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Output Gap and Neutral Interest Measures for Colombia

Abstract

Three new measures of the Colombian output gap and the real neutral interest rate are proposed. Instead of relying only on statistical filters, the proposed measures use semi-structural New Keynesian models,

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adapted for a small open economy. The output gap measures presented are in line with previous works for Colombia and capture all the turning points of the Colombian business cycle, as measured by Alfonso et al., 2011. They are also strongly correlated with inflation and precede its movements along the sample. The neutral interest rate computed indicates that the monetary policy stance has been overall countercyclical, but has failed to anticipate the output gap's movements, or at least react strongly enough to them.

Keywords: output gap, New Keynesian model, neutral interest rate.

JEL Classification: E23, E32, E43.

1. INTRODUCTION

The conduct of monetary policy requires information on the current state of the economy and a measure of the monetary stance. This information is crucial for policy makers but is by nature unobservable, and thus subject to great uncertainty, implying the need for methodologies capable to account for both things (Taylor, 1999, and Woodford, 2003a). This document uses semi-structural New Keynesian models to obtain such information for the Colombian economy in the 1994-2011 period.

The state of the economy is summarized in the output gap, defined as the difference between observed and potential output, the latter understood as the level of economic activity in absence of inflationary pressures. The output gap is therefore an indicator of inflation pressures and the dynamics of the aggregate demand.

The monetary policy stance is measured by the difference between the real interest rate and the neutral interest rate (Blinder, 1999), defined as an interest rate level at which the monetary authority exerts no influence over the behavior of the aggregate demand, in other words: "Any higher real interest rate constitutes *tight money* and will eventually imply falling inflation; and any lower real rate is *easy money* and signals eventually rising inflation" (Blinder, 1999, pp. 33). Note that the neutral rate is not equal to the natural rate of interest, for the latter is "the real rate of interest required to keep aggregate demand equal at all times to the natural rate of output" (Woodford, 2003a, p. 248). The natural rate is interpreted as a desirable level for the real interest rate, whereas the neutral rate only indicates the effect of the real interest rate over the output gap.

The output gap and the neutral interest rate must be inferred from the macroeconomic information available. For the output gap, the techniques to do so rely on the use of statistical tools such as filters, VARs, factor models, among others, that allow the decomposition of output in its trend component (associated with the potential output) and its cyclical component (associated with the output gap).[1] The neutral interest rate is more difficult to extract because its value is not necessarily related to a trend or smooth component of the real interest rate, moreover, this last variable is also unobservable, for it depends on the agents' inflation expectations.

In order to jointly estimate the desired variables it is necessary to account for the structural relations between them and other variables as the inflation rate, as well as variables that affect a small open economy, as the real exchange rate, the foreign interest rate, etc. Because of this, we expand a statistical model, the local linear trend model, with a New Keynesian model adapted for a small open economy. Three alternative specifications of the model, which differ in the way expectations are formed, are considered. This is done in order to present different measures of the output gap and the monetary stance while recognizing the lack of consensus in the literature over the way in which expectations should be modeled.[2] It must be noted that, although the use of several models helps to account for

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[1] Most of these techniques imply unwanted results over the relations of output's permanent and transitory component, making them completely correlated or orthogonal, depending on the method (Canova, 2007, Ch. 3).

[2] For example Rudebusch and Svensson (1999) argue for the use of backward looking expectations, Woodford (2003b) for forward looking expectations and Galí and Gertler (1999) for hybrid expectations.

the variability in the measurement due to model specification, it is not intended to address (nor solve) the inherent model uncertainty to which the measurement of this variables is subject (Orphanides and Williams, 2002).

In the first specification of the model, agents are assumed to follow predetermined rules when forming expectations. These rules are a function of current and lagged values of the variable over which the expectation is formed. In this way the model has a direct state space representation and the output gap can be extracted by means of the Kalman filter. In the second and third specifications, agents are assumed to have rational expectations about the future, taking into account all information available. In order to extract the output gap, the solution to the rational expectations equilibrium of the models has to be computed, and then the state space representation can be formulated.

The approach taken here is similar to the previous work of Echavarría et al. (2007) and Berg et al. (2006), and seeks to complement a literature already existing for Colombia, noting the works of González et al. (2011), Torres (2007), Rodríguez et al. (2006), Gómez and Julio (1998) and Cobo (2004) among many others. It is also closely related to various articles that seek to jointly estimate the dynamics of the output gap and the natural interest rate. This is the case of Laubach and Williams (2003), Garnier and Wilhelmsen (2009), Mesonnier and Renne (2007) and Castillo et al. (2006).

The description of the models is covered in Section 2. The models are estimated with Colombian data, this is described in Sections 3 and 4. Afterward they are used to extract the output gap measures for Colombia, this is discussed in Section 5. Finally, results for neutral interest rate estimates are presented in Section 6.

2. MODELS

Three models are used to extract information about the output gap and the neutral interest rate for Colombia in the 1994-2011 period. All models are built on top of a local linear trend model, introducing the neutral interest rate, and a more elaborate definition of the output gap, using a semi-structural new-Keynesian model for a small open economy. The models differ in the way expectations are defined. One of the models has backward looking expectations, another has forward looking expectations and in the last one expectations are formed in a hybrid manner, taking into account both past and future values of the variables.

The motivation for these models is twofold. First, they give economic structure to the output gap, and introduce the notion of a neutral interest rate, as opposed to the use of a purely statistical model. This allows to extract information from series other than the GDP when computing the output gap, and infer the dynamics of the neutral rate. This same strategy was used by González et al.(2011) for computing a measure of the Colombian natural interest rate, showing the differences between purely statistical and macroeconomic models. Second, since there is some degree of uncertainty regarding the mechanisms by which agents form inflation expectations we consider necessary to present different measures of the output gap and the neutral interest rate using different approaches to agents expectations in line with previous literature on New Keynesian models.

The remaining of this Section presents the main features of each model, Appendix A contains complete set of equations.

2.1 Local Linear Trend Model

The local linear model will be used as a base for building the more elaborate macroeconomic models that are shown below. It is a purely statistical model that decomposes output (y) into a trend component with an stochastic drift $(\overline{y}_t)$ and the output gap $(\tilde{y}_t)$.

The output gap is given by:

1 $$\tilde{y}_t = y_t - \overline{y}_t \cdot$$

The output trend component is assumed to follow a random walk with a stochastic drift:

$$\overline{y}_t = \overline{y}_{t-1} + g_t + \varepsilon_t^{\overline{y}} . \quad (2)$$

The drift $\left(g_t\right)$ is the growth rate of the trend component of output and is given by:

$$g_t = \left(1-\tau\right)\overline{g}_{ss} + \tau g_{t-1} + \varepsilon_t^{g} , \quad (3)$$

both $\varepsilon_t^{\overline{y}}$ and ε_t^{g} are i.i.d. Gaussian disturbances. The shocks' variances $\left(\sigma_{\overline{y}}^2, \sigma_g^2\right)$ and τ are parameters to be estimated.

Note that $\varepsilon_t^{\overline{y}}$ and ε_t^{g} account for permanent shocks to the level of potential output, providing an explanation for movements in that variable. This feature allows to use data on the GDP level when estimating the output gap. However, the local linear model does not give any economic structure to the output gap, and does not include other variables, also relevant for monetary policy. Because of that, this model is complemented with economic structural relations as described in the models below.

2.2 Backward Looking Semi-structural Model

The model consists in equations 1, 2, and 3, an IS curve, a Phillips curve, an uncovered interest parity (UIP) condition, and equations for the dynamics of the real interest rate and the real exchange rate.

The IS curve is given by:

$$\tilde{y}_t = \beta_1 \tilde{y}_{t-1} - \beta_2\left(r_{t-1} - \overline{r}_{t-1}\right) + \beta_3 \tilde{q}_{t-1} + z_t^{y} . \quad (4)$$

According to this representation, the output gap depends on its past value, the real interest rate gap (being $\overline{r}_t$ the neutral rate of interest), the real exchange rate gap $\left(\tilde{q}_t\right)$ and an exogenous variable z_t^{y} that stands for the effects of demand shocks $\left(\varepsilon_t^{y}\right)$ in the is curve. z_t^{y} is assumed to follow an AR(1) process:

5 $$z_t^y = \rho_y z_{t-1}^y + \varepsilon_t^y .$$

Note that when the real interest rate r_t is equal to $\overline{r_t}$ the term of the is curve involving the real interest rate is canceled, thus eliminating the effect of the real interest rate over the output gap. This is why the variable $\overline{r_t}$ is taken as the neutral interest rate.

The Phillips curve for the annualized quarterly inflation rate, is given by:

6 $$\pi_t = \pi_{t+1|t}^e + \lambda_2 \tilde{y}_{t-1} + \lambda_3 \left(q_t - q_{t-1}\right) + z_t^\pi ,$$

where $\pi_{t+1|t}^e$ denotes the period t expectations over period $t+1$ inflation, q_t is the real exchange rate level, and z_t^π is an exogenous variable that stands for the effects of supply shocks $\left(\varepsilon_t^\pi\right)$ over the Phillips curve. As before, z_t^π is assumed to follow an AR(1) process:

7 $$z_t^\pi = \rho_\pi z_{t-1}^\pi + \varepsilon_t^\pi .$$

Inflation expectations are defined as an average between the inflation target $\left(\overline{\pi}\right)$ and lagged annual inflation $\left(\pi_{4,\,t-1}\right)$, this is:

8 $$\pi_{t+1|t}^e = \lambda_1 \overline{\pi} + \left(1 - \lambda_1\right) \pi_{4,\,t-1} ,$$

as for the annual inflation $\left(\pi_{4,t}\right)$ it follows from the definition of π_t that:

9 $$\pi_{4,t} = \frac{1}{4}\left(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}\right) ,$$

where π_t stands for the period to period change in prices.

The model is complemented by three sets of equations characterizing the dynamics of the real interest rate, the foreign real interest rate and the real exchange rate.

The real interest rate must satisfy two equations. The Fisher equation 10, and an uncovered interest parity condition 11:

10 $$r_t = i_t - \pi^e_{t+1|t},$$

11 $$r_t - r^*_t = \left(\overline{r}_t - \overline{r}^*_t\right) + 4\left(q^e_{t+1|t} - q_t\right) + \varepsilon^r_t,$$

where r^*_t is the foreign real interest rate, $\overline{r}^*_t$ its neutral value at period t, and $q^e_{t+1|t}$ is the one period ahead expected value of the real exchange rate. ε^r_t is a shock that affects the UIP.

The neutral interest rate is assumed to follow an AR(1) process, this means that it is an exogenous factor for the model, nevertheless its value can be extracted from the model, since the relation between the neutral rate and other variables is well defined by IS curve 4, and the UIP condition 11. Since all equations operate simultaneously in the equilibrium, the value of the neutral rate depends implicitly on the foreign interest rate, the real exchange rate and the overall state of the economy.[3]

12 $$\overline{r}_t = \rho_r \overline{r}_{t-1} + \left(1 - \rho_r\right)\overline{r}_{ss} + \varepsilon^{\overline{r}}_t.$$

The real exchange rate gap is defined as the difference between its realized and its trend value:

13 $$\tilde{q}_t = q_t - \overline{q}_t,$$

its trend is assumed to follow a random walk:

14 $$\overline{q}_t = \overline{q}_{t-1} + \varepsilon^q_t$$

and the expected real exchange rate is assumed to be an average between the trend, and the lagged value of the exchange rate:

[3] The relation between the neutral interest rate and the potential output's growth rate $\left(g_t\right)$ is not included explicitly, as is done by Laubach and Williams (2003), Mesonnier and Renne (2007) and Echavarría et al. (2007). Nevertheless, an extra exercise was carried out modifying the definition of the neutral interest rate. The potential output's growth rate recovered was very stable and implied little changes over the neutral rate, with respect to the results presented in Section 5.

