The formation of a core periphery structure in heterogeneous financial networks

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joint with
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Views expressed are those of the authors and do not necessarily represent those of DNB.

CEMLA, Mexico City, 10 December 2014
Introduction

In the last decade great advances have been made on the role of the network of interbank exposures, i.e. the financial network, on financial stability.

- Theoretical and simulation analysis of financial contagion (Allen & Gale, 2000; Gai & Kapadia, 2010; and many others)
- Stress tests on empirically derived network structures (Upper, 2011; Solorzano et al., 2013)
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In these analyses it is assumed that the financial network is exogenously fixed. However, trading partners are consciously chosen by profit-maximizing banks. Hence the network structure changes over time, depending on the global financial circumstances and the incentives that banks have to create or delete links.
Introduction

It is important to obtain a better understanding on the factors driving the formation of financial networks, in particular

- What are the incentives (costs and benefits) for banks to form or sever links?
- How does the financial network structure change if incentives or market circumstances change?

Answering these questions will help us better understand:
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Answering these questions will help us better understand:

- What is the risk of systemic liquidity hoarding?
- What are the costs and benefits for banks of financial regulation, such as limiting large exposures?
- How does financial regulation affect the structure of the financial network and its financial stability?
- How does optimal financial regulation look like, if we take into account the effect on network structure?
Introduction

In this paper, we perform a theoretical network formation analysis of an interbank market in order to understand the formation of a core-periphery network structure in financial networks.
Example of a core-periphery network
It turns out that many financial networks have a core-periphery network structure

- Germany: Craig & Von Peter (2014)
- eMID interbank market: Fricke & Lux (2012)
- The Netherlands: Van Lelyveld & In ’t Veld (2014)
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- Germany: Craig & Von Peter (2014)
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- The Netherlands: Van Lelyveld & In ’t Veld (2014)
- After the crisis the fit became less.

We try to understand why this is the case.
Introduction

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- Trade opportunities can be realized directly or indirectly through an intermediator (broker)
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- Trade opportunities can be realized directly or indirectly through an intermediator (broker)
  - Most efficient network is to all directly or all trade indirectly through a single intermediator
- Imperfect competition between intermediators
  - The more intermediators, the lower the intermediation benefits
- Free entry of intermediators
Main Results

In this trade intermediation model we show that:

- If agents are *ex-ante homogenous*, then a core-periphery network is typically *not* an equilibrium outcome.

- A core-periphery network structure is an equilibrium outcome, if the banks in the core are bigger, that is, have more (or more valuable) trade opportunities than banks in the periphery.
Related Literature

- Network formation theory (Jackson & Wolinsky, 1996; Bala & Goyal, 2000; see Goyal, 2007, for textbook review)
- Network formation of financial networks (Babus, 2010; Farboodi, 2014)
- Network formation, intermediation benefits and structural holes (Burt, 1994; Kleinberg et al., 2007; Goyal & Vega-Redondo, 2007; Buskens & Van de Rijt, 2008)
- Networks and trade (Kranton & Minehart, 2001; Coraminas-Bosch, 2004; Blume et al. 2009)
- Intermediation and bargaining on networks (Siedlarek, 2011; Manea, 2012; Gofman, 2011)
- Network formation and core-periphery networks (Galeotti & Goyal, 2010; Persitz, 2012)
Trade and distribution

Consider an undirected network $g$ between $n$ agents. $g_{ij} = g_{ji} = 1$ denotes a link (trading relationship), and $g_{ij} = g_{ji} = 0$ the absence of a link.

Payoffs of agents are determined as follows:
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Payoffs of agents are determined as follows:

- Suppose $i$ has a liquidity shortage, and at the same time $j$ a liquidity surplus.
- $i$ and $j$ have an opportunity to trade liquidity ($i$ borrows from $j$), which would generate a total surplus of 1 to $i$ and $j$ (e.g. the wedge between deposit and lending rate of central bank)
Trade and distribution

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- if $i$ and $j$ are directly connected, then they each receive half of the surplus
- if $i$ and $j$ are indirectly connected by $m_{ij}$ intermediators, then $i$, $j$ and the intermediators share the trade surplus
  - $i$ and $j$ each receive $f_e(m_{ij}, \delta)$
  - each of the $m_{ij}$ intermediators receive $f_m(m_{ij}, \delta)$
Trade and distribution

Consider intermediated trade for some pair \((i,j)\).
We assume

- if there is one intermediator, then the three parties split the surplus evenly

\[ f_e(1, \delta) = f_m(1, \delta) = \frac{1}{3}. \]
Trade and distribution

Trade between $i$ and $j$: $m = 3$, $\delta = 0$
Trade and distribution

Trade between $i$ and $j$: $m = 3, \delta = 3/5$
Trade and distribution

Trade between $i$ and $j$: $m = 3$, $\delta = 1$
Payoff function

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- Trading relationships are mutual and maintaining such a relationship involves a cost \( c \) for both partners.

