtained for the period 1992Q1 to the last quarter of available data (National Accounts). This time-series is called “distortion-free” M1 and can be interpreted as an estimate of what M1 would have been if no changes in the data-generating process had occurred in the 1990s. Second, this series is regressed on the components of the monetary aggregates. This step relates the distortion-free money to the observed money data released every month. Adjusted M1 is thus a weighted sum of components’ levels.

Unfortunately, adjusted M1 is not free of problems. Some serious deficiencies are associated with each step of the procedure. In the first step, the choice of the estimation period is problematic. 1993 was chosen as the end of the sample under the assumption that most financial innovations occurred after this period. However, M1 was probably distorted before this date. Calculating “distortion-free” M1 from stable money demand is another problem, since it implies that structural changes over the 1990’s affected only the money supply, but money demand could also have shifted in response to these changes.

The way adjusted M1 is constructed it may lose valuable information as a money measure for analysis. The construction mutes some of the predictive power of money. For example, fundamental movements can be removed while attempting to remove distortions. In

7. See Aubry and Nott (2000) for a detailed discussion on financial innovations.
8. See Adam and Hendry (2000) for details on the development of adjusted M1.
9. The M1-VECM is developed in Hendry (1995).

addition, we find that the weights on the components are unstable and very sensitive to the choice of the sample in the second step. Some weights are also counter-intuitive (e.g. the weight on currency is above 1).

Finally, adjusted M1 is a model-dependant money measure which is quite dangerous. If the model is wrong, then adjusted M1 may not measure transactions money. All things considered, this approach has not been as successful as hoped. This leads us to now consider a completely different approach that does not rely so fundamentally on a specific economic theory.

3. A New Approach to Measuring Money

3.1 Dynamic Factor Analysis (DFA)

A factor is an index that can be used to indicate the evolution of an activity. Indexes are already familiar to economists and statisticians. Brillinger (1975) in introducing the technique used in this paper quotes Bowley (1920):

> *"Index numbers are used to measure the change in some quantity which we cannot observe directly, which we know to have a definite influence on many other quantities which we can observe, tending to increase all, or diminish all, while this influence is concealed by the action of many causes affecting the separate quantities in various ways."*

In recent years, economists have made increasing use of DFA (sometimes called dynamic latent variables) for estimating "underlying" processes. These processes may correspond closely to the economic concepts which macro economists have in mind when they build models. The techniques have been used to propose better measures of underlying inflation[10], applied to the real side of the economy[11], and used in arbitrage pricing theory models of financial decision making[12]. Despite the conceptual appeal of the techniques, to our knowledge no one has used these methods to measure transactions and monetary savings activities. One reason may be that the deposit data have not been organized in a suitable way for applying these techniques. Our first job was to fix that problem by adjusting money components to account for changes such as acquisitions that occurred in the financial sector. Previously this was only done for the aggregates and not for the components.

In broad terms, DFA is a branch of multivariate statistical analysis in which the observed variables x_i (i = 1, 2,..., p) at each period t are expressed in terms of r factors (or latent variables) f_j, where $r < p$, and idiosyncratic terms e_i (residuals). The model is given by the equation:

10. See for example Bryan and Cecchetti (1993).
11. See for example Forni and Reichlin (1996), Geweke and Singleton (1980), Quah and Sargent (1994) and Stock and Watson (1999).
12. See for example Conner an Korajczyk (1988), Garcia and Renault (1999) and Roll and Ross (1980).

$$x_i = \sum_{j=1}^{r} A_{ij} f_j + e_i \tag{1}$$

at each period *t*, or in matrix form

$$x_t = A f_t + e_t \tag{2}$$

where *A* is a *p x r* matrix of weights.

There are *p*r* unknown weights (also known as *factor loadings*) and *r* factor series to be estimated with only *p* observed series. All these weights and factors are estimated simultaneously. In a following section, the constraints imposed so as to obtain a unique solution are discussed.

3.2 Intuition

The new approach has some resemblance to weighted aggregates but, in fact, it is not an aggregation at all. Rather, it is an attempt to measure the common underlying (or latent) factors (of which *transactions money* and *savings money* are the two most important) that influence the use of currency and the money in different types of accounts.

A narrow aggregate is an attempt to add up currency and deposits used as transactions money. A weighted aggregate would attempt to divide deposits into the portion used for transactions and the portion used for savings. Because measuring an item requires a complete coverage of the data, the intuition of aggregation is that, if everything is measured and allocated correctly we would have an exact measure. In contrast, factors are latent variables which cannot be measured directly. This approach treats *transactions* and *savings* as two fundamental underlying activities in the economy. Data on currency and a wide range of deposits are used to estimate the two activities, and each measured monetary instrument (i.e. deposit type and currency) can be expressed in terms of these factors. This can be written as

$$currency = w_1\ transactions + w_2\ savings + e_{currency}$$

$$demand = w_3\ transactions + w_4\ savings + e_{demand}$$

$$notice = w_5\ transactions + w_6\ savings + e_{notice}$$

.

$$\vdots$$

$$mutual\ funds = w_{n-1}\ transactions + w_n\ savings + e_{mutual\ funds}$$

where the weights w_i are estimated simultaneously with the savings and transactions processes. Each type of deposit is a weighting of the two factors, not the other way around as is done in aggregation. Intuitively, we would expect that in the case of currency, for example, transactions activity has the heaviest weighting and savings activity a minimal weighting. The idiosyncratic process e_{xxx} indicates amounts specific to a particular measured monetary instrument and not explained by the factors.

On the real side of the economy there are considerably more data associated with underlying factors than is the case on the monetary side. (Stock and Watson, 1999, use thousands of variables.) However, our application has the advantage that we expect very few factors, while on the real side one expects many factors to be important. The idea is explained above in terms of savings and transactions activities but it is possible, for example, that corporate transactions and personal transactions should be distinguished as different factors. It is even possible that financial institution transaction activity, which we now think of as a “distortion” to the aggregates, is a separate factor. Thus we may find more than two factors, but we would be surprised if we find many more factors than this.

In this approach, each deposit provides an additional measure of the underlying factors. (We must have more monetary instruments than factors in order to solve the problem mathematically.) More deposit types provide more measurements and thus more precision. Omitted deposit types mean fewer measurements and thus less precision. In the aggregation approach, by contrast, omitted deposit types mean something is missing and the aggregate is not correct in an accounting sense.

As mentioned previously, one result of financial innovations is that an account type may start to be used in a different way. Modelling this phenomena is challenging. In the new approach, any changes affecting many of the measured variables should result from the factors, but the idiosyncratic components mean the measured variables can include changes that are not a result of factors. They flag anomalies (or distortions) since they should usually be small. A persistently important idiosyncratic component signals that the usage of a deposit may have changed, and suggests the need to reconsider the weights used for that measured variable. Thus weights will vary over time and the necessity of a change in the weights is more clearly indicated.