15 $$q^{e}_{t+1|t} = \varphi \bar{q}_t + (1-\varphi) q_{t-1} .$$

Finally, the nominal interest rate responds to a contemporaneous Taylor rule,[4] the rule's intercept is given by the neutral interest rate plus the inflation target, following Taylor(1993) and Woodford (2003a) and it is assumed that the foreign neutral interest rate and the foreign interest rate gap evolve exogenously following AR(1) processes:

16 $$i_t = \gamma_1 i_{t-1} + (1-\gamma_1)\left((\bar{r}_t + \bar{\pi}) + \gamma_2 (\pi_{4,t} - \bar{\pi}) + \gamma_3 \tilde{y}_t\right) + \varepsilon^{i}_t ,$$

17 $$\bar{r}^{*}_t = \rho_{r^*} \bar{r}^{*}_{t-1} + (1-\rho_{r^*}) \bar{r}^{*}_{ss} + \varepsilon^{\bar{r}^*}_t ,$$

18 $$r^{*}_t - \bar{r}^{*}_t = \kappa \left(r^{*}_{t-1} - \bar{r}^{*}_{t-1}\right) + \varepsilon^{r^*}_t .$$

All ε^{j} variables, with $j \in \{y, \pi, r, q, \bar{r}, \bar{r}^*, r^*\}$, are assumed to be i.i.d. Gaussian disturbances with mean zero and constant variance.

2.3 Hybrid Semi-structural Model

The second model is built on top of the adaptive expectations model and differs from it in the way inflation and real exchange rate expectations are formed, and in the dynamics of the nominal interest rate, for which it is now possible to assume a forward looking Taylor rule. Additionally a forward looking component is introduced into the is curve.

The IS curve 4 is modified and is given by:

19 $$\tilde{y}_t = \beta_1 \tilde{y}_{t-1} - \beta_2 (r_{t-1} - \bar{r}_{t-1}) + \beta_3 \tilde{q}_{t-1} + \beta_4 E_t \{\tilde{y}_{t+1}\} + z^{y}_t .$$

[4] As in Laubach and Williams (2003) and Mesonnier and Renne (2007) the equilibrium is well defined in the absence of a Taylor rule, and the nominal interest rate can be taken as an exogenous variable. The Taylor rule is included for comparison with the rational expectations models, where it plays a crucial role for equilibrium determinacy (see Taylor, 1999), and Woodford, 2003a).

Inflation expectations (Equation 8) are also modified and are now given by the average between expected and lagged annual inflation:

$$\pi^{e}_{t+1|t} = \lambda_1 E_t \left\{ \pi_{4,t+4} \right\} + \left(1-\lambda_1\right) \pi_{4,t-1} . \tag{20}$$

Exchange rate expectations formulation is also modified, and is the average between expected and lagged exchange rate. The relative importance of each component is given by the parameter φ. The equation that characterizes the expectations is:

$$q^{e}_{t|t+1} = \varphi E_t \left\{ q_{t+1} \right\} + \left(1-\varphi\right) q_{t-1} . \tag{21}$$

The Fisher equation 10 is defined in terms of the expected inflation corresponding to rational expectations:

$$r_t = i_t - E_t \left\{ \pi_{t+1} \right\} . \tag{22}$$

Finally, the Taylor rule is modified to include the four-periods expected value of inflation, taking into account the lagged effect of monetary policy:

$$i_t = \gamma_1 i_{t-1} + \left(1-\gamma_1\right)\left(\left(\overline{r}_t + \overline{\pi}\right) + \gamma_2 E_t \left(\pi_{4,t+4} - \overline{\pi}\right) + \gamma_3 \tilde{y}_t \right) + \varepsilon^{i}_t . \tag{23}$$

2.4 Forward Looking Semi-structural Model

The last model can be represented as a special case of the hybrid model, restricted so that the IS curve (Equation 19), the inflation expectations (Equation 20), and the exchange rate expectations (Equation 21) are only forward looking. This means restricting the parameters so that $\beta_1 = 0$, $\lambda_1 = 1$ and $\varphi = 1$.

3. DATA

A set of five macroeconomic variables is used for the estimation and filtering process. All variables are used in quarterly frequency with a sample that ranges from the first quarter of 1994 to the last quarter of 2011, thus the sample has 72 observations.

The series used are the natural logarithm of the seasonally adjusted GDP, total CPI inflation (seasonally adjusted), and the nominal interest rate, taken as the average rate of the 90 days certificate of deposit (CDT). As for foreign variables, the real interest rate is taken as the 90 days certificate of deposit rate for the USA,[5] and the real exchange rate corresponds to the bilateral exchange rate between Colombia and the USA, computed with the average bilateral nominal exchange rate and the CPI indices for both countries (all items included).

Two things are worthwhile mentioning. The first is that, as in Mesonnier and Renne(2007), the real interest rate is computed in-model, in a way consistent with the models' inflation expectations. The second is that the Colombian economy experienced a disinflation period in the 2000s, with a decreasing inflation target. Since the models take the nominal series as stationary, we shall work with the domestic inflation and nominal interest rate series relative to the inflation target; this eliminates the trend from the series and makes them compatible with the models definitions. Two parallel exercises were conducted incorporating a time varying inflation target, assuming AR(1) and random walk dynamics; the results are robust to this changes.

4. PARAMETRIZATION

The parameters are divided in two sets. One is fixed and is composed mainly by those of the steady state, and the other one is to be estimated. The estimation is done by means of Bayesian techniques.

4.1 Fixed Parameters

The parameters that determine the long run values of the variables in the models are fixed according to the characteristics

[5] The real rate is computed ex post with the US CPI inflation; the CPI is seasonally adjusted and all items are included.

of Colombian data. The long run rate of output growth is fixed at 4% in annual terms $\left(\overline{g}_{ss}=0.04\right)$. The inflation target is set at 3% $\left(\overline{\pi}=0.03\right)$ accordingly to the mid point of the long run target band for inflation of the Banco de la República. Since Colombia is a small open economy, its real interest rate is given in the long run by the foreign interest rate, hence the domestic and foreign real interest rates are set to 2.5% in the steady state $\left(\overline{r}_{ss}=\overline{r}_{ss}^{*}=0.025\right)$. This fact along with the absence of drift in the equilibrium exchange rate process imply that there is no depreciation in steady state.

4.2 Estimation

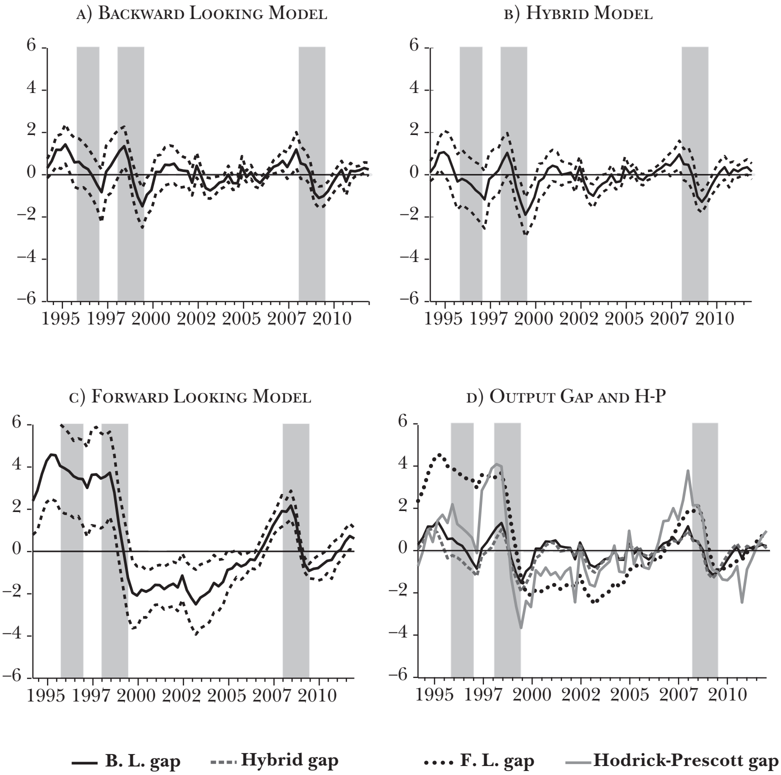
Parameters that are not fixed are estimated by means of Bayesian techniques, combining prior information with the model's likelihood function (computed with the Kalman filter). These techniques have been applied with great success to the estimation of DSGE models in the literature (e.g., Smets and Wouters, 2007), and, as noted in An and Schorfheide(2007), have many advantages when dealing with short time series and identification issues (common in semi-structural models), they also provide a natural benchmark for model comparison (the model's marginal likelihood).

Two chains of 100,000 draws are used when computing the parameters' posterior distributions. There are three types of prior distributions used. For bounded parameters (between zero and one) a beta distribution is used, the mean is set to the mid point of the interval. For unbounded parameters a gamma distribution is used, the mean is set to 0.3 in accordance to previous estimations of semi-structural models. Finally the shocks' variances are all associated with an inverse-gamma prior distribution. Appendix B summarizes the prior distributions used for the estimation of the models.

The results of the estimation procedure for each model are presented in Appendices C, D and E respectively. The estimation was made using the Dynare software (Adjemian et al., 2011).

Figure 1

OUTPUT GAP MEASURES



Output gap given by the backward looking, hybrid and forward looking models (HPD regions at 90%), and the cyclical component of output obtained from the Hodrick-Prescott filter with $\lambda = 1{,}600$. Grey areas correspond to peak-to-trough periods of the Colombian business cycle according to Alfonso et al. (2011). Series are all quarterly for the period 1994-2011. Calculations were made using Dynare.

5. THE OUTPUT GAP

After the estimation the parameters are set to their posterior mode values. Then each model is used to extract the output gap from the data. The proposed output gap measure is obtained with the Kalman smoother for variable $\tilde{y}$ in each model. Since the Hodrick-Prescott filter (henceforth HP filter) can be represented as a special case of the local linear trend model it is used as a benchmark for the results (see Harvey and Jaeger, 1993, and Canova, 2007). Figure 1d presents the results for this exercise. Panels 1a, 1b and 1c show the output gap obtained from each model with its respective higher posterior density (HPD) region at 90%. This region accounts for the uncertainty in the parameter estimates. Grey areas correspond to peak-to-trough periods of the Colombian business cycle according to Alfonso et al. (2011).

There is a clear difference between the ability of the forward looking model (Panel 1c) to capture the dynamics of the output gap, and the other two models. Even with an HPD at 90% the forward looking model is able to capture the boom experienced in Colombia in the 1990s, the subsequent recession and that affected most of the 2000s and the last cycle (2006-2007 boom and the international financial crisis of 2008).

Panel 1d presents the output gap measures and the Hodrick-Prescott filter for the Colombian GDP. Note that, although all three measures comove they are not equal, showing that the economic models have additional information when compared to the statistical filter.

The most notorious differences are in the 1994-1996, 2000-2004 and 2006-2009 periods. In the first period the forward looking model presents a higher (positive) output gap than the other models (joining the HP filter only until 1997). In the second period, the backward looking and hybrid models identify a closed output gap whereas the forward looking and the HP filter still show a negative cyclical component. In the second period the models, specially the backward looking model, fail to recognize a great increase in the output gap, as opposed to the forward looking model and the HP filter, that identify a strong positive cycle.