Hence, the payoff function, \( \pi_i(g) \), for an agent \( i \) is:
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Hence, the payoff function, $\pi_i(g)$, for an agent $i$ is:

$$\pi_i(g) = \sum_{j \in N_i^1} \left( \frac{1}{2} \alpha_{ij} - c \right) + \sum_{j \in N_i^2} \alpha_{ij} f_e(m_{ij}, \delta) + \sum_{k,l \in N_i^1 | g_{kl}=0} \alpha_{kl} f_m(m_{kl}, \delta)$$

direct trade  indirect trade  brokerage benefits

where $N_i^r$ denotes the set of nodes at distance $r \geq 1$ from $i$ in $g$ and $n_i^r = |N_i^r|$ its size.
Interpretation in interbank market

- Links are long-term (preferred) trade relationships (Cocco et al., 2009; Fecht & Braüning, 2013; Afonso et al., 2013). No relationship, no (direct) trade
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- Links are long-term (preferred) trade relationships (Cocco et al., 2009; Fecht & Braüning, 2013; Afonso et al., 2013). No relationship, no (direct) trade.
- The cost involved in maintaining a relationship might be monitoring costs.
- Intermediation benefits result from bargaining process.
  - See Siedlarek (2011) for explicit process, such that

\[
f_e(m, \delta) = \frac{m - \delta}{m(3 - \delta) - 2\delta} \quad \text{and} \quad f_m(m, \delta) = \frac{1 - \delta}{m(3 - \delta) - 2\delta}
\]
Unilateral stability

Network formation analysis requires a concept of a network being in *equilibrium*. We consider *unilateral stability* (adapted version of the concept introduced by Buskens & Van de Rijt, 2008)
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- A network $g$ is *unilaterally stable* if there is no agent $i$, such that simultaneously
  - agent $i$ strictly benefits from severing (some of) its existing links or proposing new links involving $i$
  - none of the agents to which $i$ proposes a new link is worse off
Unilateral stability

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Unilateral stability allows for entry of intermediators

- a network is unilaterally unstable if an agent has an incentive to create many links to other agents to become a broker.
A star network

\[ \delta = 3/5, \, c = 1/6. \]

Is a star network unilaterally stable?
A complete core-periphery network

\[ \delta = \frac{3}{5}, \ c = \frac{1}{6}. \]

As \( \frac{7}{3} > \frac{5}{3} \) and \( \frac{11}{6} > \frac{5}{3} \), star is not unilaterally stable.
Core-Periphery network

Definition
A network $g$ is a core-periphery network $g^{CP}$ if there is a set of agents $K \subset N$ with $k = |K| : 2 \leq k \leq n - 2$, such that

- the core agents $K$ form a completely connected clique
  - $\forall i, j \in K, i \neq j : g_{ij} = 1$
- there are no links between periphery agents $N \setminus K$
  - $\forall i, j \in N \setminus K : g_{ij} = 0$
- each core agent is connected to at least one periphery agent and vice versa
  - $\forall i \in K \exists j \in N \setminus K : g_{ij} = 1$ and $\forall j \in N \setminus K \exists i \in K : g_{ij} = 1$
Complete core-periphery network

Definition

A core-periphery network $g^{CP}$ is a complete core-periphery network if in addition
- every core agent is linked to all periphery agents
  - $\forall i \in K, j \in N \setminus K : g_{ij} = 1$
Example of a core-periphery network
Example of a complete core-periphery network
Core-periphery network with homogeneous agents

Consider first the case of *homogeneous agents*

- trade surplus is 1: $\forall i, j, i \neq j : \alpha_{ij} = 1$

**Theorem**

*For any $c$ and $\delta$, any complete core-periphery network of size $n$ and core size $k$ with $2 \leq k \leq n - 2$ is not unilaterally stable.*
Suppose a complete core-periphery network is unilaterally stable.
It is better for agents 3 and 4 to trade indirectly through the 2 core agents.
But then it should be better for agents 1 and 2 to trade indirectly through 4 agents.
Incomplete core-periphery networks

Can incomplete core-periphery networks be unilaterally stable?
Incomplete core-periphery networks

Can incomplete core-periphery networks be unilaterally stable? Yes, in special cases. However, if $n$ is large enough, then a core-periphery network becomes unilaterally unstable.