Even though balances are shifting around, the objective of this approach is to get a transactions money measure which avoids noisy fluctuations coming from financial innovations that cause measurement problems due to their effect on deposit accounts. Savings money growth should also be more stable in this sense. Variable weights are required to absorb the effects of shifts due to innovations. However, given the large number of unknown parameters, it is impossible to solve the system of equations mathematically with continuously variable weights. Eventually, as a first step to address this, we will identify break points, that is, periods when financial innovations modified the usage of certain accounts.[13] For example, the elimination of differential reserve requirements on business demand and notice deposits in the early 1990's removed the incentive for banks to distinguish between these two types of accounts. Using the new methodology, in response to this financial innovation, demand and notice deposits should have comparable weights on transactions and savings factors while before this change, notice deposits were used more as a saving account than demand deposits.

3.3 Data Problems

The proposed methodology for measuring transactions and savings money helps solve certain kinds of measurement problems, but more importantly, it should help to quickly pin-point new problems so that corrections can be applied. This sub-section discusses certain types of problems which occur, what there effect will be, how they are dealt with in this paper, and how they might eventually be resolved.

It is important to distinguish between two modes in the process of collecting data. One is the usual *operational mode* which is the situation when new data is obtained but the weights in the "data measurement model" are fixed and not being estimated. The second is *estimation mode*, which is the situation when the weights are initially estimated and occasionally re-estimated. Data problems are not corrected in operational mode, but the calculation of the measures should flag problems quickly and well before they have a substantial effect on the measurements. The problems can then be corrected in a timely way.

13. Aubry and Nott (2000) discuss the major financial innovation waves in Canada. This could be used to determine the dates of the changes.

The first type of data problem is a shift in the usage of a certain deposit classification. For example, demand deposits previously paid little interest and were rarely used for savings deposits. Now they often pay attractive interest rates and are sometimes used for savings. This kind of structural break will require a re-estimation of the weights. This is slightly different from the effect of a structural break on aggregates. Firstly, there is an explicit error (idiosyncratic) term which provides an automatic mechanism to partially ignore the effect for some time. That is, the change affects the error term much more than it affects the measure. Secondly, the error term quickly flags the break. Thirdly, there is a specific mechanism for making the eventual correction: re-estimate the weights for the problematic data classification. By contrast, there is no simple mechanism to deal with known structural breaks in the current aggregates.

The second type of data problem is a shift among data classifications. For example, Canadian savings bonds decreased in popularity in the second half of the 1990s and at least part of that was a shift into mutual funds, which increase substantially in the same period. This shift has more to do with availability or marketing of different types of financial instruments than it does with the underlying phenomena of interest. One simple way to compensate for this problem is to amalgamate the data classifications involved. Then the shift is internal to the classification and does not show in the data at the level of aggregation of the components that are used. This is the approach used in the example in this paper, because of its simplicity. It is probably not the best way to handle this problem. A second simple way to compensate is to omit the affected classifications. As mentioned earlier, the methodology requires only samples and not a complete accounting, so omitting some classifications is a possibility. A more satisfying way to deal with this kind of problem is to build a second level into the data measurement model, one that accounts for shifts among classifications. This additional level of complexity is not discussed in this paper but eventually will be necessary. There is additional information which can be used at this level, so the second level does not depend only on the data and techniques as discussed here.

The third type of data problem is a shift of market share among institutions. In the current calculation of the aggregates this is only a problem if it is a shift between institutions included and not included in the aggregates. However, an additional level of sophistication, which will not be discussed elsewhere in this paper, entails adding a breakdown by institution. In order to do this it is necessary to build a third level into the data measurement model, one that accounts for shifts among institutions. In this paper that is not necessary because we are using data aggregated across institutions (and we will ignore shifts between institutions included and not included in the aggregates).

Finally, there is a distinction between problems with initial estimation and problems (identified in operational mode) which lead to re-estimation. In the later case, underlying factors will already be established for large parts of the sample and the timing and nature of a new breakpoint will have been identified (in operational mode). During initial estimation there is no established baseline for the factors, and structural break points also need to be established. There are several possibilities for dealing with the special problems at this initial stage. One, used in this paper, is to amalgamate some problematic data classifications. Another, not yet investigated, is to begin with sample periods when structural changes appear to be less problematic.

This paper does not elaborate on the details of the data measurement model outlined above but focuses on a somewhat simplified version of one data measurement model. The above details will eventually become very important, and there are ways to deal with them, but there are more fundamental issues to address first.

3.4 Identification Issues

The term "factor analysis" is sometimes used in a generic sense to describe several techniques including *principal components analysis* (PCA) and it is sometimes used in a more specific sense to describe a special interpretation of equation (2). (See, for example, Basilevsky, 1994.) Specifically, the factors should result in an idiosyncratic term e_t with a diagonal correlation matrix. That is, factor analysis attempts first to explain common movements in the measurements rather than the most variation as in PCA. Of course, explaining as much variation as possible is also interesting, so this difference is really one of relative emphasis. An important difference is that principal components are uncorrelated (orthogonal) but factors are not necessarily. One would not expect transactions and savings to be uncorrelated, so in the current problem factors are more logical than principal components. PCA is sometimes suggested as a technique for estimating factors (see, for example, Johnson and Wichern, 1998). This results in orthogonal factors which can then be rotated to find "oblique factors." The problem is then to find the appropriate rotation. That approach is not attempted here as it seems more natural to apply constraints on the estimation, which will then result in the oblique factors.

Perhaps the most difficult and technically controversial aspects of estimating DFA models are the specification of the objective function and the imposition of identifying (or uniqueness) constraints. It is necessary to determine constraints which make the statistical estimation well defined, so that there are not multiple solutions. This is a common problem in econometric work, but here we have the additional objective of trying to do this in a way which is relatively neutral with respect to economic theories. That is, we would like to achieve measures of factors which are economically interesting but do not require imposing too much (potentially controversial) theory in order to achieve the measurement. In other words, the measures should be good for a wide range of economic theories.

One aspect of this identification problem is that any invertible matrix G defines new factors $(G f_t)$ and weights (AG^{-1}) and the equation

$$x_t = (AG^{-1})\,(G f_t) + e_t \tag{3}$$

gives identical measured variables x_t and idiosyncratic terms e_t as in equation (3). Thus these models cannot be distinguished statistically and some otherwise motivated constraint must be imposed. The simplest example of this is simply a different relative scaling of the factors and weights. Since the factors are treated as an index this scaling problem can be resolved by specifying that the factors have value 1.0 in the first period. (And thus they should only be interpreted in growth rates not in levels.) However, rotations preserving the magnitude are still a concern. A second aspect is that different idiosyncratic terms e_t may result in similar objective function values and thus cannot be distinguished in the estimation.

A possible constraint which may be related to the rotation problem is that the factors and weights should be positive. This is consistent with the way we intuitively think of the concept of money. Another possible consideration for a constraint is a “roughness penalty” as used in the functional data analysis theory of Ramsay and Silverman (1997). This is similar in some respects to a filter, but the penalty is on rapid variation of the underlying factor rather than the measurements themselves as would be typical with a filter. In this regard it is closer to a Kalman filter, but there is no attempt here to model the underlying dynamics as with a Kalman filter. (Modelling the underlying dynamics may be interesting in the future, but is an economic modelling problem and the present work is focused primarily on measurement issues.) The theoretical justification for this roughness penalty is that the underlying phenomena of interest for economic modelling and policy should be smoother than the measured data. The disadvantage is that too high a penalty may obscure rapid variations that are important.