Besides the differences between the proposed measures for the output gap and the one given by the HP filter, there are also differences between those measures and the consensus among the experts. According to them, the gap should have been positive at the beginning of the sample (as in the forward looking model) and more negative at the 1998-1999 recession. The models fail to reproduce these facts because of two reasons. First, the Kalman filter is initialized at an arbitrary point, that does not necessarily reflect the true value of the states. In the previous exercise the filter was initialized as if the gap was equal to zero –its steady state value– in 1994Q1.[6] Second, the local linear trend model, on top of which the proposed models are built, understands the data in the 1998-1999 period as a change in output's trend –this means that the model is attributing part of the recession to a decrease in potential output, thus generating a *less negative* output gap. It is important to note that most of the models designed to extract output's cyclical component fail to recognize a strong negative output gap in the 1998-1999 period,[7] but, unlike most of them, the use of the Kalman filter allows us to incorporate additional information about the output gap, for the estimation and filtering process.[8]

Because of the above discussion a second exercise is carried out. The models are now estimated using the same database and prior distributions for the parameters, while allowed to observe the output gap level given by the experts for the first four observations of the sample and the fourth quarter of 1999 (Table 1). This information is subject to measurement error, whose variance is estimated along with all the other parameters. The results of the estimation are summarized in Appendices C.2, D.2 and E.2. All models turn out to assign little variance to the measurement error of the output gap additional information,

[6] In Figure 1 the output gap is not equal to zero at the first period because the gap measure is given by the Kalman filter smoother, which takes into account the whole sample for determining the gap value at each period. Only the forward looking model interpreted the data as to get the positive output gap at the beginning of the sample.

[7] An exception to this is the measure proposed in Cobo (2004), based on the production function approach.

[8] The methodology presented in Julio (2011) represents an exception to this, allowing the introduction of *priors* as linear restrictions on the Hodrick-Prescott filter.

Table 1

OUTPUT GAP PRIOR INFORMATION
(percentage)

Period	*Value*	*Period*	*Value*	*Period*	*Value*	*Period*	*Value*	*Period*	*Value*
1994Q1	1.35	1994Q2	1.62	1994Q3	2.03	1994Q4	2.38	1999Q4	–5.72

as reflected in the recovered output gap measure (Figure 2) and the posterior mode of the parameter (Tables 6, 8 and 10).

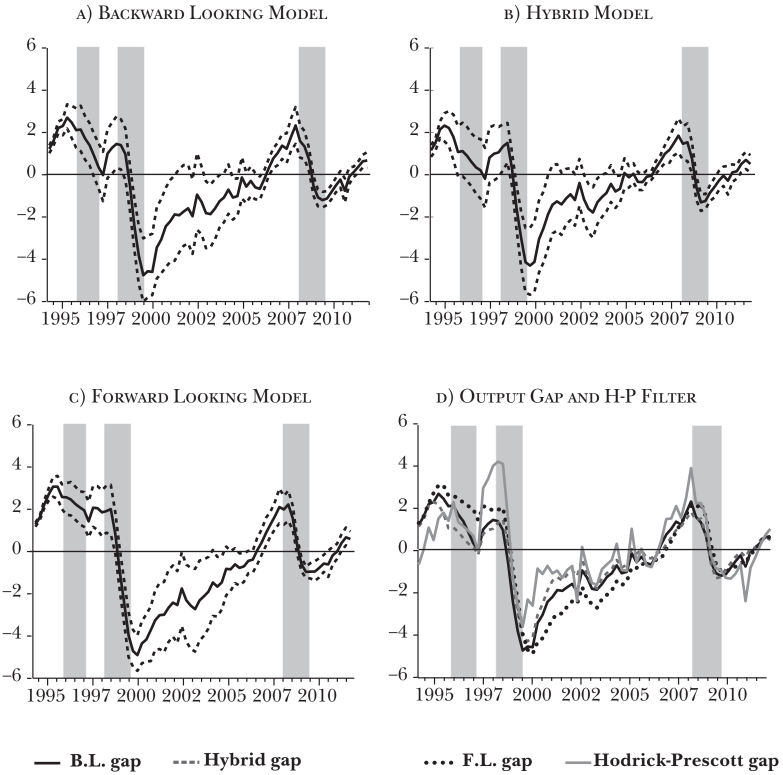
The output gap measures recovered from this exercise are able to recognize both a positive gap between 1994 and 1998, as well as a more negative and persistent gap following the 1998-1999 recession, up until the mid 2000s (although the HPD regions in panels 2a and 2b include zero after 2001). They also present a somewhat higher gap at the end of the sample and in the 2007-2008 period. As before there is a comovement between the three measures, with differences in the timing and magnitude of the cycles, because of the additional information given to the models the differences in the recovered output gap are now fewer. In order to assess the models goodness of fit, we use the marginal likelihood value. It is found that among the conditioned estimation, the forward looking model is the one with the higher marginal likelihood and the hybrid model is the worst. In the case of the non-conditioned estimation, the backward looking model is the one with the highest marginal likelihood and the worst is once again the hybrid model. The values are presented in the Appendix.

The gaps presented also match previous findings on the Colombian business cycle. As shown in Figure 3, all measures identify all the peaks and troughs presented by Alfonso et al. (2011), who use an accumulated diffusion index, computed with 24 Colombian series,[9] in order to obtain a chronology of the business cycle. Before the additional information was introduced only the forward looking matched this turning points (Panel 1c).

[9] The only variable in common between this exercise and the one of Alfonso et al. (2011) is the nominal interest rate.

Figure 2

OUTPUT GAP MEASURES - PRIOR INFORMATION



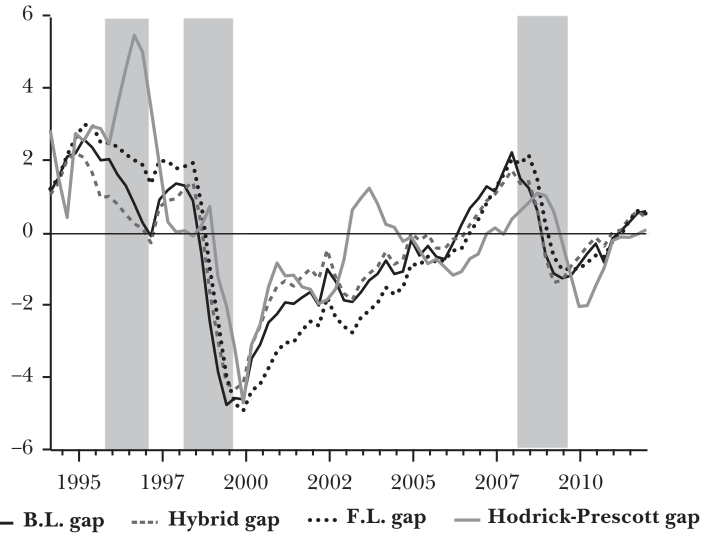
Output gap given by the backward looking, hybrid and forward looking models (HPD regions at 90%), and the cyclical component of output obtained from the Hodrick-Prescott filter with $\lambda = 1{,}600$. Grey areas correspond to peak-to-trough periods of the Colombian business cycle according to Alfonso et al. (2011). Series are all quarterly for the period 1994-2011. Calculations were made using Dynare.

Figure 3 also makes clear that there is a strong positive correlation between the output gap measures and the core inflation, defined as the CPI less food items inflation.[10] Moreover, the output gap precedes the movements in the core inflation

[10] Core inflation gap is defined as the current level of inflation less the target.

Figure 3

OUTPUT GAP MEASURES VS CORE CPI INFLATION



Output gap given by the backward looking, hybrid and forward looking models with prior information about the output gap level in 1994 and 1999Q4, and CPI less food items inflation relative to the inflation target. Grey areas correspond to peak-to-trough periods of the Colombian business cycle according to Alfonso et al. (2011). Series are all quarterly for the period 1994-2011. Calculations were made using Dynare.

to some extent. Note for example the inflation's peaks after the 1995, 1997 and 2007 peaks in the output gap, as well as the falls in inflation after the 1998 and 2009 falls in the output gap. This can also be seen when computing the correlation between the core inflation and the current and lagged values of the output gap (Table 2), the correlation is always above halve and is greater for the first and second lagged values than for the contemporaneous one (except in the forward looking model).

Finally, the output gap can be decomposed into the effects of the shocks by using the state space representation of the model (Canova, 2007). The historical decomposition for the output gap measures is computed and reported in Figure 4. The exercise consists in identifying which shocks affected the

Table 2

OUTPUT GAP AND CORE INFLATION GAP CORRELATION

Model	$corr(\tilde{y}_t, \pi_t)$	$corr(\tilde{y}_{t-1}, \pi_t)$	$corr(\tilde{y}_{t-2}, \pi_t)$	$corr(\tilde{y}_{t-4}, \pi_t)$	$corr(\tilde{y}_{t-5}, \pi_t)$
Backward looking	0.660	0.727	0.764	0.749	0.676
Hybrid	0.603	0.667	0.694	0.667	0.590
Forward looking	0.745	0.753	0.724	0.659	0.566

economy in the sample period using the observed macroeconomic series, along with the economic structure of the models. After the shocks have been identified it is possible to compute their individual impact over the output gap.

Since the identification and impact depend on the model's structure, the decomposition is different for the backward looking, hybrid and forward looking models. Nevertheless there are common features between them. The most important one is that the output gap is explained mostly by the effect of shocks to the IS curve (demand shocks). This is very useful if one wishes to interpret the output gap as a measure of demand pressures in the economy. Another common characteristic is the low and short-lived effect of the filter's initial values over the output gap, it can be seen that this effect is only determinant in the first period and that only lasts for approximately 12 periods. Another common feature is the effect of the Phillips curve shock (supply shocks) after the 1999 recession. Because of the large drop in inflation that followed the first quarters of 1999, the models identify a Phillips curve shock that helps to explain such drop, as a consequence positive pressures over the output gap were created.

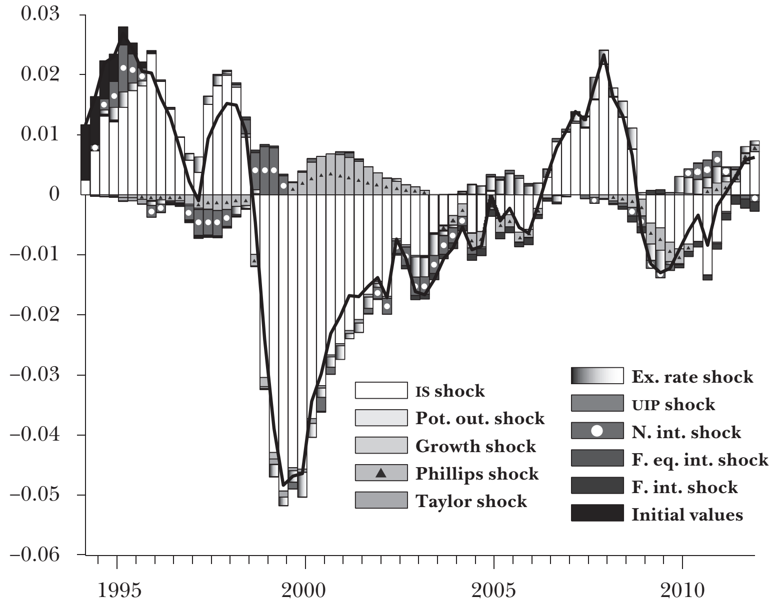
There are three other shocks that appear significant in the historical decomposition. The first is to the foreign interest rate, this shock is more relevant in Panels 4b and 4c and has a negative effect over the output gap for the 2000s' period. During this period the foreign interest rate was low and the models identify this as a negative shock, associated to a real appreciation of the exchange rate. Nevertheless there must be caution over this result, for the models are biased toward the negative effects of the shock, since they do not take into account the positive effects of appreciation and a cheaper debt over the aggregate demand.