**Theorem**

*For any $c$, $\delta$ and $k$, there exists a $\bar{n}$, such that any (complete or incomplete) core-periphery network of size $n > \bar{n}$ and core size $k$ is not unilaterally stable.*

Intuition: **intermediation benefits** of being part of the core *increase quadratically* with $n$, whereas **linking costs increase linearly*. Hence, for large $n$ intermediation benefits exceed linking costs, and peripheral banks have incentive to become a core bank.
Stable networks

If the core-periphery network is not (unilaterally) stable, what kind of networks are?

We consider a dynamic process:

- Initial network is the empty network
- Round-robin best feasible action dynamics (Kleinberg et al., 2008)
- This process always converges for any \( c \) and \( \delta \), and the resulting network is unilaterally stable
- There may be more unilateral stable networks to which the network does not converge to
Absorbing states dynamic process. $n = 8$. 

![Diagram showing different network structures: Complete network, Multipartite networks, Star, Empty network. The graph plots $c$ against $\delta$. The x-axis ranges from 0 to 1, and the y-axis ranges from 0 to 1.6. The graph includes points for Homogeneous agents and Heterogeneous agents.]
Example of convergence to a multipartite network

\[ \delta = \frac{3}{5}, \ c = \frac{1}{6}. \]

The dynamics lead to a core of \( k = 2 \), resulting in a stable bipartite network with groups 1, 2 and 3, 4, 5, 6.
Extensions

Why are core-periphery networks unstable in the framework above? What additional assumption are necessary to explain the existence of core-periphery networks?
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Why are core-periphery networks unstable in the framework above? What additional assumption are necessary to explain the existence of core-periphery networks?

- Instability follows from homogeneity banks
  - If banks in the periphery want to trade indirectly through core banks, then banks in the core want to trade indirectly through periphery banks as well!
Heterogenous agents

Consider now a case in which banks are heterogeneous

- Two types of banks: $k$ big banks and $n - k$ small banks
- Big banks generate more trading opportunities $\alpha_{ij}$:

\[
\begin{array}{c|cc}
\alpha_{ij} & L & S \\
\hline
L & \alpha^2 & \alpha \\
S & \alpha & 1
\end{array}
\]

with some $\alpha > 1$. 
Theorem

Suppose

\[ c \geq \frac{1}{2} - f_e(k, \delta) + (n-k-2) \min\left\{ \frac{1}{2} f_m(k+1, \delta), f_e(k+1, \delta) - f_e(k, \delta) \right\}. \]

Then, for any \( n, k \) and \( \delta \), there exists an \( \bar{\alpha} > 0 \), such that for all \( \alpha > \bar{\alpha} \) the complete core-periphery network with \( k \) big banks in the core is unilaterally stable.

Intuition:

- if \( \alpha \) is very large, then it is more attractive for core banks to trade directly with each other.
Dynamic process with heterogenous agents

Do core-periphery networks also arise as part of a dynamic process? We consider same dynamic process as before:

- 4 banks: 2 big, 2 small
- Initial network is the empty network
- Round-robin best feasible action dynamics (Kleinberg et al., 2008)
- Big banks are first in queue to decide, then small banks
Absorbing states dynamic process. $n = 4$, $k = 2$, $\alpha = 1.5$. 
Absorbing states dynamic process. $n = 4$, $k = 2$, $\alpha = 2$. 
A calibration to the Dutch Interbank market

We calibrate $n$, $k$ and $\alpha$ to the Dutch interbank market

- Analysis Dutch interbank market in Van Lelyveld & In ’t Veld (2013)
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A calibration to the Dutch Interbank market

We calibrate $n$, $k$ and $\alpha$ to the Dutch interbank market

- Analysis Dutch interbank market in Van Lelyveld & In ’t Veld (2013)
- Around $n = 100$ banks
- Core-periphery structure with around $k = 15$ banks in the core
- Large heterogeneity in asset size. We consider $\alpha = 10$.
- We look at the absorbing states of the dynamic process
Absorbing states dynamic process. $n = 100$, $k = 15$, $\alpha = 10$. 

\[ \text{Complete network} \]

\[ \text{Core periphery network} \]

\[ \text{Other networks} \]
Conclusions

In this paper we ask ourselves: why do financial networks have a core-periphery structure? We focus on the role of intermediation benefits.

We find that:

- A core-periphery network is not stable if agents are homogenous.
- If there is enough heterogeneity in trade surplus, then a core-periphery network with big banks in the core can be stable.
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We find that:

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- If there is enough heterogeneity in trade surplus, then a core-periphery network with big banks in the core can be stable

This result suggests that we cannot abstract away *heterogeneity* in banking size if we want to understand financial networks.
Future research

In future research we would like to

- Endogenize heterogeneity in trade surplus.
- Introduce default probabilities in order to understand the role of network formation on systemic risk and financial stability.