The best combination of constraints and penalties (or objectives) is a matter of ongoing investigation. This is complicated by the fact that the estimation algorithms are being investigated simultaneously. The estimates are typically done by an iterative procedure and can be very slow. As previously mentioned, both the weights and the factor series are parameters in the estimation, so there are a very large number of parameters (over 700 in the preliminary experiments discussed below). This means the estimation is fairly difficult even when the problem is not ill-conditioned, and some combinations or too few constraints do give ill-conditioned problems.

To summarize, possible constraints and objectives being considered include

(i) factors set to 1.0 at the first period (or something equivalent like 1.0 in January 2000).

(ii) e_t has a diagonal correlation matrix (or covariance matrix) $E(e_{ti}, e_{tj}) = 0$ for i not equal j, where i and j indicate different financial instruments.

(iii) Minimum diagonal of the covariance of e_t.

(iv) Factors should not be correlated with the idiosyncratic term $E(f_t, e_t) = 0$.

(v) Factors and weights should be positive.

(vi) Roughness penalties.

The relative importance of each of these remains to be determined.

3.5 Estimation with Simulated Data

This section reports results of simulation experiments used to test the estimation algorithms. Data was generated by adding noise to two known factors multiplied by known weights to give artificial measurement data. The estimation algorithms were then tested to see if they would recover the original factors. This sort of simulation experiment should be treated with some caution as it really only shows that estimation works in the single artificial situation used to generate that data. Nonetheless, it does help eliminate many problems, especially coding problems. It also suggests that the algorithms work for small samples, and theoretical small sample results are typically very difficult to obtain.

There are some further caveats which should be mentioned. The example here is one estimation technique which worked fairly well, but there were many that did not. Hopefully this is an indication of a reasonable estimation/identification combination, but given the nature of these experiments at this stage it could also be a random draw. Also, the two simulated factors are relatively uncorrelated, which may help estimation. Furthermore, good starting conditions were known. We intend to do considerably more testing of the algorithms with simulated data, including simulations which mimic the actual data more closely. This is necessary in order to better understand the technique and the interpretation of factors.

The simulated data is shown in Figure 1. Twelve series were generated using two factors. Figure 2 shows the actual factors used to generate the data (solid) and the estimated factors (dashed). Table 1 shows the estimated weights together with the true weights in brackets.

The estimation was done using a minimization routine[14] and objective function defined by summing together two objectives and a roughness penalty. The first objective was to minimize the square of the elements of the covariance matrix of the idiosyncratic components. The second was to minimize the square of the elements of the covariance between the idiosyncratic components and the factors, so idiosyncratic components will then be residuals in the sense that they cannot be explained by the factors. These covariance elements are squared because it is important that the objective associated with off-diagonal elements is not negative, and because the objective function should be differentiable for this optimization routine. Using the squared elements of the covariance matrix of the idiosyncratic components serves dual objectives of minimizing the off-diagonal elements (so factors explain correlated movements in the data) and minimizing the variance (so factors explain as much variation as possible). However, using the squares may distort this intuitive objective by putting disproportionate weight on larger elements. The roughness penalty was defined by the sum of the square of the second difference in the factor series. The relative importance of the roughness penalty is controlled by a scale factor which was

14. Option "L-BFGS-B" of the optim function in the programming language R (Ihaka and Gentleman, 1996, see <http://www.r-project.org/>). Code and specific details are available from the authors and will eventually be on the web site <http://www.bank-banque-canada.ca/pgilbert>

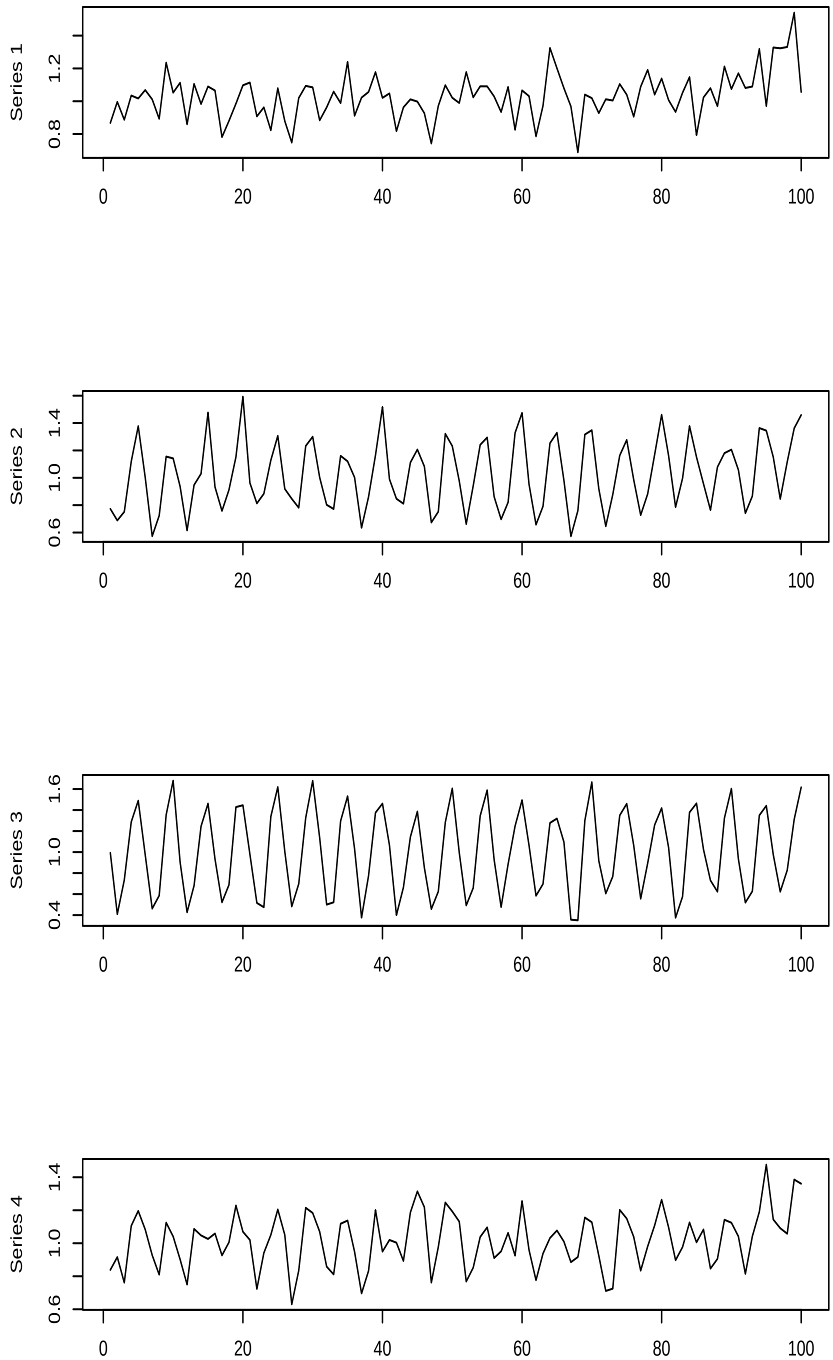
set to 1x10^{-8}. The effect of this scaling penalty is to eliminate sharp variations in the factors. In this example, a larger penalty decreases the difference between the peaks of the true and estimated factors in the second panel of Figure 2.