The second shock is to the real exchange rate trend, it is expansive in 2004 and 2010, both periods of real exchange rate appreciation. The reason for this is that the models interpret these appreciations as changes in the real exchange rate trend. When the trend is lowered the exchange rate gap becomes

Figure 4a

OUTPUT GAP HISTORICAL DECOMPOSITION

BACKWARD LOOKING MODEL



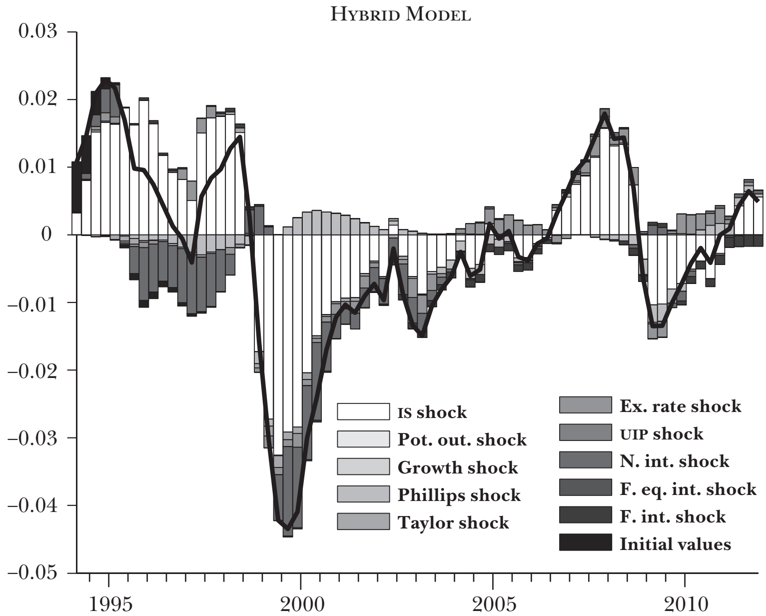
Output gap historical decomposition in shocks given by the adaptive and rational expectations models with prior information about the output gap level in 1994 and 1999Q4. The series are all of quarterly frequency for the period 1994-2011. Calculations were made using Dynare.

positive, hence increasing the output gap. However this effect is not of great magnitude, relative to the effect of other shocks.

The third shock is to the neutral interest rate. Note that for the models this variable is completely exogenous and only influenced by this shock. Because the models are able to extract both the level of the real interest rate and of the interest rate gap, the neutral rate of interest can be computed. The negative effect over the output gap of the neutral interest rate shock in the early 2000s is explained by a decrease in the neutral rate from the high levels of the late 1990s, which lowered the interest rate gap. More about the neutral rate is discussed in the following Section.

Figure 4b

OUTPUT GAP HISTORICAL DECOMPOSITION



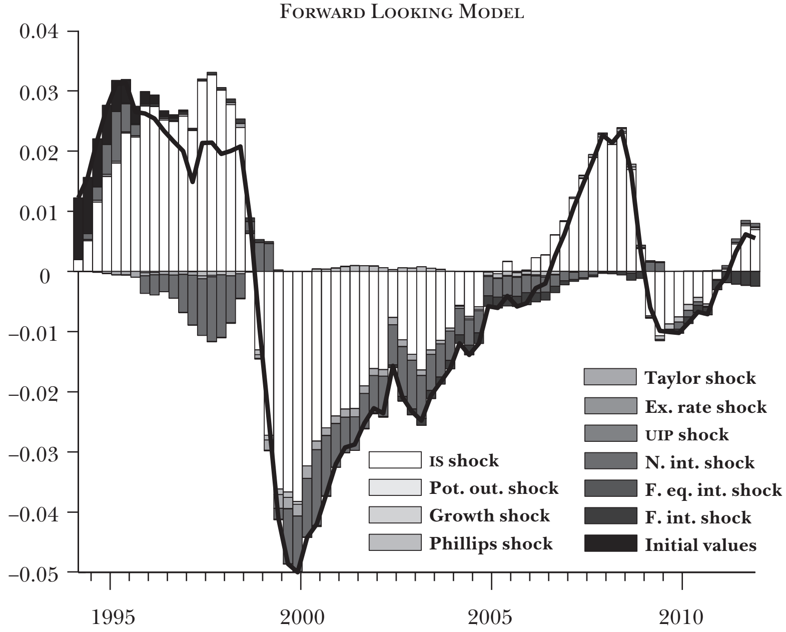
Output gap historical decomposition in shocks given by the adaptive and rational expectations models with prior information about the output gap level in 1994 and 1999Q4. The series are all of quarterly frequency for the period 1994-2011. Calculations were made using Dynare.

6. THE NEUTRAL INTEREST RATE

Before discussing the models' implications over the neutral interest rate, it is important to examine the behavior of the real interest rate. Recall that this variable is computed in-model, given the nominal interest rate and the inflation expectations, nevertheless all three models generate similar measures (Figure 5) that are also in line with the movements and level of the expost real interest rate. The period under consideration is characterized by high and volatile levels of the real interest rate before the 2000, followed by a more stable period with a lower interest rate level. This is clear from the mean and standard

Figure 4c

OUTPUT GAP HISTORICAL DECOMPOSITION



Output gap given by the backward looking, hybrid and forward looking models with prior information about the output gap level in 1994 and 1999Q4, and CPI less food items inflation relative to the inflation target. Grey areas correspond to peak-to-trough periods of the Colombian business cycle according to Alfonso et al. (2011). Series are all quarterly for the period 1994-2011. Calculations were made using Dynare.

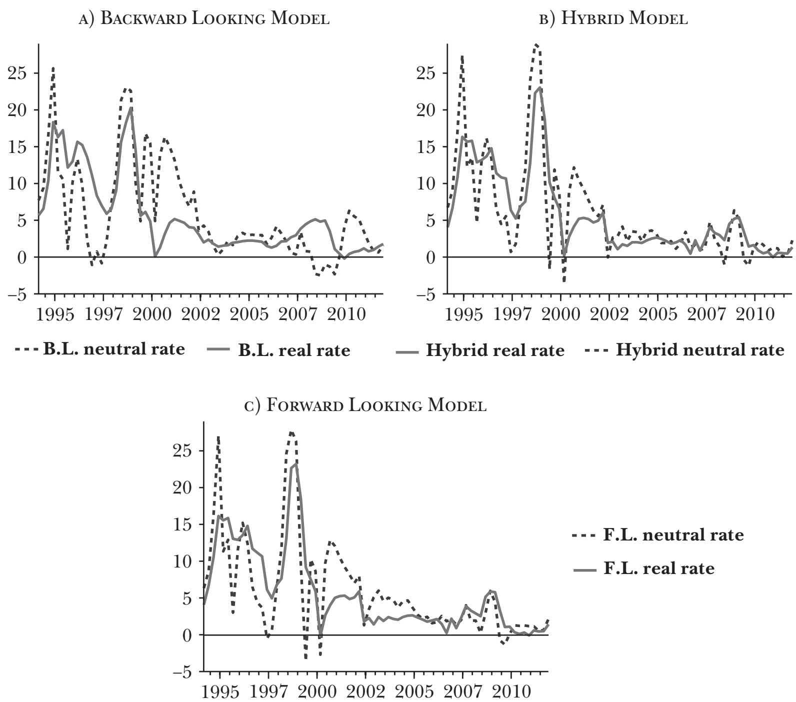
deviation of the real interest variable in all models, shown in the first two columns of Table 3.

The volatility; and subsequent stabilization, of the real interest rate is probably explained by changes in the Colombian monetary policy; we refer to Giraldo et al. (2011) and the references therein for a review of Colombia's recent monetary history. Overall there are no drastic changes in the real interest rate, save from the great increase that coincides with the 1998-1999 recession, which is explained by the large drop in inflation that followed the crisis (see Figure 4).

Turning to the neutral interest rate, Figure 5 presents the neutral and real interest rate that each model recovered from the data. Note that all measures of the neutral interest rate are even more volatile than the real interest rate measures (compare the standard deviations of both variables in the second and fourth columns of Table 3). The volatility of the real interest rate, although present only before the 2000, influences the neutral interest rate in the whole sample, generating a changing measure of neutrality for the last decade.

Figure 5

NEUTRAL INTEREST RATE



Neutral and real interest rate measures given by the backward looking, hybrid and forward looking models with prior information about the output gap level in 1994 and 1999Q4 (HPD regions at 90%). The series are all of quarterly frequency for the period 1994-2011. Calculations were made using Dynare.

All models imply that there was a positive interest rate gap before the 1999 recession and a negative gap afterward, with a slow convergence of the neutral rate to the levels that the real interest rate has presented after the crisis. Then, the interest rate gap turns positive in the 2007-2008 period, although considerably more (and sooner) in the backward looking model. At the end of the sample the interest rate gap becomes negative, again in a larger amount in the backward looking model.

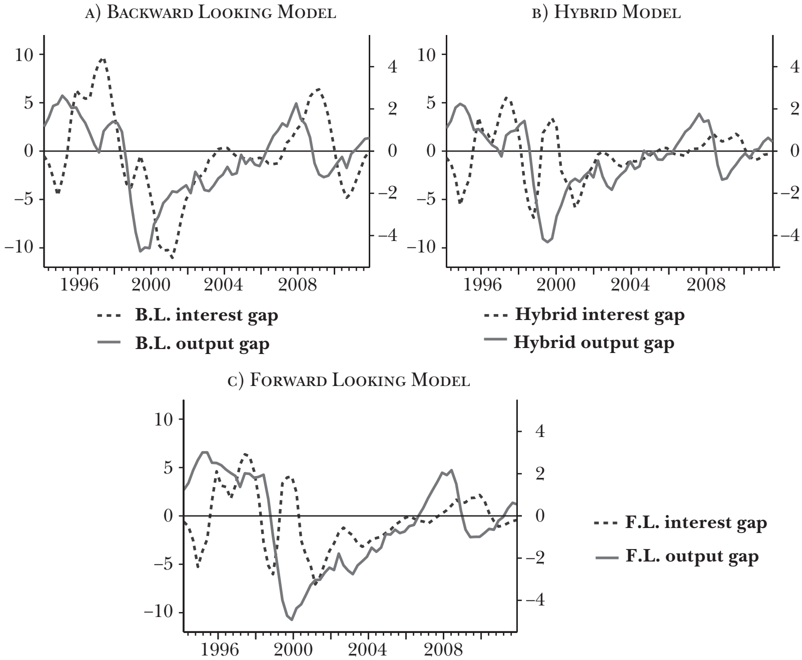
Note, from Figure 6, that the behavior of the interest rate gap is countercyclical almost everywhere. It exerts a negative pressure over the output gap while it was positive in the pre-1999 period, and has expansive effects afterward, up until the 2007-2008 period, in which the output gap is again positive. Finally, the interest rate gap has positive effects after 2008, when there is a drop in the output gap, associated to the international turbulence that followed the recent us financial crisis. The countercyclality of the interest rate gap is clearly interrupted in the hybrid and forward looking models between 1999 and 2001 (Panels 6b and 6c). In this period the interest rate gap turns positive while the output gap remains negative. This is attributed to a drop in the neutral interest rate (see Panels 5b and 5c), since this variable is exogenous for the model, this means that the model identifies the need of a positive interest gap in order to explain the drop in the output gap in those periods. This is reflected in the negative correlation between the output and interest rate gaps for the 1994-2000 period for the hybrid and forward looking models (Table 3).

Yet, it must be mentioned that the interest gap reaction is lagged with respect to the output gap movements,[11] this is clearer in Panel 6a for the backward looking model, and post-2006 period in Panels 6b and 6c, where the interest rate gap turns positive a year after the output gap does and then remains

[11] Recall that the interest rate gap presented in Figure 5 is smoothed with a fourth order moving average. This is done for clarity since the neutral rate measures are too volatile, and does not affect the findings.

Figure 6

NEUTRAL INTEREST GAP VS. OUTPUT GAP



Smoothed real interest rate gap and output gap given by the backward looking, hybrid and forward looking models with prior information about the output gap level in 1994 and 1999Q4. The real interest rate gap is smoothed with a fourth order moving average. The series are all of quarterly frequency for the period 1994-2011. Calculations were made using Dynare.

positive while the output gap fells in the 2008 financial crisis. In other words, given that the monetary authority can influence the real interest rate, the monetary policy, although countercyclical, has failed to anticipate the changes of the output gap, or at least to react strongly enough to them.

7. FINAL REMARKS

This document presents three new measures of the Colombian output gap and neutral interest rate. Both variables are crucial

Table 3

INTEREST RATE MOMENTS

	Real interest rate		Neutral interest		Interest rate gap		
	Mean	*Std*	*Mean*	*Std*	*Mean*	*Std*	$corr(\tilde{y}_t, \tilde{r}_t)$
				Backward looking model			
1994-2011	7.961	5.299	8.413	6.526	−0.451	5.293	0.484
1994-2000	14.056	4.889	13.000	7.792	1.056	6.696	0.343
2000-2011	4.914	1.505	6.119	4.303	−1.205	4.318	0.630
				Hybrid model			
1994-2011	8.103	5.507	8.548	6.609	-0.445	4.234	0.232
1994-2000	14.307	5.227	13.425	8.614	0.882	6.299	−0.106
2000-2011	5.001	1.720	6.110	3.386	−1.109	2.513	0.644
				Forward looking model			
1994-2011	8.086	5.486	8.433	6.859	−0.348	3.883	−0.077
1994-2000	14.352	5.114	14.304	8.475	0.047	6.024	−0.286
2000-2011	4.953	1.611	5.498	3.085	−0.545	2.213	0.399

for the conduct of monetary policy and their measurement is subject to a great deal of uncertainty. Because of this the results presented here are not to be taken as final, but as an extra input, useful for policy evaluation and academic research. The models deliver an output gap coherent with previous works for Colombia, as Echavarría et al. (2007), and is capable to identify all the turning points of the Colombian business cycle, as measured by Alfonso et al. (2011). The Colombian output gap begins with a positive, although variable, level from 1994 to 1997, when there is a large drop that starts with the 1998-1999 crisis, after this drop the output gap remains negative until 2006. The gap turns positive in the 2006-2008 period and then drops in 2009, after the international turmoil that followed the US financial crisis of 2008. Both models imply that the gap has recovered from its last drop and is positive since 2011, although still close to zero.

As for the neutral interest rate, the models are more heterogeneous in the results, but all imply a somewhat countercyclical behavior of the monetary policy during most of the sample period, except at the time of the 1999 recession. They also imply a delay between the movements of the output gap and those of the interest rate gap, specially the hybrid and forward looking models. This may correspond to a lack of anticipation of the monetary authority, or the need of stronger reaction to the economy's condition.

Finally, it is important to expand the methodology to account for model uncertainty to which the output and neutral interest rate measurement is subject to; this implies the use of more advanced techniques that go beyond the scope of this paper. It is also noted that the methodology presented relies in semi-structural models to take into account the relations between several macroeconomic aggregates, and there are still efforts to be done in order to compute a micro-founded measure of the output gap, and the natural interest rate, in the spirit of Woodford (2003a) and Christiano et al. (2010a,b). These new measures can potentially improve our understanding of the shocks that affect the economy, and the design of monetary policy.

Appendix

A. Equations

A.1 Adaptive Expectations Model

A.1 $$y_t = \tilde{y}_t + \overline{y}_t$$

A.2 $$\overline{y}_t = \overline{y}_{t-1} + g_t + \varepsilon_t^{\overline{y}}$$

A.3 $$g_t = (1-\tau)\overline{g}_{ss} + \tau g_{t-1} + \varepsilon_t^g$$

A.4 $$\tilde{y}_t = \beta_1 \tilde{y}_{t-1} - \beta_2 (r_{t-1} - \overline{r}_{t-1}) + \beta_3 \tilde{q}_{t-1} + z_t^y$$

A.5 $$\pi_t = \pi_{t+1|t}^e + \lambda_2 \tilde{y}_{t-1} + \lambda_3 (q_t - q_{t-1}) + z_t^\pi$$

A.6 $$\pi_{t+1|t}^e = \lambda_1 \overline{\pi}_t + (1-\lambda_1)\pi_{4,t-1}$$

A.7 $$\pi_{4,t} = \frac{1}{4}(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})$$

A.8 $$i_t = \gamma_1 i_{t-1} + (1-\gamma_1)\left((\overline{r}_t + \overline{\pi}) + \gamma_2 (\pi_{4,t} - \overline{\pi}) + \gamma_3 \tilde{y}_t\right) + \varepsilon_t^i$$

A.9 $$r_t = i_t - \pi_{t+1|t}^e$$

A.10 $$r_t - r_t^* = 4(q_{t+1|t}^e - q_t) + (\overline{r}_t - \overline{r}_t^*) + \varepsilon_t^r$$

A.11 $$\overline{r}_t = \rho_r \overline{r}_{t-1} + (1-\rho_r)\overline{r}_{ss} + \varepsilon_t^{\overline{r}}$$

A.12 $$r_t^* = \overline{r}_t^* + \kappa(r_{t-1}^* - \overline{r}_{t-1}^*) + \varepsilon_t^{r^*}$$

A.13 $$\overline{r}_t^* = \rho_{r^*} \overline{r}_{t-1}^* + (1-\rho_{r^*})\overline{r}_{ss}^* + \varepsilon_t^{\overline{r}^*}$$

A.14 $$q_t = \tilde{q}_t + \overline{q}_t$$

A.15 $$\overline{q}_t = \overline{q}_{t-1} + \varepsilon_t^q$$

A.16 $$q_{t+1|t}^e = \varphi \overline{q}_t + (1-\varphi) q_{t-1}$$

A.17 $$z_t^y = \rho_y z_{t-1}^y + \varepsilon_t^y$$

A.18 $$z_t^\pi = \rho_\pi z_{t-1}^\pi + \varepsilon_t^\pi$$

A.2 Rational Expectations Semi-structural Model

A.19 $$y_t = \tilde{y}_t + \overline{y}_t$$

A.20 $$\overline{y}_t = \overline{y}_{t-1} + g_t + \varepsilon_t^{\overline{y}}$$

A.21 $$g_t = (1-\tau)\overline{g}_{ss} + \tau g_{t-1} + \varepsilon_t^g$$

A.22 $$\tilde{y}_t = \beta_1 \tilde{y}_{t-1} - \beta_2 (r_{t-1} - \overline{r}_{t-1}) + \beta_3 \tilde{q}_{t-1} + \beta_4 E_t \{\tilde{y}_{t+1}\} + z_t^y$$

A.23 $$\pi_t = \pi_{t+1|t}^e + \lambda_2 \tilde{y}_{t-1} + \lambda_3 (q_t - q_{t-1}) + z_t^\pi$$

A.24 $$\pi_{t+1|t}^e = \lambda_1 E_t \{\pi_{4,t+4}\} + (1-\lambda_1)\pi_{4,t-1}$$

A.25 $$\pi_{4,t} = \frac{1}{4}(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})$$

A.26 $$i_t = \gamma_1 i_{t-1} + (1-\gamma_1)\left((\overline{r}_t + \overline{\pi}) + \gamma_2 E_t(\pi_{4,t+4} - \overline{\pi}) + \gamma_3 \tilde{y}_t\right) + \varepsilon_t^i$$

A.27 $$r_t = i_t - E_t\{\pi_{t+1}\}$$

A.28 $$r_t - r_t^* = 4(q_{t+1|t}^e - q_t) + (\overline{r}_t - \overline{r}_t^*) + \varepsilon_t^r$$

A.29 $$\overline{r}_t = \rho_r \overline{r}_{t-1} + (1-\rho_r)\overline{r}_{ss} + \varepsilon_t^{\overline{r}}$$

A.30 $$r_t^* = \overline{r}_t^* + \kappa(r_{t-1}^* - \overline{r}_{t-1}^*) + \varepsilon_t^{r^*}$$

A.31 $$\overline{r}_t^* = \rho_{r^*} \overline{r}_{t-1}^* + (1-\rho_{r^*})\overline{r}_{ss}^* + \varepsilon_t^{\overline{r}^*}$$

A.32 $$q_t = \tilde{q}_t + \overline{q}_t$$

A.33 $$\overline{q}_t = \overline{q}_{t-1} + \varepsilon_t^q$$

A.34 $$q_t^e = \varphi E_t\{q_{t+1}\} + (1-\varphi) q_{t-1}$$

A.35 $$z_t^y = \rho_y z_{t-1}^y + \varepsilon_t^y$$

A.36 $$z_t^\pi = \rho_\pi z_{t-1}^\pi + \varepsilon_t^\pi$$

B. Prior Distributions

Table A.1

PRIOR DISTRIBUTIONS

Parameter	*Description*	*Distribution*	*Mean*	*Standard deviation*
σ_i	Shock *i* standard deviation	Inv. gamma	0.0125	∞
τ	Persistence of the growth process	Beta (0,1)	0.5	0.15
λ_1	Inflation expectations	Beta (0,1)	0.5	0.15

λ_2	Elasticity of inflation to output gap	Gamma	0.30	0.25
λ_3	Elasticity of inflation to depreciation	Gamma	0.30	0.25
β_1	Elasticity of output gap to its lag	Beta (0,1)	0.5	0.15
β_2	Elasticity of output gap to real interest gap	Gamma	0.30	0.25
β_3	Elasticity of output gap to exchange rate gap	Gamma	0.30	0.25
β_4	Elasticity of output gap to expectations	Gamma	0.30	0.25
φ	Exchange rate expectations	Beta (0,1)	0.5	0.15
κ	Persistence of foreign interest rate gap	Beta (0,1)	0.5	0.15
ρ_r	Persistence of natural interest rate	Beta (0,1)	0.5	0.15
ρ_{r^*}	Persistence of foreign natural interest rate	Beta (0,1)	0.5	0.15
ρ_y	Persistence of IS shock	Beta (0,1)	0.5	0.15
ρ_π	Persistence of Phillips curve shock	Beta (0,1)	0.5	0.15