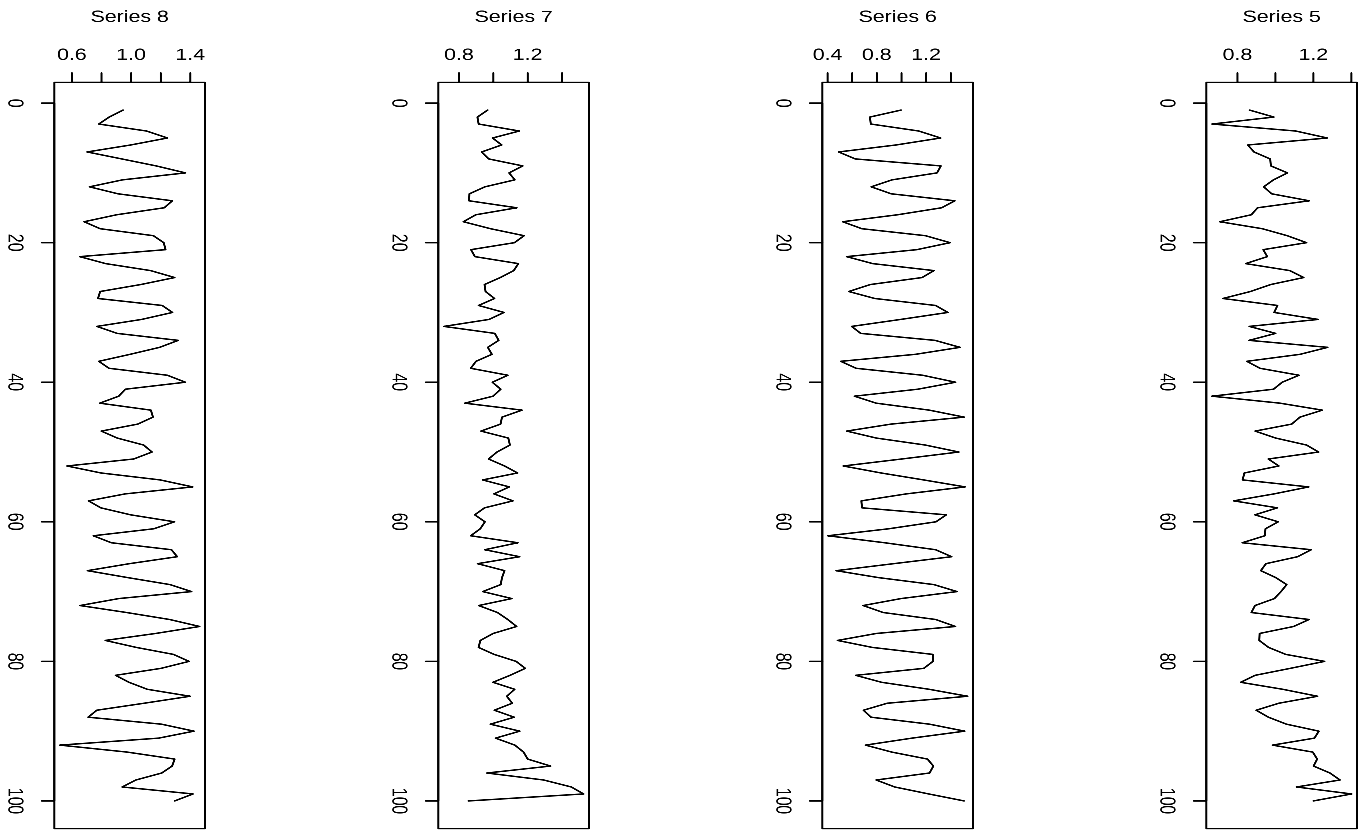
While several problems remain, the overall impression from these experiments is that the techniques can work, potentially quite well when adequately refined.

Table 1: Estimated weights (true in brackets)

component	factor 1	factor 2
Series 1	0.9711753 (0.9)	0.02132972 (0.1)
Series 2	0.5787789 (0.5)	0.34110143 (0.5)
Series 3	0.2179484 (0.1)	0.62290401 (0.9)
Series 4	0.7867802 (0.7)	0.16416504 (0.3)
Series 5	0.8677186 (0.8)	0.09618929 (0.2)
Series 6	0.4158194 (0.3)	0.46269413 (0.7)
Series 7	0.9953030 (0.9)	0.00000000 (0.1)
Series 8	0.6367933 (0.5)	0.30587756 (0.5)
Series 9	0.2263639 (0.1)	0.63344682 (0.9)
Series 10	0.7769181 (0.7)	0.17132445 (0.3)
Series 11	0.8724282 (0.8)	0.10439224 (0.2)
Series 12	0.4024281 (0.3)	0.47958831 (0.7)

Figure 1. Simulated data




Series 8
0.6
1.0
1.4
Series 7
0.8
1.2
Series 6
0.4
0.8
1.2
Series 5
0.8
1.2
0
20
40
60
80
100