C. Estimation Results – Backward Looking Model

C.1 Unconditioned Estimation

Table A.2

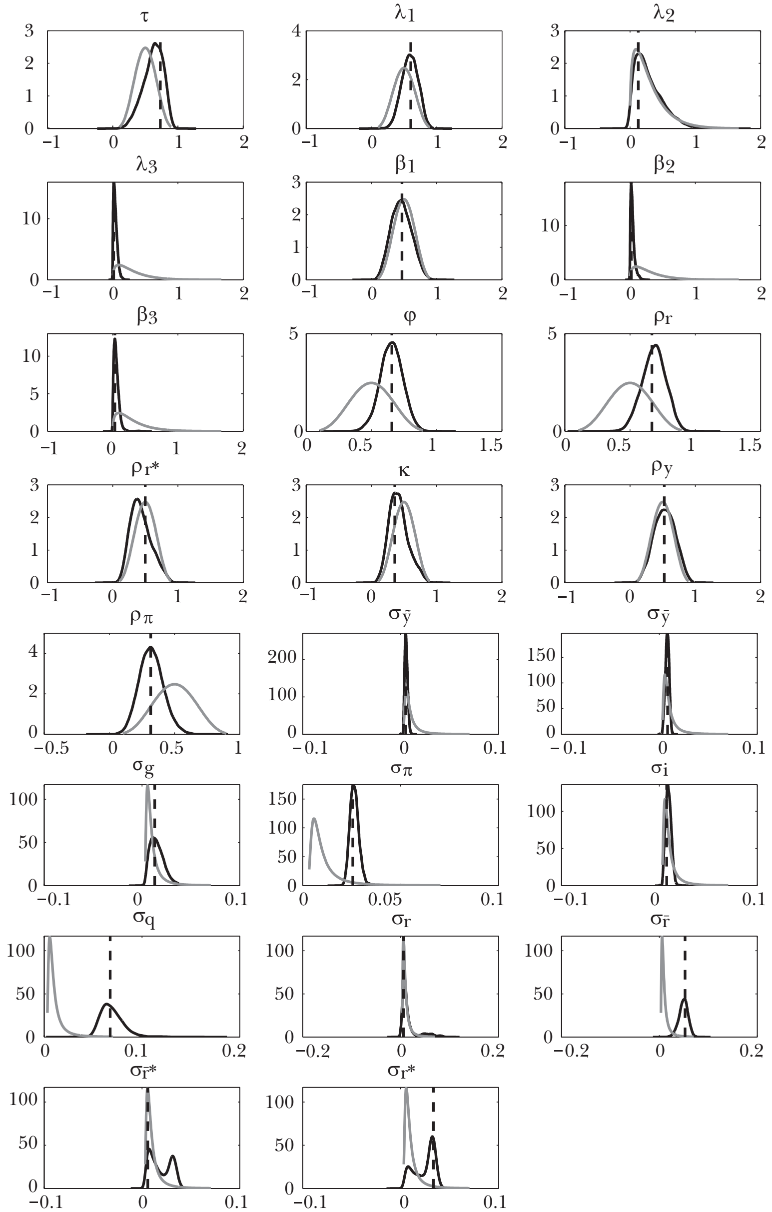
ESTIMATION RESULTS – BACKWARD LOOKING MODEL

Parameter		Prior		Posterior			HPD 90%	
		Mean	*SD*	*Mean*	*Mode*	*SD*	*Lower*	*Upper*
Persistence of the growth process	τ	0.50	0.15	0.62	0.73	0.13	0.37	0.86
Inflation expectations	λ_1	0.50	0.15	0.59	0.60	0.13	0.38	0.79
Elasticity of inflation to output gap	λ_2	0.30	0.25	0.29	0.13	0.19	0.00	0.59
Elasticity of inflation to depreciation	λ_3	0.30	0.25	0.04	0.01	0.02	0.00	0.08
Elasticity of output gap to its lag	β_1	0.50	0.15	0.46	0.47	0.19	0.20	0.71
Elasticity of output gap to real interest gap	β_2	0.30	0.25	0.04	0.02	0.03	0.00	0.08
Elasticity of output gap to exchange rate gap	β_3	0.30	0.25	0.06	0.03	0.04	0.00	0.11
Exchange rate expectations	φ	0.50	0.15	0.65	0.66	0.08	0.52	0.80
Persistence of natural interest rate	ρ_r	0.50	0.15	0.68	0.67	0.09	0.54	0.84
Persistence of foreign natural interest rate	ρ_{r^*}	0.50	0.15	0.43	0.50	0.18	0.18	0.69
Persistence of foreign interest rate gap	κ	0.50	0.15	0.42	0.36	0.09	0.19	0.67

Persistence of IS shock	ρ_y	0.50	0.15	0.53	0.52	0.19	0.28	0.80
Persistence of Phillips curve shock	ρ_π	0.50	0.15	0.32	0.32	0.09	0.16	0.47
Standard deviation of IS curve shock	σ_y	0.013	∞	0.006	0.005	0.002	0.003	0.008
Standard deviation of potential output shock	$\sigma_{\bar{y}}$	0.013	∞	0.008	0.008	0.002	0.005	0.011
Standard deviation of growth shock	σ_g	0.013	∞	0.016	0.013	0.006	0.004	0.026
Standard deviation of Phillips curve shock	σ_π	0.013	∞	0.026	0.026	0.002	0.023	0.030
Standard deviation of nominal interest rate shock	σ_i	0.013	∞	0.009	0.008	0.003	0.005	0.014
Standard deviation of potential exchange rate shock	σ_q	0.013	∞	0.069	0.068	0.010	0.052	0.086
Standard deviation of UIP shock	σ_r	0.013	∞	0.016	0.006	0.002	0.003	0.047
Standard deviation of natural real interest rate shock	$\sigma_{\bar{r}}$	0.013	∞	0.049	0.053	0.008	0.034	0.064
Standard deviation of foreign natural interest rate shock	$\sigma_{\bar{r}^*}$	0.013	∞	0.019	0.006	0.003	0.004	0.034
Standard deviation of foreign interest rate shock	σ_{r^*}	0.013	∞	0.024	0.034	0.003	0.006	0.037
Marginal likelihood	784.9973							

Figure 7

Prior & Posterior: Backward Looking Model



Prior and posterior density functions for the estimated parameters, prior functions are in gray and posteriors functions in black, the dashed vertical line indicates the parameter posterior mean. Calculations were made with Dynare.

C.2 Conditioned Estimation

Table A.3

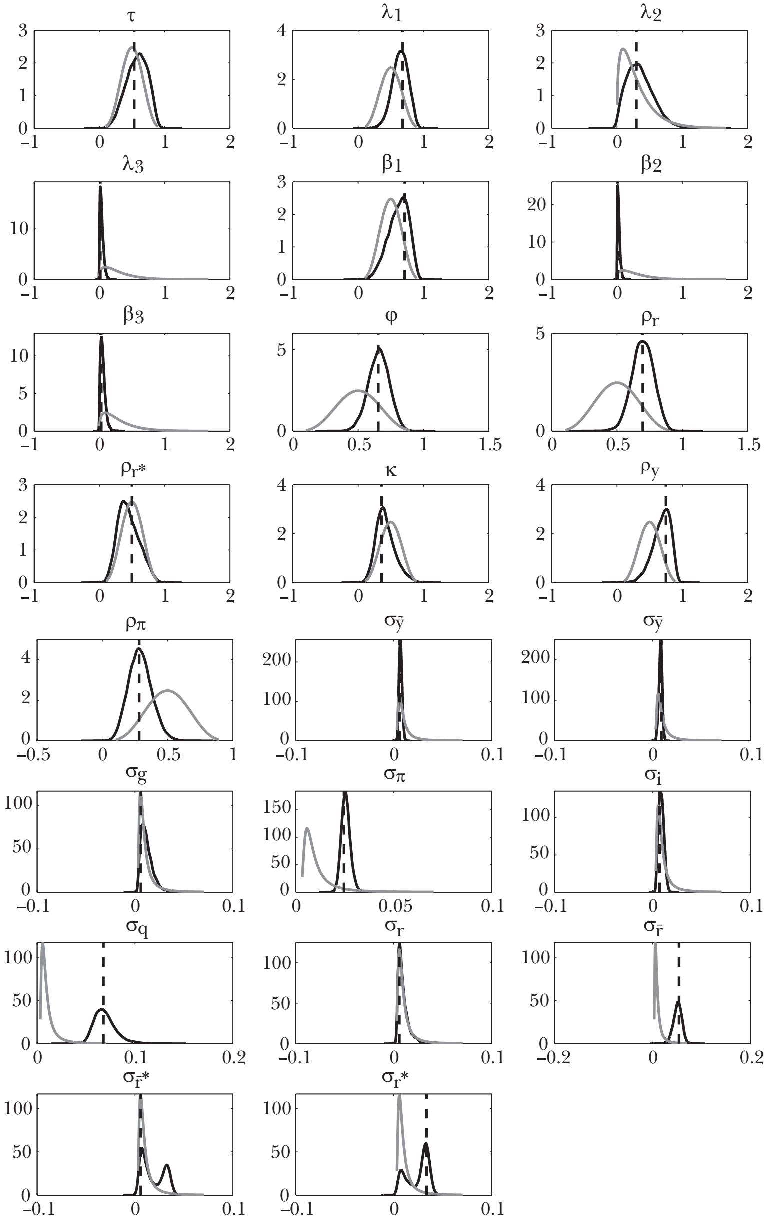
ESTIMATION RESULTS - BACKWARD LOOKING MODEL

Parameter		Prior		Posterior			HPD 90%	
		Mean	*SD*	*Mean*	*Mode*	*SD*	*Lower*	*Upper*
Persistence of the growth process	τ	0.5	0.15	0.57	0.53	0.20	0.32	0.84
Inflation expectations	λ_1	0.5	0.15	0.64	0.68	0.13	0.46	0.85
Elasticity of inflation to output gap	λ_2	0.3	0.25	0.35	0.30	0.20	0.03	0.65
Elasticity of inflation to depreciation	λ_3	0.3	0.25	0.04	0.01	0.02	0.00	0.07
Elasticity of output gap to its lag	β_1	0.5	0.15	0.61	0.71	0.14	0.37	0.86
Elasticity of output gap to real interest gap	β_2	0.3	0.25	0.03	0.01	0.01	0.00	0.05
Elasticity of output gap to exchange rate gap	β_3	0.3	0.25	0.05	0.03	0.03	0.00	0.10
Exchange rate expectations	φ	0.5	0.15	0.66	0.65	0.08	0.53	0.79
Persistence of natural interest rate	ρ_r	0.5	0.15	0.70	0.70	0.08	0.56	0.83
Persistence of foreign natural interest rate	ρ_{r^*}	0.5	0.15	0.45	0.50	0.18	0.18	0.70
Persistence of foreign interest rate gap	κ	0.5	0.15	0.42	0.36	0.09	0.19	0.66
Persistence of IS shock	ρ_y	0.5	0.15	0.68	0.75	0.12	0.47	0.89

Persistence of Phillips curve shock	ρ_π	0.5	0.15	0.29	0.28	0.09	0.15	0.43
Standard deviation of IS curve shock	σ_y	0.013	∞	0.007	0.006	0.001	0.005	0.009
Standard deviation of potential output shock	$\sigma_{\bar{y}}$	0.013	∞	0.008	0.009	0.001	0.005	0.011
Standard deviation of growth shock	σ_g	0.013	∞	0.012	0.006	0.003	0.003	0.021
Standard deviation of Phillips curve shock	σ_π	0.013	∞	0.026	0.025	0.002	0.022	0.029
Standard deviation of nominal interest rate shock	σ_i	0.013	∞	0.009	0.007	0.003	0.004	0.013
Standard deviation of potential exchange rate shock	σ_q	0.013	∞	0.069	0.068	0.011	0.052	0.086
Standard deviation of UIP shock	σ_r	0.013	∞	0.009	0.006	0.002	0.003	0.017
Standard deviation of natural real interest rate shock	$\sigma_{\bar{r}}$	0.013	∞	0.051	0.054	0.008	0.037	0.065
Standard deviation of foreign natural interest rate shock	$\sigma_{\bar{r}^*}$	0.013	∞	0.019	0.006	0.003	0.004	0.035
Standard deviation of foreign interest rate shock	σ_{r^*}	0.013	∞	0.025	0.034	0.003	0.005	0.037
Standard deviation of measurement error	σ	0.013	∞	0.009	0.005	0.002	0.003	0.017
Marginal likelihood		796.8164						

Figure 8

PRIOR & POSTERIOR: BACKWARD LOOKING MODEL



Prior and posterior density functions for the estimated parameters, prior functions are in gray and posteriors functions in black, the dashed vertical line indicates the parameter posterior mean. Calculations were made with Dynare.

D. Estimation Results - Hybrid Model

D.1 Unconditioned Estimation

Table A.4

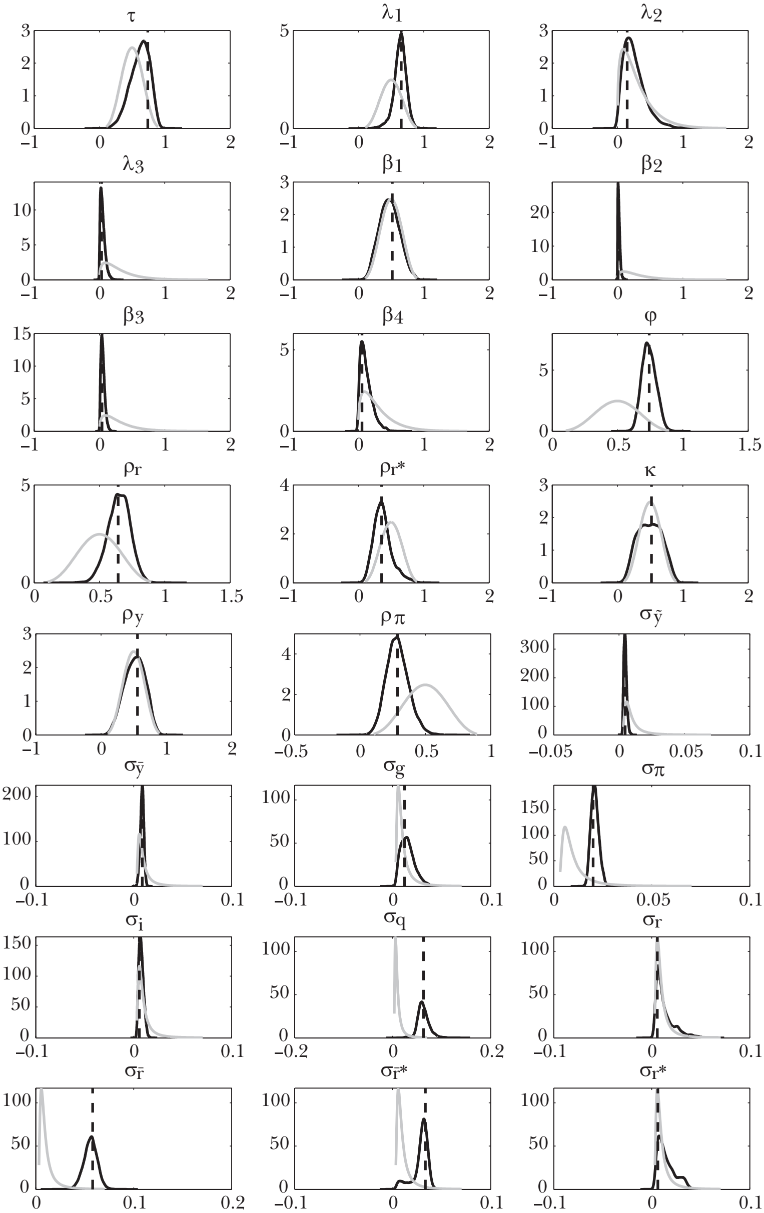
ESTIMATION RESULTS – HYBRID MODEL

Parameter		*Prior*		*Posterior*			*HPD 90%*	
		Mean	*SD*	*Mean*	*Mode*	*SD*	*Lower*	*Upper*
Persistence of the growth process	τ	0.5	0.15	0.62	0.74	0.13	0.39	0.85
Inflation expectations	λ_1	0.5	0.15	0.64	0.66	0.07	0.49	0.78
Elasticity of inflation to output gap	λ_2	0.3	0.25	0.26	0.15	0.15	0.01	0.48
Elasticity of inflation to depreciation	λ_3	0.3	0.25	0.05	0.03	0.04	0.00	0.10
Elasticity of output gap to its lag	β_1	0.5	0.15	0.47	0.52	0.20	0.22	0.71
Elasticity of output gap to real interest gap	β_2	0.3	0.25	0.12	0.05	0.07	0.00	0.25
Elasticity of output gap to exchange rate gap	β_3	0.3	0.25	0.02	0.01	0.01	0.00	0.05
Elasticity of output gap to expectations	β_4	0.3	0.25	0.05	0.04	0.04	0.01	0.10
Exchange rate expectations	φ	0.5	0.15	0.74	0.74	0.05	0.65	0.83
Persistence of natural interest rate	ρ_r	0.5	0.15	0.65	0.64	0.08	0.51	0.79
Persistence of foreign natural interest rate	ρ_{r^*}	0.5	0.15	0.37	0.36	0.09	0.14	0.58
Persistence of foreign interest rate gap	κ	0.5	0.15	0.49	0.52	0.19	0.20	0.78

Persistence of IS shock	ρ_y	0.5	0.15	0.52	0.56	0.22	0.26	0.77
Persistence of Phillips curve shock	ρ_π	0.5	0.15	0.29	0.29	0.09	0.15	0.42
Standard deviation of IS curve shock	σ_y	0.013	∞	0.005	0.005	0.001	0.003	0.007
Standard deviation of potential output shock	$\sigma_{\bar{y}}$	0.013	∞	0.009	0.009	0.002	0.005	0.011
Standard deviation of growth shock	σ_g	0.013	∞	0.015	0.012	0.006	0.004	0.026
Standard deviation of Phillips curve shock	σ_π	0.013	∞	0.021	0.02	0.002	0.018	0.024
Standard deviation of nominal interest rate shock	σ_i	0.013	∞	0.008	0.006	0.002	0.004	0.011
Standard deviation of potential exchange rate shock	σ_q	0.013	∞	0.063	0.063	0.010	0.046	0.080
Standard deviation of UIP shock	σ_r	0.013	∞	0.013	0.006	0.002	0.003	0.027
Standard deviation of natural real interest rate shock	$\sigma_{\bar{r}}$	0.013	∞	0.056	0.058	0.006	0.044	0.067
Standard deviation of foreign natural interest rate shock	$\sigma_{\bar{r}^*}$	0.013	∞	0.029	0.033	0.003	0.014	0.040
Standard deviation of foreign interest rate shock	σ_{r^*}	0.013	∞	0.015	0.006	0.003	0.003	0.030
Marginal likelihood	782.029							

Figure 9

PRIOR & POSTERIOR: HYBRID MODEL



Prior and posterior density functions for the estimated parameters, prior functions are in gray and posteriors functions in black, the dashed vertical line indicates the parameter posterior mean. Calculations were made with Dynare.

D.2 Conditioned Estimation

Table A.5

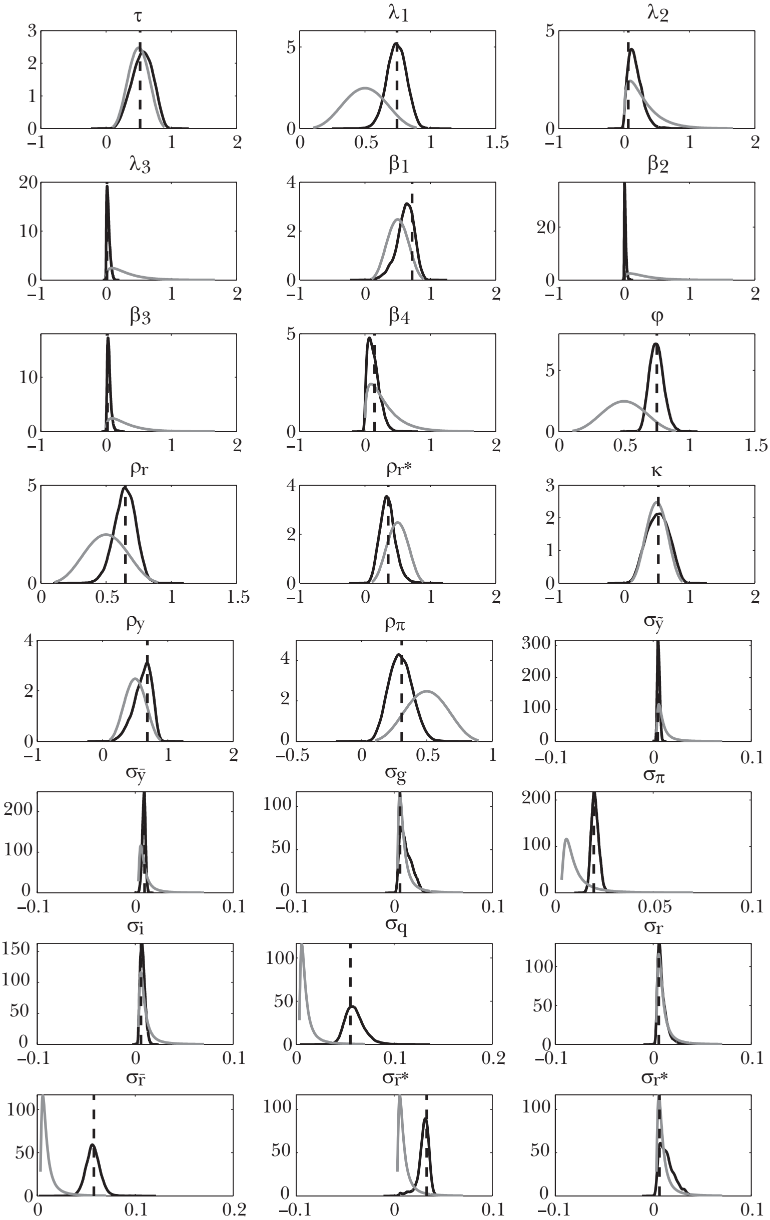
ESTIMATION RESULTS - HYBRID MODEL

Parameter		*Prior*		*Posterior*			*HPD 90%*	
		Mean	*SD*	*Mean*	*Mode*	*SD*	*Lower*	*Upper*
Persistence of the growth process	τ	0.5	0.15	0.55	0.52	0.19	0.30	0.81
Inflation expectations	λ_1	0.5	0.15	0.74	0.75	0.07	0.62	0.86
Elasticity of inflation to output gap	λ_2	0.3	0.25	0.17	0.06	0.07	0.01	0.32
Elasticity of inflation to depreciation	λ_3	0.3	0.25	0.03	0.01	0.02	0.00	0.07
Elasticity of output gap to its lag	β_1	0.5	0.15	0.61	0.72	0.10	0.40	0.83
Elasticity of output gap to real interest gap	β_2	0.3	0.25	0.13	0.15	0.13	0.00	0.26
Elasticity of output gap to exchange rate gap	β_3	0.3	0.25	0.02	0.01	0.01	0.00	0.04
Elasticity of output gap to expectations	β_4	0.3	0.25	0.05	0.02	0.02	0.01	0.09
Exchange rate expectations	φ	0.5	0.15	0.75	0.75	0.05	0.66	0.83
Persistence of natural interest rate	ρ_r	0.5	0.15	0.64	0.65	0.08	0.51	0.77
Persistence of foreign natural interest rate	ρ_{r^*}	0.5	0.15	0.35	0.35	0.09	0.15	0.53
Persistence of foreign interest rate gap	κ	0.5	0.15	0.52	0.52	0.19	0.24	0.78

Persistence of IS shock	ρ_y	0.5	0.15	0.61	0.69	0.14	0.39	0.82
Persistence of Phillips curve shock	ρ_π	0.5	0.15	0.30	0.31	0.09	0.15	0.44
Standard deviation IS curve shock	σ_y	0.013		0.006	0.005	0.001	0.003	0.008
Standard deviation of potential output shock	$\sigma_{\bar{y}}$	0.013		0.009	0.010	0.001	0.006	0.011
Standard deviation of growth shock	σ_g	0.013		0.012	0.006	0.003	0.004	0.022
Standard deviation of Phillips curve shock	σ_π	0.013		0.020	0.020	0.002	0.017	0.023
Standard deviation of nominal interest rate shock	σ_i	0.013		0.008	0.006	0.002	0.004	0.011
Standard deviation of potential exchange rate shock	σ_q	0.013		0.060	0.055	0.01	0.045	0.076
Standard deviation of UIP shock	σ_r	0.013		0.009	0.006	0.002	0.003	0.015
Standard deviation of natural real interest rate shock	$\sigma_{\bar{r}}$	0.013		0.057	0.058	0.006	0.045	0.068
Standard deviation of foreign natural interest rate shock	$\sigma_{\bar{r}^*}$	0.013		0.030	0.033	0.003	0.022	0.038
Standard deviation of foreign interest rate shock	σ_{r^*}	0.013		0.014	0.006	0.003	0.004	0.025
Standard deviation of measurement error	σ	0.013		0.010	0.005	0.002	0.003	0.018
Marginal likelihood	793.8896							

Figure 10

PRIOR & POSTERIOR: HYBRID MODEL



Prior and posterior density functions for the estimated parameters, prior functions are in gray and posteriors functions in black, the dashed vertical line indicates the parameter posterior mean. Calculations were made with Dynare.