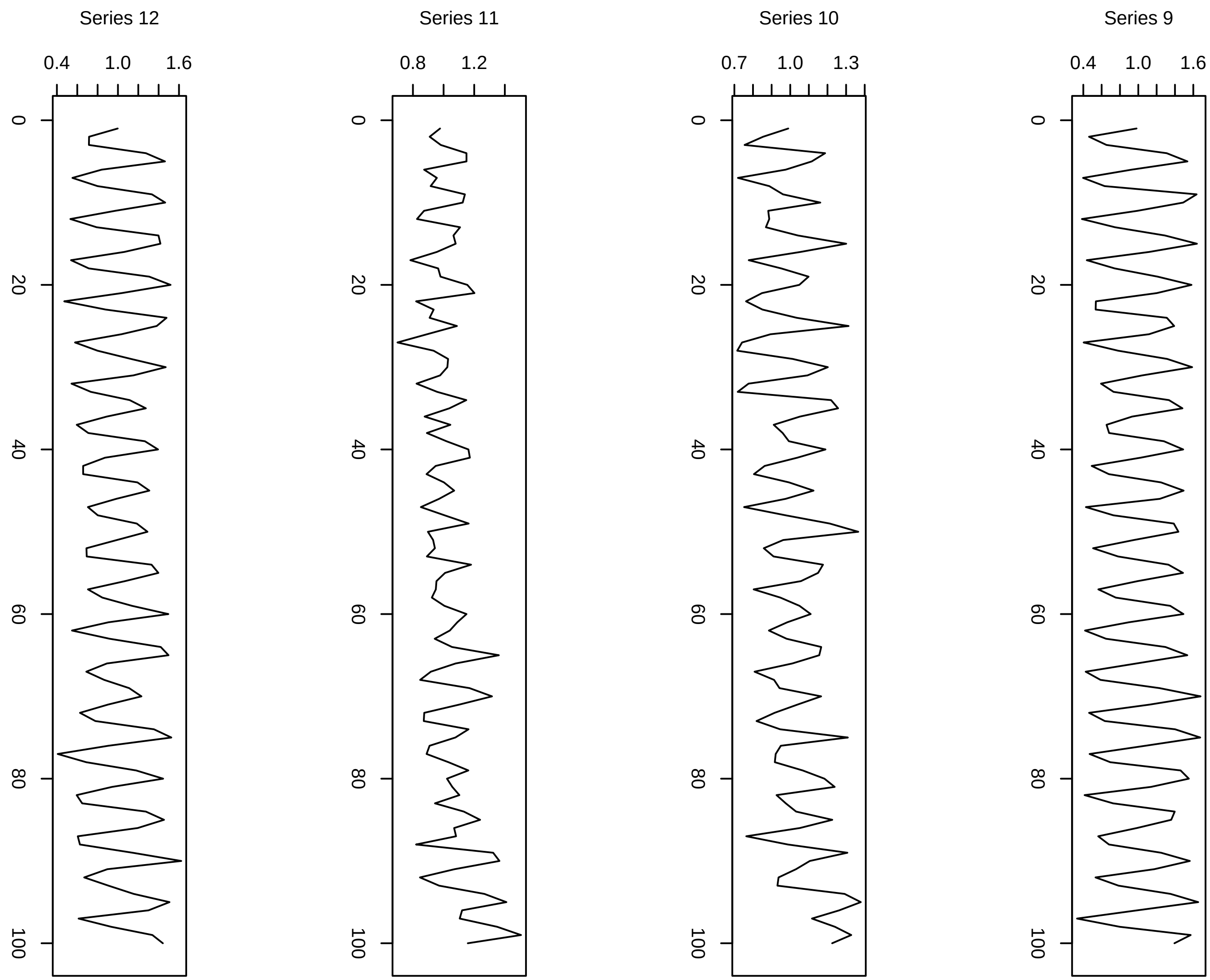
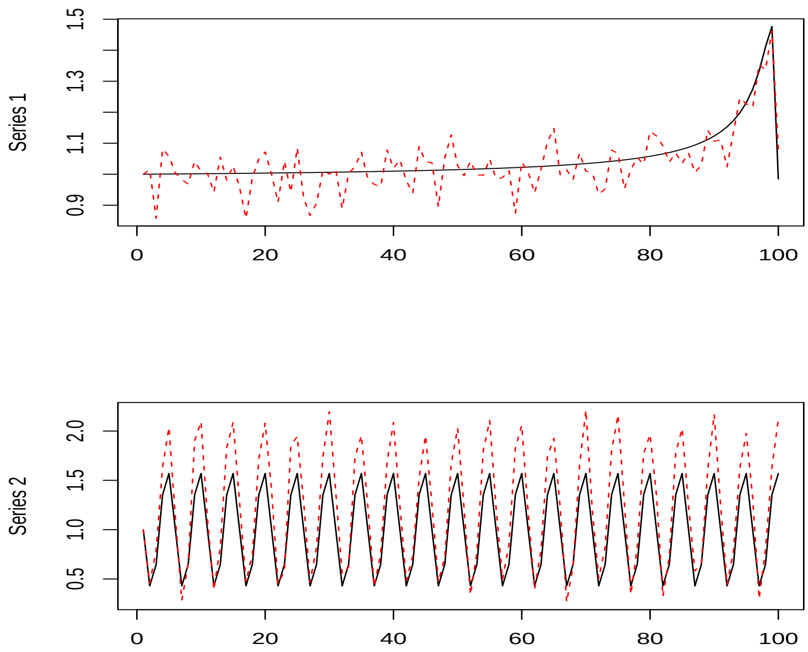

Series 12
0.4
1.0
1.6
Series 11
0.8
1.2
Series 10
0.7
1.0
1.3
Series 9
0.4
1.0
1.6
0
20
40
60
80
100

Figure 2. True Factors (solid) and Estimated Factors (dashed)



3.6 Preliminary Experiments with Canadian Data

This section summarizes preliminary results using Canadian data on currency and several different deposit types. The results are preliminary in several respects:

- The most appropriate identification constraints, as discussed above, are still a matter of ongoing investigation.
- The estimation algorithms and convergence criteria need to be refined.
- The component data has been adjusted for institutional take-overs and some reporting errors by banks. Previously these adjustments have only been done for aggregates and not for the components, however, the DFA methodology requires that the adjustments be made to the components. This has been done but the dataset is still preliminary.
- The results are based on assuming two and only two factors are important, and that has not yet been properly established. We will need to test for the appropriate number of factors given the phenomena we are trying to measure.
- It is possible that there are large structural breaks due to shifts in usage of some deposit classifications and thus different weights need to be established for different parts of the sample.

Once weights are established the idiosyncratic component helps identify shifts in usage, but that is not true during initial estimation.

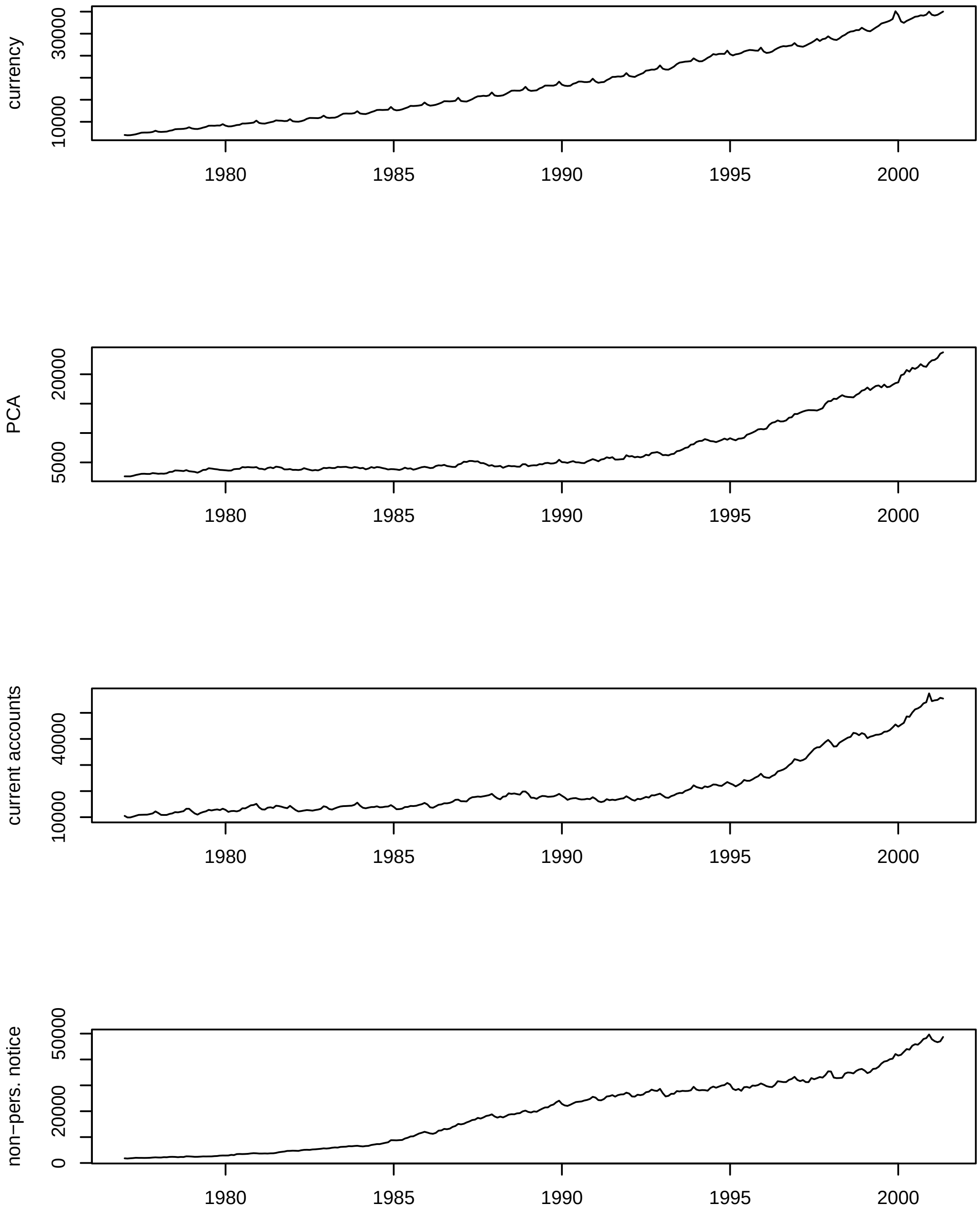
- As discussed previously, shifts among data classifications can substantially affect the effort to estimate factors representing the phenomena of interest. One instance of this is that mutual funds have become very popular in recent years. Possibly related is the fact that Canadian savings bonds have decreased substantially in popularity at the same time. As a temporary measure these categories have been added together, however this is not a completely satisfactory solution.