E. Estimation Results - Forward Looking Model

E.1 Unconditioned Estimation

Table A.6

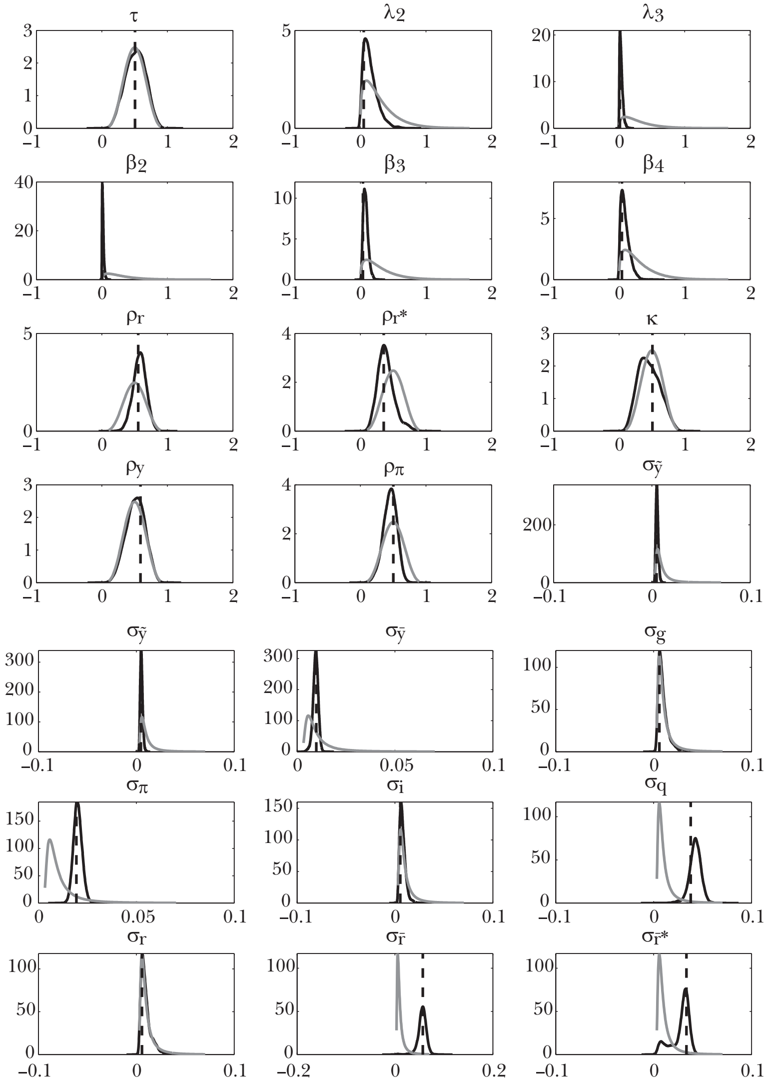
ESTIMATION RESULTS – FORWARD LOOKING MODEL

Parameter		*Prior*		*Posterior*			*HPD 90%*	
		Mean	*SD*	*Mean*	*Mode*	*SD*	*Lower*	*Upper*
Persistence of the growth process	τ	0.5	0.15	0.51	0.51	0.18	0.25	0.75
Elasticity of inflation to output gap	λ_2	0.3	0.25	0.15	0.05	0.05	0.00	0.30
Elasticity of inflation to depreciation	λ_3	0.3	0.25	0.03	0.01	0.02	0.00	0.06
Elasticity of output gap to real interest gap	β_2	0.3	0.25	0.09	0.04	0.06	0.00	0.18
Elasticity of output gap to exchange rate gap	β_3	0.3	0.25	0.02	0.01	0.01	0.00	0.04
Elasticity of output gap to expectations	β_4	0.3	0.25	0.08	0.04	0.03	0.02	0.14
Persistence of natural interest rate	ρ_r	0.5	0.15	0.57	0.55	0.10	0.41	0.73
Persistence of foreign natural interest rate	ρ_{r^*}	0.5	0.15	0.39	0.36	0.09	0.17	0.57
Persistence of foreign interest rate gap	κ	0.5	0.15	0.46	0.51	0.18	0.20	0.73
Persistence of IS shock	ρ_y	0.5	0.15	0.51	0.59	0.15	0.28	0.76

Persistence of Phillips curve shock	ρ_π	0.5	0.15	0.45	0.5	0.09	0.28	0.62
Standard deviation of IS curve shock	σ_y	0.013	∞	0.005	0.005	0.001	0.003	0.007
Standard deviation of potential output shock	$\sigma_{\bar{y}}$	0.013	∞	0.010	0.010	0.001	0.007	0.012
Standard deviation of growth shock	σ_g	0.013	∞	0.010	0.006	0.002	0.003	0.017
Standard deviation of Phillips curve shock	σ_π	0.013	∞	0.020	0.019	0.002	0.016	0.024
Standard deviation of nominal interest rate shock	σ_i	0.013	∞	0.008	0.005	0.002	0.004	0.012
Standard deviation of potential exchange rate shock	σ_q	0.013	∞	0.042	0.038	0.007	0.033	0.052
Standard deviation of UIP shock	σ_r	0.013	∞	0.010	0.006	0.002	0.003	0.017
Standard deviation of natural real interest rate shock	$\sigma_{\bar{r}}$	0.013	∞	0.056	0.057	0.006	0.043	0.069
Standard deviation of foreign natural interest rate shock	$\sigma_{\bar{r}^*}$	0.013	∞	0.028	0.033	0.003	0.008	0.039
Standard deviation of foreign interest rate shock	σ_{r^*}	0.013	∞	0.016	0.006	0.003	0.004	0.032
Marginal likelihood	782.7967							

Figure 11

PRIOR & POSTERIOR: FORWARD LOOKING MODEL



Prior and posterior density functions for the estimated parameters, prior functions are in gray and posteriors functions in black, the dashed vertical line indicates the parameter posterior mean. Calculations were made with Dynare.

E.2 Conditioned Estimation

Table A.7

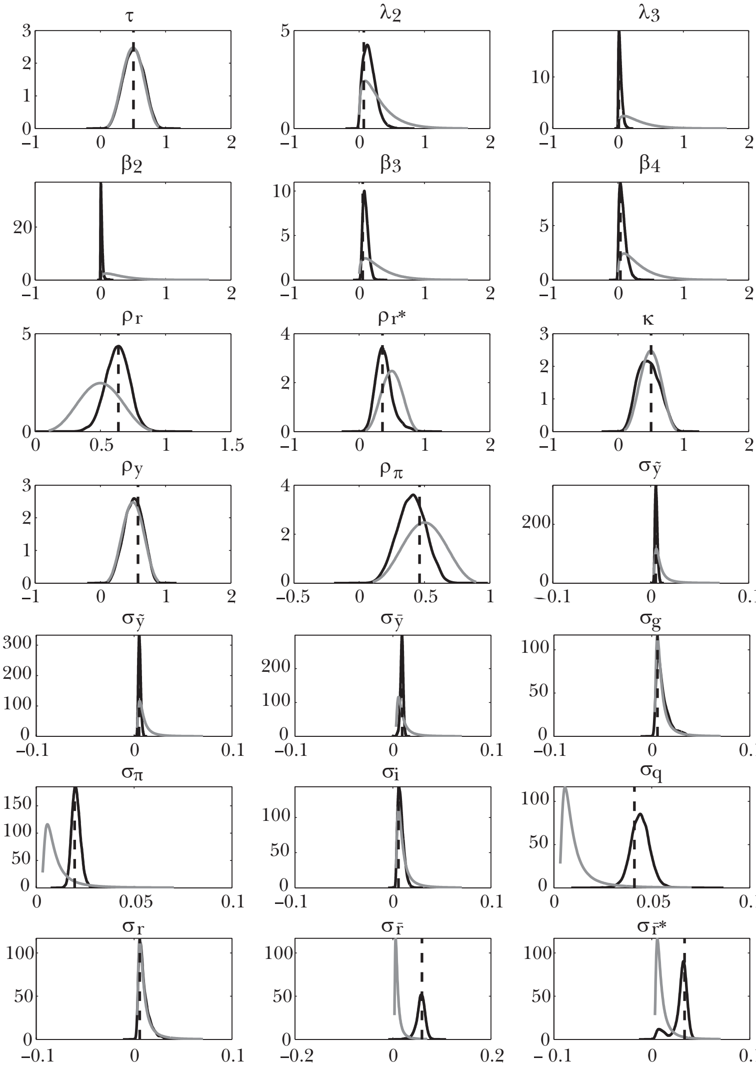
ESTIMATION RESULTS – FORWARD LOOKING MODEL

Parameter		*Prior*		*Posterior*			*HPD 90%*	
		Mean	*SD*	*Mean*	*Mode*	*SD*	*Lower*	*Upper*
Persistence of the growth process	τ	0.5	0.15	0.51	0.51	0.18	0.25	0.74
Elasticity of inflation to output gap	λ_2	0.3	0.25	0.16	0.07	0.07	0.01	0.29
Elasticity of inflation to depreciation	λ_3	0.3	0.25	0.04	0.01	0.02	0.00	0.07
Elasticity of output gap to real interest gap	β_2	0.3	0.25	0.07	0.04	0.05	0.00	0.15
Elasticity of output gap to exchange rate gap	β_3	0.3	0.25	0.02	0.01	0.01	0.00	0.04
Elasticity of output gap to expectations	β_4	0.3	0.25	0.10	0.05	0.03	0.03	0.16
Persistence of natural interest rate	ρ_r	0.5	0.15	0.63	0.64	0.08	0.48	0.78
Persistence of foreign natural interest rate	ρ_{r^*}	0.5	0.15	0.38	0.36	0.09	0.17	0.58
Persistence of foreign interest rate gap	κ	0.5	0.15	0.47	0.51	0.18	0.20	0.73
Persistence of IS shock	ρ_y	0.5	0.15	0.51	0.58	0.14	0.28	0.75
Persistence of Phillips curve shock	ρ_π	0.5	0.15	0.40	0.46	0.11	0.22	0.58

Standard deviation of IS curve shock	σ_y	0.013	∞	0.006	0.005	0.001	0.004	0.008
Standard deviation of potential output shock	$\sigma_{\bar{y}}$	0.013	∞	0.009	0.010	0.001	0.007	0.012
Standard deviation of growth shock	σ_g	0.013	∞	0.011	0.006	0.002	0.003	0.020
Standard deviation of Phillips curve shock	σ_π	0.013	∞	0.020	0.020	0.002	0.017	0.024
Standard deviation of nominal interest rate shock	σ_i	0.013	∞	0.008	0.006	0.002	0.004	0.013
Standard deviation of potential exchange rate shock	σ_q	0.013	∞	0.044	0.041	0.005	0.036	0.052
Standard deviation of UIP shock	σ_r	0.013	∞	0.010	0.006	0.002	0.003	0.018
Standard deviation of natural real interest rate shock	$\sigma_{\bar{r}}$	0.013	∞	0.057	0.059	0.006	0.043	0.070
Standard deviation of foreign natural interest rate shock	$\sigma_{\bar{r}^*}$	0.013	∞	0.029	0.033	0.003	0.012	0.040
Standard deviation of foreign interest rate shock	σ_{r^*}	0.013	∞	0.014	0.006	0.003	0.003	0.030
Standard deviation of measurement error	σ	0.013	∞	0.008	0.005	0.002	0.003	0.013
Marginal likelihood	796.9809							

Figure 12

PRIOR & POSTERIOR: FORWARD LOOKING MODEL



Prior and posterior density functions for the estimated parameters, prior functions are in gray and posteriors functions in black, the dashed vertical line indicates the parameter posterior mean. Calculations were made with Dynare.

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