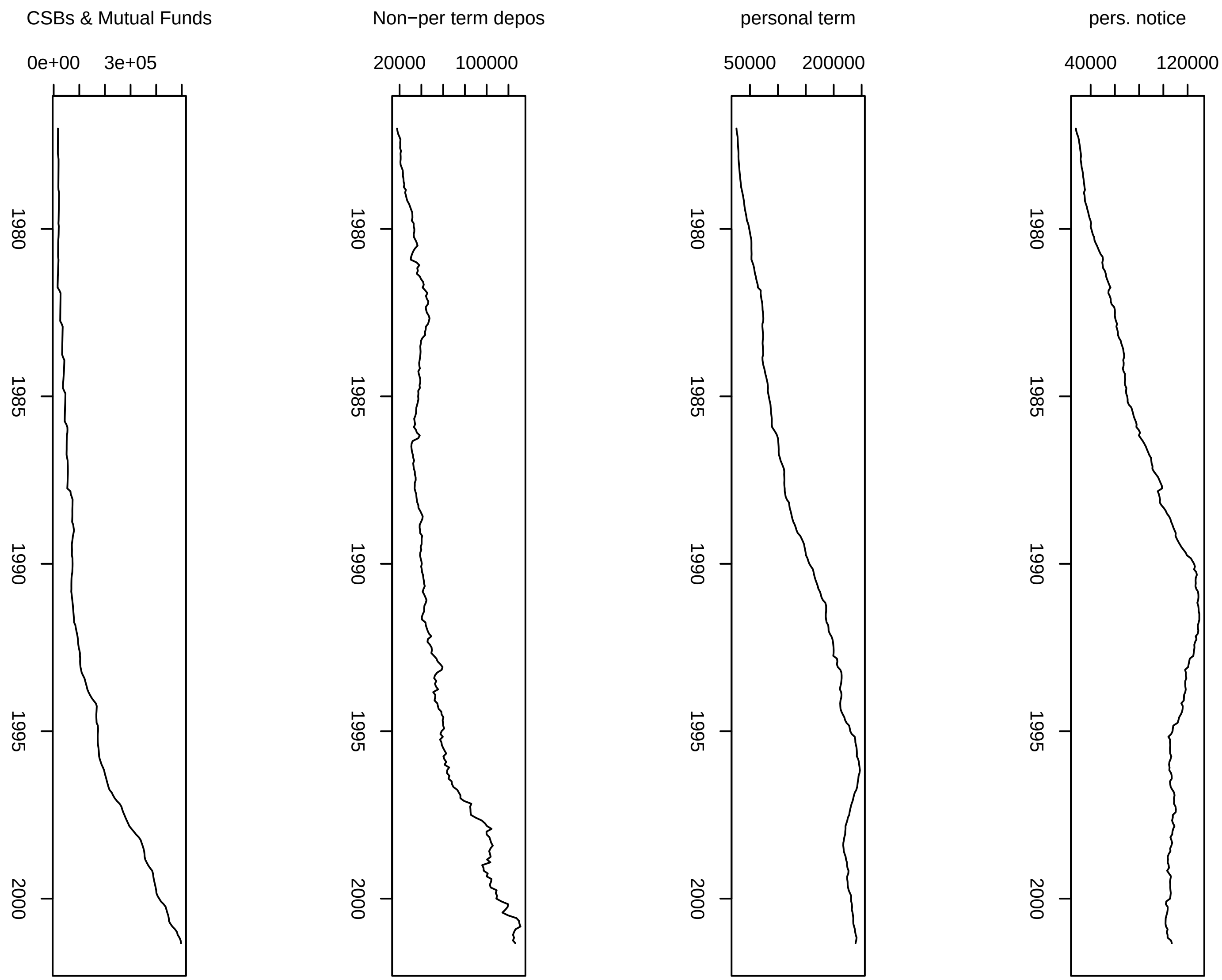
- The sample used here begins only in January 1977 because some components begin then. However, many of the components begin in 1968 and some even earlier. One of the advantages of the proposed measure over aggregation techniques is that it should be possible to extend it in a consistent way, even when some of the sampled deposit categories change. For simplicity, this extension has not been considered yet.

Figure 3 shows the component series data. One feature of the data is that a certain portion of the growth can be explained by population growth. This would be common to both the savings and the transaction factors, but in estimation the effect on both may tend to be accumulated in a single factor. Therefore, at least for estimation purposes, calculations have been done with per capita data.

Over the sample period our knowledge of the Canadian economy suggests we would expect that savings and transaction money have grown most of the time, with a possibility of some levelling off and possibly some short periods of contraction. The decline in the series "personal notice" and to some extent "personal term," as shown in Figure 3, cannot be explained by these factors. We know that some deposit types included in this "personal notice" series have been largely abandoned for deposits with more attractive features. Deposits have shifted into all purpose checking accounts which offer comparable interest. Since the data is a weighting of the factors in the model (2) it is relatively important, at least during estimation, that the data does not have features clearly at odds with all the factors we are trying to measure. Once weights are estimated then anomalies, such as those caused by shifts between deposit categories, will be evident in the idiosyncratic component. However, during estimation the procedures may try to find factors to explain these features rather than the transactions and savings phenomena of interest. These shifts between categories are difficult to deal with and some accommodation for them does need to be done before the estimation will work properly.

Figure 3. Components of Canadian Monetary Aggregates





CSBs & Mutual Funds
0e+00
3e+05
Non−per term depos
20000
100000
personal term
50000
200000
pers. notice
40000
120000
1980
1985
1990
1995
2000

Figure 4 shows the estimated factors and Figure 5 the component data with the dashed line indicating the portion explained by the two factors. The estimation was done minimizing an objective function defined by adding together two objectives. The first was the sum of the squares of the elements of the covariance of the idiosyncratic component and the second is the sum of the squares of the elements of the covariance between the idiosyncratic component and the factors. There was no roughness penalty in this example. The factors and weights were constrained to be positive. Currency and personal term deposits were used as the basis for the initial starting values of the two factors in the iterative estimation (but this choice should be of relatively little importance other than to speed convergence). The covariance matrix of the idiosyncratic component has some fairly large elements, so this part of the specified objective has not been obtained as well as one might expect. The relative importance of the diagonal and off-diagonal parts of the covariance of the idiosyncratic component requires more consideration. As mentioned previously, the best combination of objectives and constraints is the subject of ongoing research. Some experimentation has also been done using an EM algorithm, but simultaneous optimization of weights and factors has been more successful.

Figure 4. Estimated Factors.

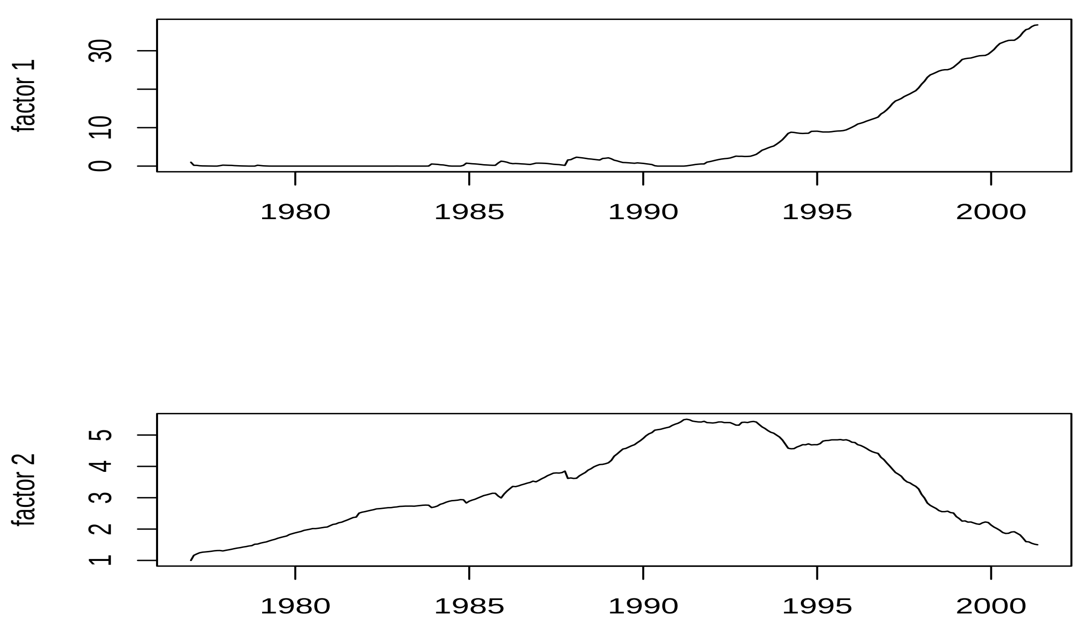
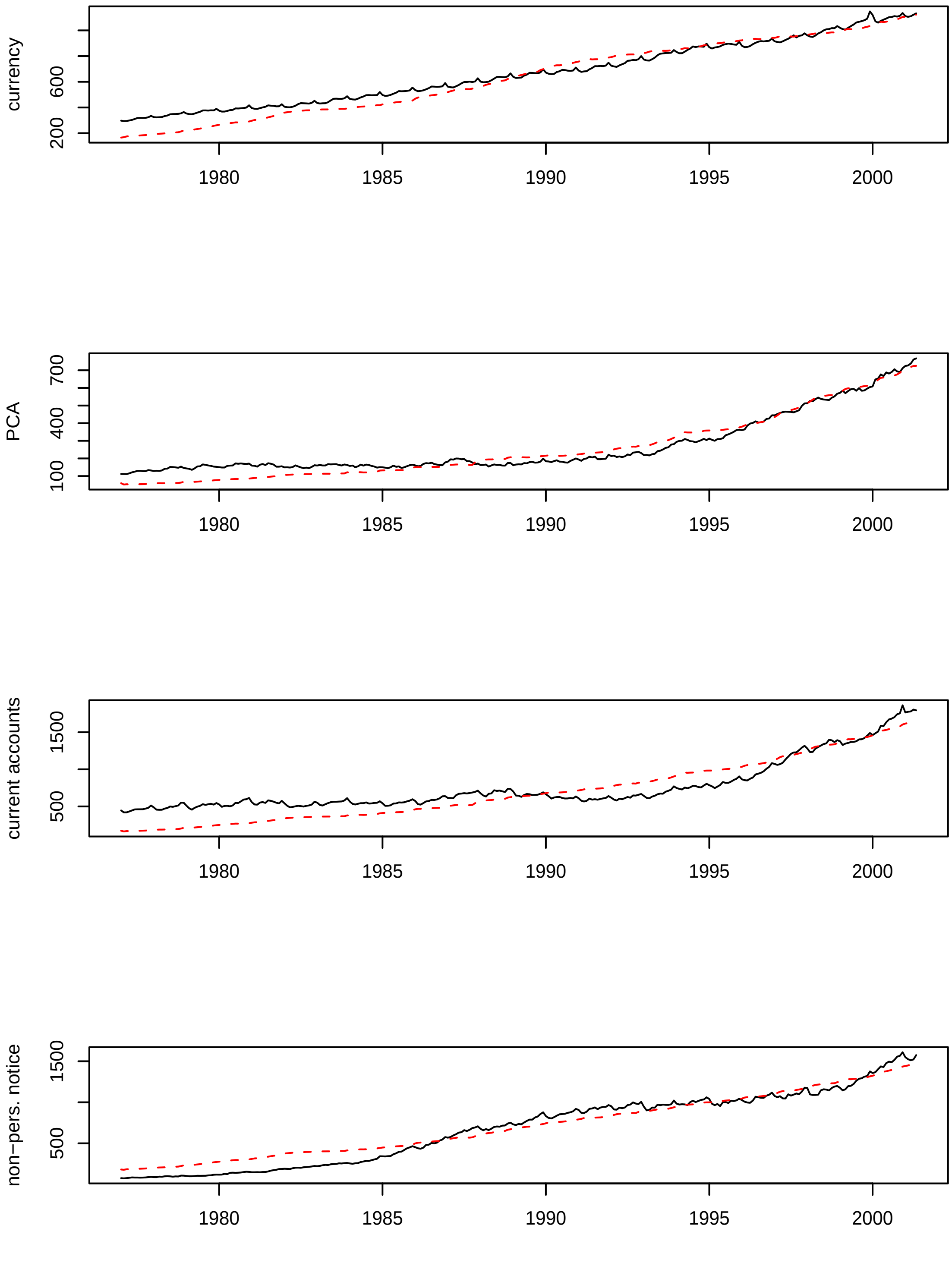
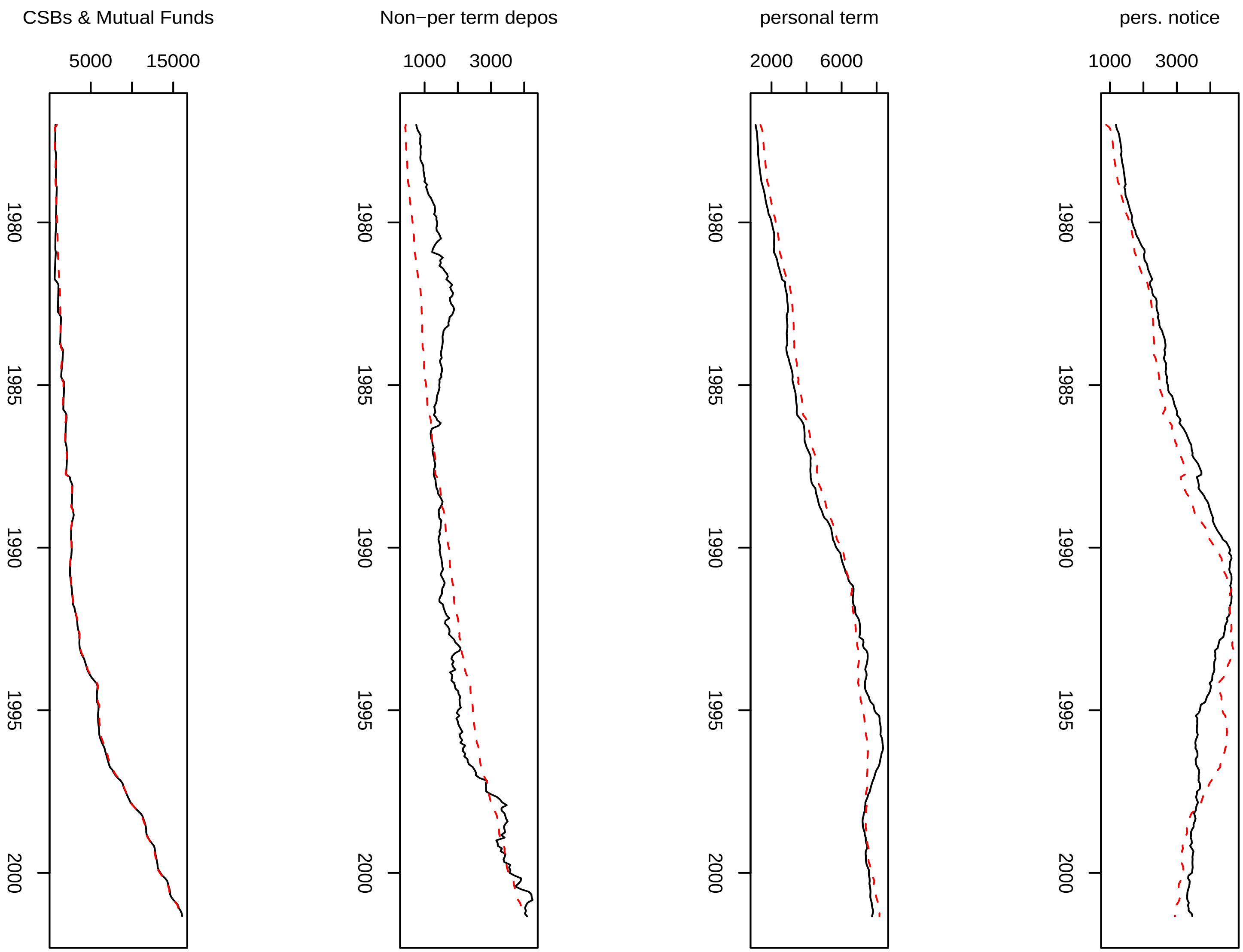


Figure 5. Per capita components (solid) and portion explained by factors (dashed)





CSBs & Mutual Funds
5000
15000
Non−per term depos
1000
3000
personal term
2000
6000
pers. notice
1000
3000
1980
1985
1990
1995
2000

The estimated weights are shown in Table 2. While initial conditions have some influence, the estimation procedure does not guarantee which factor should be interpreted as the transaction money component. That can be discovered from the estimated weights on components such as currency, which we expect to be more heavily weighted on the transaction component. Focusing on currency, it seems likely that the second factor is transactions. However, the weighting is also fairly heavy on the second factor for deposits normally associated with savings.

At this point, we have to keep in mind that these results are very preliminary. Other disturbances in the data during initial estimation can prevent the factors from capturing the economic phenomena. There is a decline in certain deposit types, possibly due among other things to a phasing out of these deposit types by the banks. As mentioned earlier, it is important that this type of structural break be accommodated in some way during estimation so that the estimated factors are not trying to explain the change in structure. Also, we impose only two factors, but three or four may be necessary. Further tests will be done to determine the location of structural breaks and the number of factors.

Finally, the constraints and penalties in the estimation are not yet very refined and it is likely that they do not yet restrict the estimated factors sufficiently to impose the interpretation we would like. This is the subject of ongoing work.

Table 2: Factor Loadings

component	factor1	factor2
currency	24.81976	140.85855
personal chequing accounts	18.05468	41.47066
current accounts	39.55737	133.09486
non-personal notice	34.01550	147.22757
personal notice	45.86092	840.47074
personal term	173.34623	1194.59430
Non-personal term deposits	94.60341	339.61453
Mutual Funds & Canadian Savings bonds	415.91869	485.70514

It is also important to understand that the saving process, in the present work, is very narrow and not a measure of aggregate saving in the economy. *Savings*, in the context of this paper, is only money. The broader concept of saving (as defined in the National Accounts, for example) includes other financial assets such as stocks, bonds,[15] and also real assets, accumulated by households and firms. By including mutual funds we have included one portion of savings which expanded rapidly in the 1990's. It seems clear from the relative weights that the first factor is trying to explain this phenomena, but this is not a complete picture of savings portfolios.

In terms of monetary policy, we are particularly interested in extracting the transaction money process. We think that this index will be a purer measure of the underlying phenomena and a better indicator for output and inflation forecasts than current monetary aggregates.

15. A small portion of these assets will be included in our estimation, because a certain quantity of stocks and bonds is held through mutual funds.

4. Conclusions

If this approach to measuring transaction and savings money proves successful it would be the most fundamental reformulation in the way money is measured since the introduction of monetary aggregates a half century ago. The results presented here are preliminary. The conceptual formulation is intriguing, both statistically and economically very interesting, and preliminary indications are that the method can work. However, many technical and practical problems still need to be overcome.

The next steps are to refine the estimation procedure, objective function and constraints. The procedures need to be robust, or at least any sensitivities well understood. The estimation objectives and constraints need to make sense in the context of different economic theories. Anomalies in the data which can be attributed to other causes may need to be accommodated in some way; if they are too important they, rather than the phenomena of interest, will be estimated as the factors.

Eventually there will be efforts to validate the new measures. The validation should not rely too heavily on specific economic theories. Possibilities include comparisons with current aggregates, comparisons of the weights with prior information about deposit type usage, and comparison of break points and corresponding weight changes with known structural changes. The ultimate test, at least from a policy perspective, is whether the estimated measures provide better information for forecasting inflation and output.

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